

# Reply to “Comment on ‘Texture Zeros and WB Transformations in the Quark Sector of the Standard Model’”

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We respond to the comments of S. Sharma et al. [Phys. Rev. D 91, 038301 (2015)] on my recent paper, Y. Giraldo [Phys. Rev. D 86, 093021 (2012)]. In their comments, they begin discussing a particular class of five-zero texture non-Fritzsch-like quark mass matrix, which was proposed by me, and questioning its validity. Then, they put in doubt the phases included in the unitary matrices used for diagonalizing the quark mass matrices, from which they claim that the CP violating parameter  $\epsilon_k$  obtained does not agree with the experimental value. Because of these comments, finally, they recommend being careful while analyzing the implications of Weak Basis transformations on textures. Other minor points are also discussed by them.

In the following, I will show that the mentioned five-zero texture non-Fritzsch-like quark mass matrices is completely valid and generates all the physical quantities involved, including the CP violating parameter  $\epsilon_k$ , for which is required the inclusion of phases in the unitary matrices used for diagonalizing the quark mass matrices in order to put the CKM matrix in standard form. These phases can be rotated away so they do not have any physical meaning. Finally, the relevance of the weak basis transformation is appreciated: which is *complete* and facilitates calculations, especially of textures zeros.

## I. INTRODUCTION

In Standard Model (SM), the most general weak basis (WB) transformation [1], that leaves the physical content invariable and the up- and down-quark mass matrices  $M_u$  and  $M_d$  Hermitian <sup>1</sup>, is

$$\begin{aligned} M_u &\longrightarrow M'_u = U^\dagger M_u U, \\ M_d &\longrightarrow M'_d = U^\dagger M_d U, \end{aligned} \quad (1.1)$$

where  $U$  is an arbitrary unitary matrix. We say that the two quark mass matrices  $M_{u,d}$  and  $M'_{u,d}$  are equivalent each other. So, this implies that the number of equivalent mass matrices is infinity. Hence, we are able to explicitly construct texture zeros in quark mass matrices through WB transformations. If these texture zeros exist, the WB transformation is able to find them. The reason is that, as was shown in my paper [2], the WB transformation is exhaustive (complete) finding all possibilities, included possible four and five-zero textures.

Through WB transformations, Branco et al. [1] show that is always possible to find, at most, three zeros in quark mass matrices with no physical meanings. But, this does not restrict the number of zeros can be found by applying the WB transformation to mass matrices, the case is that the model must be put into a physical context. Therefore, we have found additional zeros (four- and even five-zero textures [2]) by using the recent quark mass and mixing data. These additional zeros now have physical meanings because they were obtained from specific experimental data.

## II. UP-QUARK MASS MATRIX IN DIAGONAL FORM

One point of discussion is that to facilitate the analysis the initial quark mass matrices used by me is as follows [2]

$$\begin{aligned} M_u &= D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix}, \\ M_d &= V D_d V^\dagger, \end{aligned} \quad (2.1)$$

where the up and down diagonal matrices  $D_u$  and  $D_d$  contain the respective quark mass eigenvalues, and  $V$  is the usual quark Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The authors' comments claim that we do not start with the most general mass matrices. But this is not true. The starting matrices (2.1), used in papers like [1, 3], are as general as any other one. The reason is that starting from arbitrary Hermitian matrices  $M_u$  and  $M_d$ , and using their respective diagonalizing matrices  $U_u$  and  $U_d$ , and performing a WB transformation (1.1) using for this case the unitary matrix  $U = U_u$ . We have

$$\begin{aligned} M_u &\longrightarrow M'_u = U_u^\dagger M_u U_u = D_u, \\ M_d &= U_d D_d U_d^\dagger \longrightarrow M'_d = U_u^\dagger (U_d D_d U_d^\dagger) U_u, \\ &= (U_u^\dagger U_d) D_d (U_d^\dagger U_u), \\ &= V D_d V^\dagger, \end{aligned}$$

where the CKM mixing matrix  $V = U_u^\dagger U_d$  was considered. Additionally, note also that the three no-physical-texture zeros mentioned above appear also in (2.1).

Although, the crux of the comments is below.

<sup>1</sup> The quark mass matrices are Hermitian due to the polar decomposition theorem, where the unitary component can be absorbed in the right-handed quark fields.

### III. PHASES AND THE CKM MIXING MATRIX

Let us resolve the problem for a particular case. Let us consider the numerical quark mass matrices given in

$$M_u = \begin{pmatrix} 0 & 0 & -92.3618 + 157.694i \\ 0 & 5748.17 & 28555.1 + 5911.83i \\ -92.3618 - 157.694i & 28555.1 - 5911.83i & 166988 \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & 13.9899 & 0 \\ 13.9899 & 0 & 424.808 \\ 0 & 424.808 & 2796.9 \end{pmatrix},$$

where their diagonalizing matrices are respectively<sup>2</sup>

$$U_u = \begin{pmatrix} 0.998779 \times e^{2.10064i} & 0.0493829 \times e^{2.10064i} & 0.00104594 \times e^{2.10064i} \\ 0.0484608 \times e^{0.20415i} & 0.983788 \times e^{-2.93744i} & 0.172662 \times e^{0.204148i} \\ -0.00955555 \times e^{0i} & 0.172401 \times e^{0i} & 0.984981 \times e^{0i} \end{pmatrix}, \quad U_d = \begin{pmatrix} 0.978718 & -0.205210 & 0.000718698 \\ 0.202880 & 0.968118 & 0.146926 \\ -0.0308464 & -0.143653 & 0.989147 \end{pmatrix}, \quad (3.1)$$

from which the authors' comments have obtained the quark mixing matrix, Eq. (7). Nevertheless, generalizing the result in (3.1), no physical phases ( $x, y, z, v$  and  $w$ ) can be added to the matrices, obtaining the following diagonalizing matrices

$$U'_u = \begin{pmatrix} 0.998779 \times e^{2.10064i} \times e^{xi} & 0.0493829 \times e^{2.10064i} \times e^{yi} & 0.00104594 \times e^{2.10064i} \times e^{zi} \\ 0.0484608 \times e^{0.20415i} \times e^{xi} & 0.983788 \times e^{-2.93744i} \times e^{yi} & 0.172662 \times e^{0.204148i} \times e^{zi} \\ -0.00955555 \times e^{0i} \times e^{xi} & 0.172401 \times e^{0i} \times e^{yi} & 0.984981 \times e^{0i} \times e^{zi} \end{pmatrix}, \quad (3.2a)$$

$$U'_d = \begin{pmatrix} 0.978718 \times e^{iv} & -0.205210 \times e^{iw} & 0.000718698 \\ 0.202880 \times e^{iv} & 0.968118 \times e^{iw} & 0.146926 \\ -0.0308464 \times e^{iv} & -0.143653 \times e^{iw} & 0.989147 \end{pmatrix}, \quad (3.2b)$$

and where there is no way to distinguish what are the ‘‘true’’ matrices of diagonalization. Even further, if you choose the values  $x = -1.30524$ ,  $y = 0.790611$ ,  $z = -0.00515513$ ,  $v = 0.785572$  and  $w = -2.14216$ , you obtain

$$U'_u = \begin{pmatrix} 0.998779 \times e^{0.795395i} & 0.0493829 \times e^{2.89125i} & 0.00104594 \times e^{2.09548i} \\ 0.0484608 \times e^{-1.10109i} & 0.983788 \times e^{-2.14683i} & 0.172662 \times e^{0.198993i} \\ 0.00955555 \times e^{1.83635i} & 0.172401 \times e^{0.790611i} & 0.984981 \times e^{-0.00515513i} \end{pmatrix},$$

$$U'_d = \begin{pmatrix} 0.978718 \times e^{0.785572i} & -0.205210 \times e^{-2.14216i} & 0.000718698 \\ 0.202880 \times e^{0.785572i} & 0.968118 \times e^{-2.14216i} & 0.146926 \\ -0.0308464 \times e^{0.785572i} & -0.143653 \times e^{-2.14216i} & 0.989147 \end{pmatrix},$$

two diagonalizing matrices that are equally valid. As a result, a quark CKM mixing matrix compatible with the recent mixing data [5] is derived

$$V_{ckm} = U_u'^{\dagger} \cdot U_d' = \begin{pmatrix} 0.974276 & 0.225334 & 0.00124462 - 0.0032841i \\ -0.225194 - 0.000106564i & 0.973443 - 0.0000294788i & 0.0411845 \\ 0.00806881 - 0.00319789i & -0.0404056 - 0.000739786i & 0.999145 \end{pmatrix}. \quad (3.3)$$

As you can observe, the phases included in (3.2) are not against to reduce the number of free parameters. These phases come out naturally from the diagonalizing process, and it is impossible to avoid them. When you establish specific diagonalization matrices, you are choosing specific phases. These phases are just different ways to present the CKM matrix as was told in my paper [2] above equation (3.29).

Eq. (4.22) of my paper [2]. Which was also considered by the authors' comments in row (a) of Table 1. Apparently, the corresponding CKM matrix obtained is not compatible with the recent quark mixing data. The numerical quark mass matrices in discussion are (in MeV units)

Going even further these phases have an interpretation and its nature is clarified in the next section.

### IV. NO PHYSICAL PHASES

The additional phases introduced in matrices (3.2) leave the physical content invariable, including the Jarlskog invariant quantity. This can be seen by bringing the matrices (3.2) to the following products

$$U'_u = U_u f_1 \quad \text{and} \quad U'_d = U_d f_2,$$

where the diagonal matrices  $f_1 = \text{diag}(e^{xi}, e^{yi}, e^{zi})$  and  $f_2 = \text{diag}(e^{vi}, e^{wi}, 1)$ ; and  $U_u$  and  $U_d$  are given in (3.1).

<sup>2</sup> These diagonalizing matrices were obtained by using Mathematica, but different matrices are obtained by using other software like maxima, octave, . . . . Actually, the difference is just in the phases.

Such that the CKM mixing matrix obtained in (3.3) becomes

$$V_{ckm} = U_u^\dagger U'_d = (U_u f_1)^\dagger (U_d f_2) = f_1^\dagger (U_u^\dagger U_d) f_2,$$

therefore

$$U_u^\dagger U_d = f_1 V_{ckm} f_2^\dagger, \quad (4.1)$$

which implies the following two results:

1. First, the mixing matrix obtained from Eq. (3.1), i.e.  $U_u^\dagger U_d$ , apparently does not fit the standard form of the CKM mixing matrix, but is well known that the five phases present in  $f_1$  and  $f_2$  can be rotated away [4], such that both expressions for the CKM, in (4.1), are equivalent. Therefore the Jarlskog invariant, as well as any other physical quantity, is not affected by adding phases as given in (3.2). And as a result, the phases  $x, y, z, v$  and  $w$  have no physical meaning.
2. Second, if there are still doubts,  $f_1$  and  $f_2$  in (4.1) add phases to the matrix elements of  $V_{ckm}$ , where each phase of  $f_2$  is placed in the same column and each phase of  $f_1$  in the same row. Therefore, the Jarlskog invariant  $J = \text{Im}(V_{us}V_{ub}^*V_{cs}^*V_{cb})$  is not affected by adding these additional phases, because they cancel out in the same row ( $V_{us}, V_{ub}^*$ ) and ( $V_{cs}^*, V_{cb}$ ), and in the same column ( $V_{us}, V_{cs}^*$ ) and ( $V_{ub}^*, V_{cb}$ ).

Finally, the Jarlskog invariant quantity is

$$J = \text{Im}(V_{us}V_{ub}^*V_{cs}^*V_{cb}) = 2.96695 \times 10^{-5},$$

clearly inside the range given by PDG 2012 [5], i.e.,  $(2.80 - 3.16) \times 10^{-5}$ . The same for the quark masses (in MeV):  $m_d = 2.90, m_s = 66, m_b = 2860, m_u = 1.75, m_c = 638, m_t = 172100$  [5].

We can consider other phase invariant quantities like the inner angles of the CKM unitarity triangle:  $\beta = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*) = 21.6^\circ$ ,  $\alpha = \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*) = 89.1^\circ$ , and  $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*) = 69.2^\circ$ .

However, in the frame of the SM, the usual formula for the Kaon CP violating parameter  $\epsilon_k$ , is valid only in the basis where  $V_{ud}V_{us}^*$  is real [5, 6], for that reason

the phases given in (3.2) must be considered in order to transform the CKM mixing matrix into its standard convention.

## V. CONCLUSIONS

To begin with, the WB transformation is complete, so we can find all possible quark mass matrices representing the model by starting from a specific quark mass matrices. It is important to mention that, in the SM, is always possible to find a maximum of three no physical vanishing elements in the quark mass matrices by performing a WB transformation. In the process does not matter the value of physical quantities. But if we want to find additional zeroes is necessary to take into account physical considerations.

Another important result, emphasized by other authors, is that the quark mass matrix structure given in (2.1), which was called in my paper [2] as the *u-diagonal representation*, is so general as any other one. These matrices are deduced from a WB transformation and have the advantage of having available the quark masses and the CKM matrix elements.

The phases included in the diagonalizing matrices (3.2), are precisely the five phases that can be rotated away through the phase redefinition of the left-handed up and down quark fields as was shown in (4.1), and as a consequence these phases have no physical meaning. Even further, as a result, these phases does not affect the invariance of the Jarlskog quantity, and so on. Respect to the Kaon CP violating parameter  $\epsilon_k$ , it must be calculated in a basis where  $V_{ud}V_{us}^*$  is real. For that reason, phases must be included in the diagonalizing matrices in order to achieve this requirement.

Definitely, the introduction of extra additional phases is not against the basic spirit of the texture specific mass matrices which was to control the number of free parameters. These phases have no physical meaning, but their inclusion is necessary to adjust the resulting CKM matrix to its standard choice.

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- [1] G. C. Branco, D. Emmanuel-Costa and R. G. Felipe, Phys. Lett. B 477, 147 (2000) [hep-ph/9911418]; D. Emmanuel-Costa and C. Simoes, Phys. Rev. D 79, 073006 (2009) [arXiv:0903.0564 [hep-ph]].
- [2] Y. Giraldo, Phys. Rev. D 86, 093021 (2012), [arXiv:1110.5986 [hep-ph]].
- [3] H. Fusaoka and Y. Koide, Phys. Rev. D 57, 3986 (1998).
- [4] Andrija Rasin, [hep-ph/9708216].
- [5] J. Beringer et al., Particle Data Group, Phys. Rev. D 86, 010001 (2012).
- [6] G. Buchalla *et al.*, Rev. Mod. Phys. 68, 1125 (1996) [hep-ph/9512380].