Essential Idempotents in Group Algebras and Minimal Cyclic Codes

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Abstract

Let G be a finite group of order n, $\mathbb{F}_q$ the field with q elements and assume that $(n, q) = 1$. Let e be an idempotent in $\mathbb{F}_q$. For a normal subgroup H of G set $\widehat{H} = 1/|H| \sum_{h \in H} h$. If $e\widehat{H} = e$ then $\mathbb{F}_q G.e \subset \mathbb{F}_q G\widehat{H}$ and it is easy to see that, from the point of view of coding theory, this impies that the code defined by the ideal $\mathbb{F}_q G.e$ is a *repetition code*.

A primitive idempotent of $\mathbb{F}_q G$ is called *essential* if this does not happen; i.e. if $e\widehat{H} = 0$ for every normal subgroup H of G. This idempotents were first considered by Bakshi, Raka and A. Sharma in [1], where they are called non-degenerate.

If G is abelian, then G contains an essential idempotent if and only if G is cyclic. We shall give several algebraic characterizations of essential idempotents and compute their number in $\mathbb{F}_q C_n$, where C_n denotes the cyclic group of order n.

Using this concept, we are able to prove that every minimal abelian code is combinatorially equivalent to a minimal cyclic code.

Finally, if we denote by m the order of $\overline{q}$ in $\mathbb{Z}_n$, set $N = q^m - 1$ and $\ell = N/n$ we establish a correspondence between essential idempotents of $\mathbb{F}_q C_n$ and those of $\mathbb{F}_q C_N$.

These results are joint work with R. Ferraz and G. Chalom [2]

Referencias

[1] G. K. Bakshi, M. Raka, A. Sharma, Idempotent Generators of Irreducible Cyclic Codes, *Proc. Int. Conf. Number Theory and Discrete Geometry*, Ramanujan Lecture Notes, **6**, (2008), 13–18, ed. R. Balasubramanian, S. G. Dani, P. M. Gruber, R. J. Hans-Gill.

[2] G. Chalom, R. Ferraz e C. Polcino Milies, Essential Idempotents in Group Algebras and Minimal Cyclic Codes, *preprint*.