A depth-averaged, two-phase flow code for hazard mapping that satisfies both hydraulic and granular flow extremes.

By

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The Titan project

Aims

- Create realistic 2D & 3D models that simulate geophysical mass flow
- Integrate and communicate information from several linked sources including:
  - .- Simulation results
  - .- Remote sensing data
  - .- GIS data

Current applications

- One phase dense granular flows

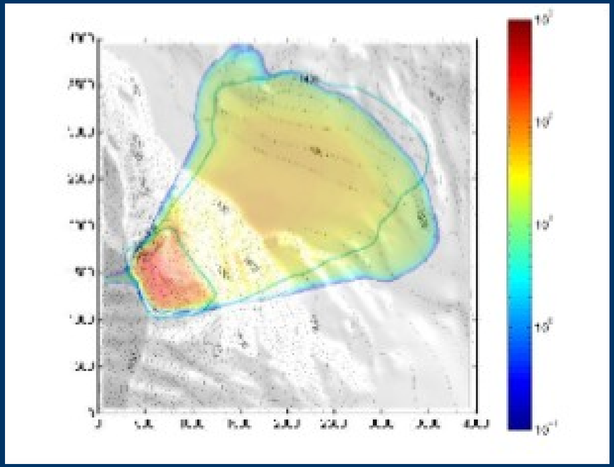
Rock avalanches
Block-and-ash flows

- Two phase dense flows

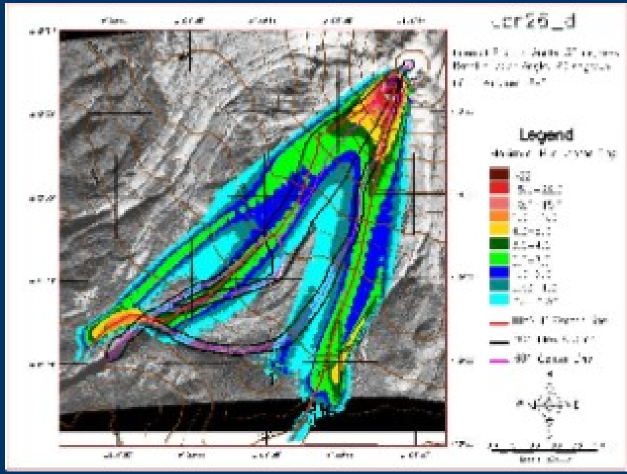
Debris flows

- Hazard maps & risk assessment

- Probabilistic analysis



Frank slide

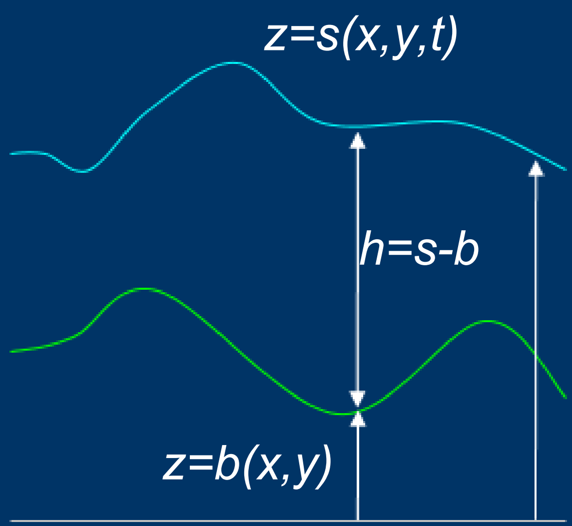


Colima 1991

Mathematical basis

Savage-Hutter (1989) Model

- Provides a framework for later, more sophisticated models.
- The model is based on assumptions:
Aspect ratio of the flow is small
Granular flows have Coulomb friction behaviour
Top surface is stress-free
Top surface and bed surface represented as functions Fh(X,t) =0 and Fb(X) = 0



The equations are depth averaged:

$$h\bar{v}_x=\int_b^s v_x dz, h\bar{v}_y=\int_b^s v_y dz,$$

geophysical mass

Problems with the one phase model

- Several natural flows are multiphase

- Savage-Hutter model is only for dry avalanches

- Interstitial fluid modelled in Titan by reducing bed and internal friction angles

However,

- Interstitial fluid can greatly alter the dynamics of the flow .

- Flows can achieve very high velocities, and can travel long distances.

- Water may be supplied by rainfall, melting of snow or ice or by overflow of a crater lake.

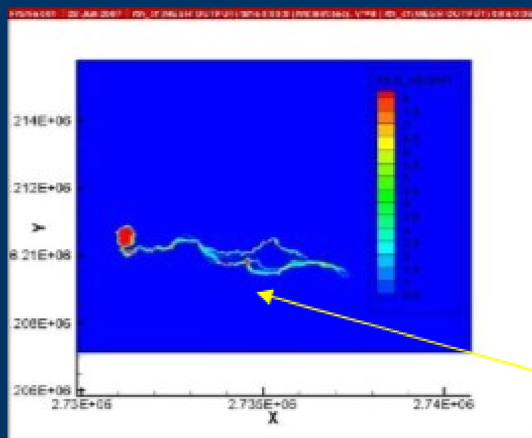
Pitman and Le model

- *Pitman and Le,* (2005) expand the applications of TITAN *to* debris flows with various water contents

- Independent equations of motion specified for each phase, along with an equation for volume fractions

- Solid-Fluid interaction modelled using a drag force term

Validation: actual prediction of the 2007 Mt Rapehu Lahar



Previously non recognized bifurcation predicted by Titan

Actual path

Bank levee

Titan prediction

Problems with Pitman model

- Numerics break down at flow-front

- Non-physical velocities, as conserved variables, are divided by very small values

- It does not work well close to the extremes of pure dry or pure water flows.

Cordoba, Pitman and Sheridan model

Characteristics of the proposed model

- No limit on the fluid phase content

- Pure dry avalanches modeled as a mixture of near maximum pack concentration of solids plus air.

- Solid phase modeled as a granular flow that obeys the Coulomb friction law.

- Fluid phase modeled as shallow water that follows the Darcy-Weisbach approach for friction.

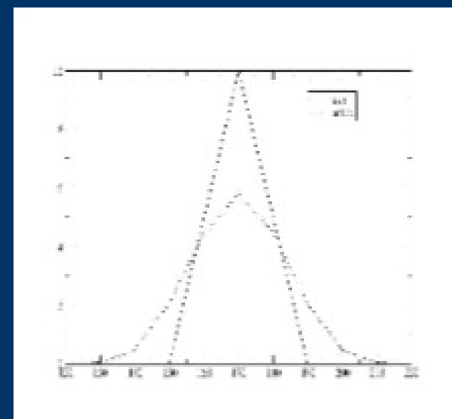
- Phase equations coupled through a drag coefficient and pressure.

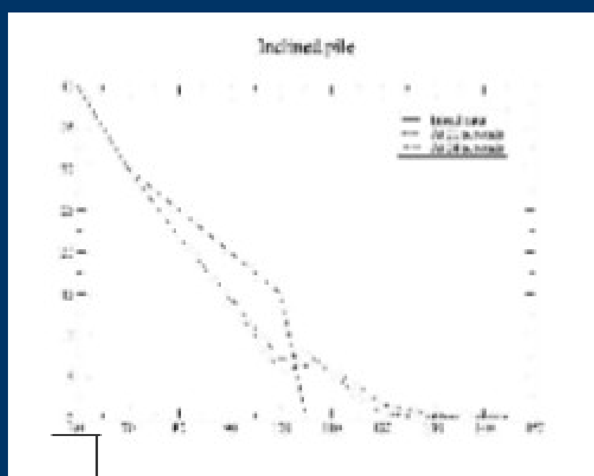
- Mathematically the equations become the Savage and Hutter (1889) approach for pure solids and to pure shallow water (e.g. see the model of Cun-hong et al., 2006) in case of zero solids concentration

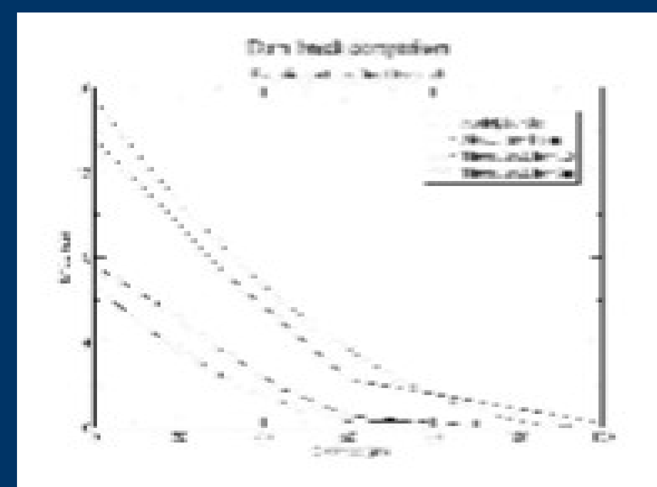
Validation

For purposes of validation and analytical testing, the one dimensional version of the equations was tested:

Symmetry

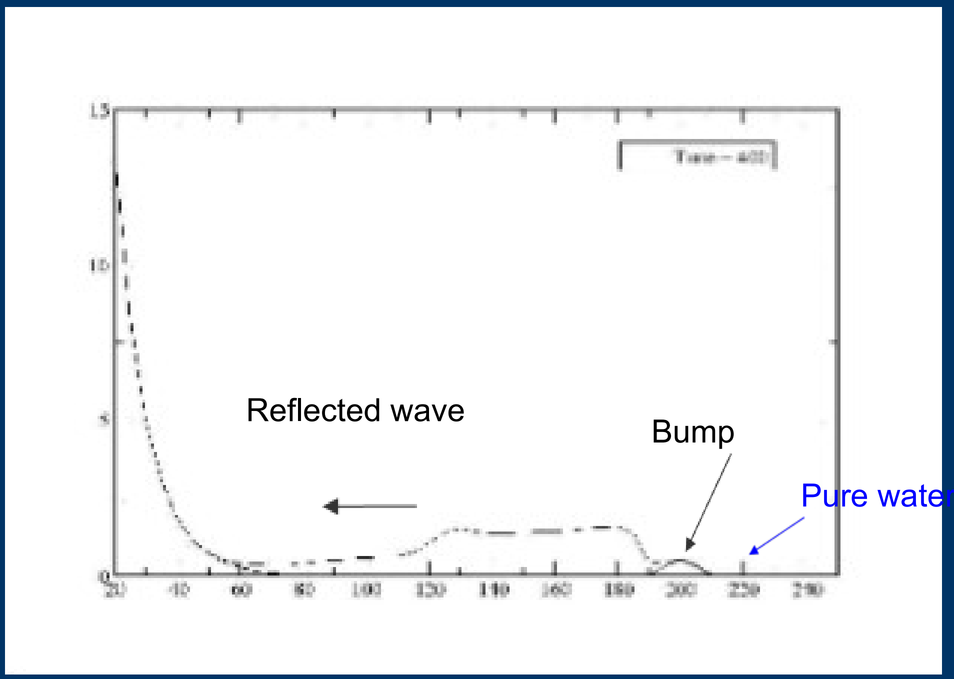




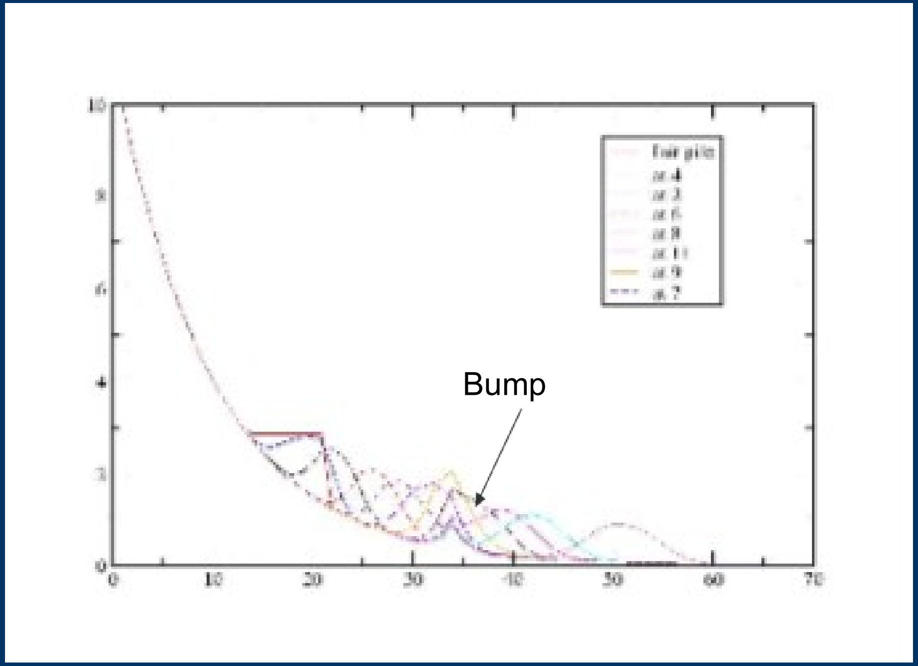


Dam break

Exponentially decaying topography with bump



Big bump



Small bump

Equation system for bi-phase flows

Conservation laws, assuming a fluid saturated mixture of solids and water:

Mass:

$$\partial_t \rho^s \varphi + \nabla \cdot (\rho^s \varphi v_i) = 0$$

$$\partial_t \rho^f (1-\varphi) + \nabla \cdot (\rho^f (1-\varphi) u_i) = 0$$

Momentum:

$$\partial_t (\rho^s \varphi v_i) + \nabla \cdot (\rho^s \varphi v_i v_j) = \nabla \cdot T^s + f^s + \rho^s \varphi g$$

$$\partial_t (\rho^f (1-\varphi) u_i) + \nabla \cdot (\rho^f (1-\varphi) u_i u_j) = \nabla \cdot T^f + f^f + \rho^f (1-\varphi) g$$

Interaction forces between phases

$$f^s = -\varphi \nabla T^f + D(v-u)$$

$$f^f = -f^s$$

Where the drag D is approached through the empirical relationship:

$$D = \frac{3}{4} C_d \frac{\rho^f \left\| v-u \right\|}{f^2 d} \varphi (1-\varphi)$$

and

$$f = f(R_e)$$

Titan2phase implementation:

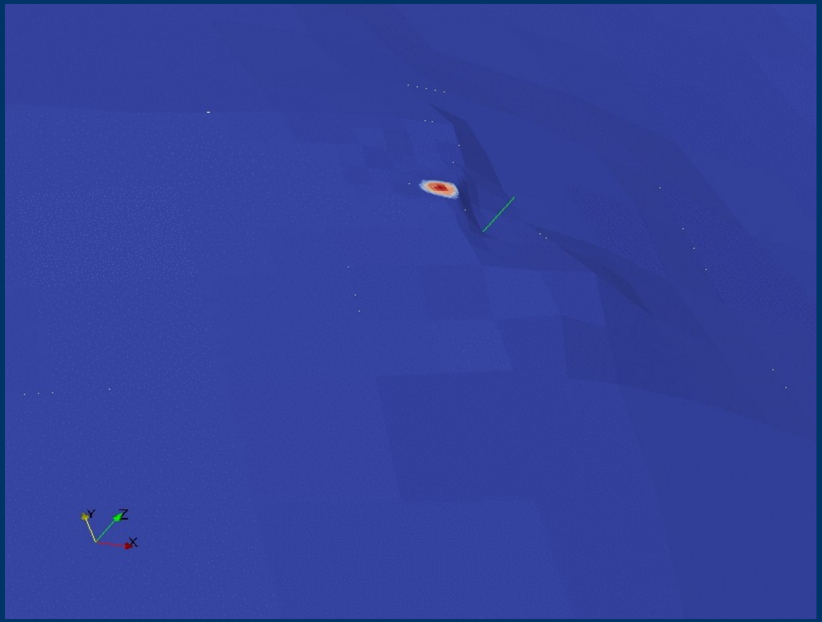
- Full two phase equation system implemented into the Titan2D frame.

- Eigen dependent time steps of the order of O(10^-4)

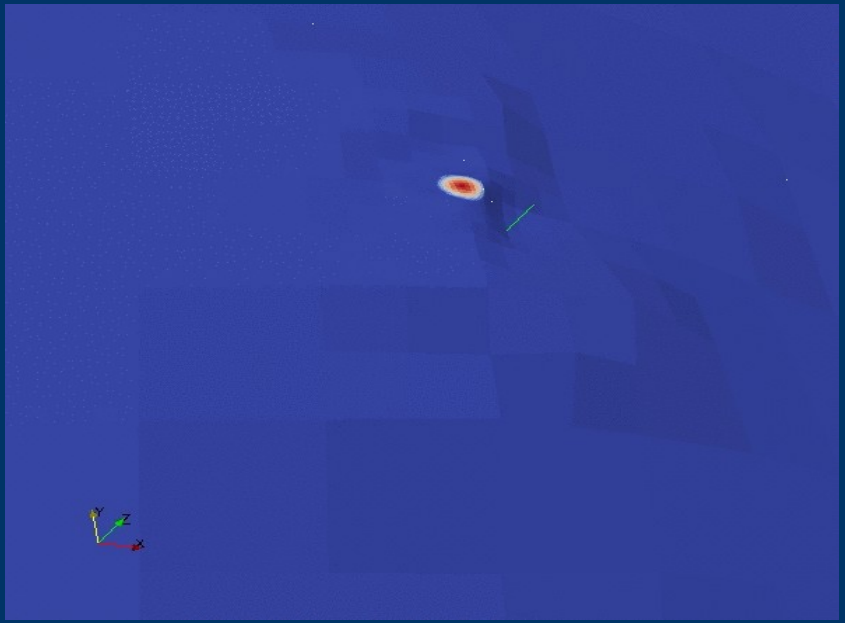
- Presence of a fluid phase reduces the bulk bed friction in a natural way.

- Currently, constant volumetric fraction of solids assumed.

Titan2D and Titan2phase



Titan2D



Titan2phase

Next steps:

- Allowing change in particle concentration in time
- Validation
- Corrections and debugging
- Validation
- Release of a beta-version
- Web distribution as free software
- Implementation on VHUB

Equation System

Equation system

Mass conservation

$$\delta_t \varphi_s + \frac{\delta(\varphi_s v_i)}{\delta x_i} = 0$$

$$\delta_t \varphi_f + \frac{\delta(\varphi_f u_i)}{\delta x_i} = 0$$

Momentum balance:

$$\delta_t(\varphi_s v_i) + v_j \frac{\delta(\varphi_s v_i)}{\delta x_j} = \frac{1}{\rho_s}(\nabla \cdot T_s + f_s) + \varphi_s g_i$$

$$\delta_t(\varphi_f u_i) + u_j \frac{\delta(\varphi_f u_i)}{\delta x_j} = \frac{1}{\rho_f}(\nabla \cdot T_f + f_f) + \varphi_f g_i$$

where

$$f_s = p_f \frac{\delta \varphi_s}{\delta x_i} + D\Delta u$$

$$f_f = p_f \frac{\delta \varphi_f}{\delta x_i} - D\Delta u$$

where $\Delta u = (v - u)$ and the interface drag coefficient D is either:

$D = \frac{\varphi_s \varphi_f^3 \Delta\rho}{\tau_s}$, where the relaxation time $\tau_s = w_{h_s} / g$, for particles

at the Stokes regime, or the empirical approach (Dobran, 2001):

$D = \frac{3}{4} C_d \frac{\varphi_s \rho_f |\Delta u|}{d\varphi_f^2}$, for other cases

Constitutive equations (after depth averaging):

For solid phase, the Coulomb rule is used (Savage and Hutter (1989), Pitman and Lee (2005)):

$$T_{s_{ij}} = \alpha_{ij} T_{s_{zz}}$$

where

$$T_{s_{zz}} = \varphi_s g h$$

and (Patra et al. (2005)):

$$\alpha_{ii} = k_{ap}$$

$$\alpha_{jz} = -\frac{v_j}{\sqrt{v_x^2 + v_y^2}}\left(1 + \frac{v_j}{r_j g}\right)$$

$$\alpha_{xy} = -sgn(\partial v_x / \partial y) k_{ap} \sin\phi_{int}$$

For the fluid phase:

$T_{f_{ij}} = \alpha_{ij} T_{f_{zz}}$, where

$$T_{f_{zz}} = \varphi_f g h$$

and

$$\alpha_{ii} = 1$$

$$\alpha_{jz} = -\frac{f_r}{8} u_j^2$$

$$\alpha_{xy} = 0.5$$