Systematic Study of 3-3-1 Models

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Abstract

We carry a systematic study of possible models based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$. Old and new models emerge from the analysis.

The Standard Model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ [1] can be extended in different ways: first, by adding new fermion fields (adding a right-handed neutrino field constitute its simplest extension, and has deep consequences as for example, the implementation of the see-saw mechanism, and the enlarging of the possible number of local abelian symmetries that can be gauged simultaneously); second, by augmenting the scalar sector to more than one higgs representation, and third by enlarging the local gauge group. In this last direction, $SU(3)_L \otimes U(1)_X$ as a flavor group has been studied previously by several authors in the literature [2]-[6] who have explored possible fermion and higgs-boson representation assignments.

In what follows we are going to present a systematic analysis of the local gauge model based on the gauge structure $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, which we call the 331 theory.

We assume that the electroweak group is $SU(3)_L \otimes U(1)_X \supset SU(2)_L \otimes U(1)_Y$. We also assume that the left handed quarks (color triplets), left-handed leptons (color singlets) and scalars, transform under the two fundamental representations of $SU(3)_L$ (the 3 and 3^*). Two classes of models will be discussed: one family models where the anomalies cancel in each family as in the SM, and family models where the anomalies cancel by an interplay between the families. $SU(3)_c$ is vectorlike as in the SM.

The most general expression for the electric charge generator in $SU(3)_L \otimes U(1)_X$

is a linear combination of the three diagonal generators of the gauge group

$$Q = aT_{3L} + \frac{2}{\sqrt{3}}bT_{8L} + xI_3, \tag{1}$$

where $T_{iL} = \lambda_{iL}/2$, being λ_{iL} the Gell-Mann matrices for $SU(3)_L$ normalized as $\mathbf{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$, $I_3 = Dg.(1,1,1)$ is the diagonal 3×3 unit matrix, and a and b are arbitrary parameters to be determined anon. Notice that we can absorb an eventual coefficient for x in its definition.

If we assume that the usual isospin $SU(2)_L$ of the SM is such that $SU(2)_L \subset SU(3)_L$, then $a = 1$ and we have just one parameter set of models, all of them characterized by the value of b. So, Eq. (1) allows for an infinite number of models in the context of the 331 theory, each one associated to a particular value of the parameter b, with characteristic signatures that make one different from other, as we will see.

There are a total of 17 gauge bosons in the gauge group under consideration, they are: one gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$ which remain massless after breaking the symmetry, and another 8 associated with $SU(3)_L$ that we may write in the following way:

$$\frac{1}{2}\lambda_{\alpha L}A^\alpha_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} D^0_{1\mu} & W^+_\mu & K^{(1/2+b)}_\mu \\ W^-_\mu & D^0_{2\mu} & K^{-(1/2-b)}_\mu \\ K^{-(1/2+b)}_\mu & K^{(1/2-b)}_\mu & D^0_{3\mu} \end{pmatrix},$$

where $D^\mu_1 = A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, $D^\mu_2 = -A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, and $D^\mu_3 = -2A^\mu_8/\sqrt{6}$. The upper indices in the gauge bosons in the former expression stand for the electric charge of the corresponding particle, some of them functions of the b parameter as they should be. Notice that the gauge bosons have integer electric charges only for $b = \pm 1/2,\ \pm 3/2, ; \pm 5/2, ..., \pm(2n+1)/2,\ n = 1, 2, 3....$ A deeper analysis shows that the negative values for b can be related to the positive one just by taking the complex conjugate in the covariant derivative of each model, which in turn is equivalent to replace $3 \leftrightarrow 3^*$ in the fermion content of each particular model. So, our first conclusion is that, if we do not want exotic electric charges in the gauge sector of our theory, then b must be equal to $1/2$. We will see next that this is also the condition for excluding exotic electric charges in the fermion sector.

Now, contrary to the SM where only the abelian $U(1)_Y$ factor is anomalous, in the 331 theory both, $SU(3)_L$ and $U(1)_X$ are anomalous ($SU(3)_c$ is vectorlike). So, special combination of multiplets must be used in each particular model in order to cancel the possible anomalies, and end with renormalizable models. The triangle anomalies we must take care of are: $[SU(3)_L]^3$, $[SU(3)_c]^2U(1)_X$, $[SU(3)_L]^2U(1)_X$, $[grav]^2U(1)_X$ and $[U(1)_X]^3$.

In order to present specific examples, let us see how the charge operator in Eq.(1) acts on the representations 3 and 3^* of $SU(3)_L$:

$$\begin{aligned} Q[3] &= Dg.(\frac{1}{2}+\frac{b}{3}+x,-\frac{1}{2}+\frac{b}{3}+x,-\frac{2b}{3}+x) \\ Q[3^*] &= Dg.(-\frac{1}{2}-\frac{b}{3}+x,\frac{1}{2}-\frac{b}{3}+x,\frac{2b}{3}+x). \end{aligned}$$

Notice from the former expressions that, if we accommodate the known left-handed quark and lepton isodoublets in the two upper components of 3 and 3^* (or 3^* and 3), and forbid the presence of exotic electric charges in the possible models, then the electric charge of the third component in those representations must be equal either to the charge of the first or second component, which in turn implies $b=\pm 1/2$. Since the negative value is equivalent to the positive one, $b=1/2$ is a necessary and sufficient condition in order to exclude exotic electric charges in the fermion sector.

1 Some examples

1.1 The Pleitez-Frampton model

As a first example let us take $b=3/2$, consecuently $Q[3]=Dg.(1+x,x,-1+x)$ and $Q[3^*]=Dg.(-1+x,x,1+x)$. Then the following multiplets are associated with the respective $(SU(3)_c,SU(3)_L,U(1)_x)$ quantun numbers: $(e^-,\nu_e,e^+)_L^T\sim(1,3^*,0)$; $(u,d,j)_L^T\sim(3,3,-1/3)$ and $(d,u,s)_L^T\sim(3,3^*,2/3)$, where j and s are isosinglets exotic quarks of electric charges $-4/3$ and $5/3$ respectively. This multiplet structure is the basis of the Pleitez-Frampton model [2] for which the anomaly-free arrangement for three families is given by:

$$\begin{aligned} \psi_L^a &= (e^a,\nu^a,e^{ca})_L^T\sim(1,3^*,0) \\ q_L^i &= (u^i,d^i,j^i)_L^T\sim(3,3,-1/3) \\ q_L^1 &= (d^1,u^1,s)_L^T\sim(3,3^*,2/3) \\ u_L^{ca} &\sim (3,1,-2/3),\ \ d_L^{ca}\sim(3,1,1/3) \\ s_L^c &\sim (3,1,-5/3),\ \ j_L^{ci}\sim(3,1,-4/3), \end{aligned}$$

where the upper c symbol stands for charge conjugation, $a = 1,2,3$ is a family index and $i = 2,3$ is related to two of the 3 families (in the 331 basis). As can be seen, there are six triplets of $SU(3)_L$ and six anti-triplets, which ensures cancellation of the $[SU(3)_L]^3$ anomaly. A power counting shows that the other four anomalies also vanish.

1.2 Other 331 family models in the literature

Let us analyze other two 331 three family models already present in the literature, for which $b = 1/2$ (they do not contain exotic electric charges). For that particular value of b we have: $Q[3] = Dg.(2/3+x, -1/3+x, -1/3+x)$ and $Q[3^*] = Dg.(-2/3+x, 1/3+x, 1/3+x)$. Then we get the following multiplets associated with the given quantun numbers: $(u,d,D)_L^T \sim (3,3,0)$, $(e^-, \nu_e, N^0)_L^T \sim (1,3^*,-1/3)$ and $(d,u,U)_L^T \sim (3,3^*,1/3)$, where D and U are exotic quarks with electric charges $-1/3$ and $2/3$ respectively. With this gauge structure we may construct the following anomaly free model for three families:

$$\begin{array}{rcl}
\psi_L^{'a} & = & (e^a, \nu^a, N^{0a})_L^T \sim (1,3^*,-1/3) \\
q_L^{'i} & = & (u^i, d^i, D^i)_L^T \sim (3,3,0) \\
q_L^{'1} & = & (d^1, u^1, U)_L^T \sim (3,3^*,1/3) \\
u_L^{ca} & \sim & (3,1,-2/3), \;\; d_L^{ca} \sim (3,1,1/3) \\
U_L^c & \sim & (3,1,-2/3), \;\; D_L^{ci} \sim (3,1,1/3),
\end{array}$$

where $a = 1,2,3$ is a family index and $i = 2,3$. This model has been analyzed in the literature in Ref. [3]. If needed, this model can be augmented with an undetermined number of neutral Weyl states $N_L^{0b} \sim (1,1,0)$, $b = 1,2,...$, without violating the anomaly constraint relations. We call this **Model A**.

The other model has the same quark multiplets used in the previous model arranged in a different way, and it makes use of a new lepton multiplet $\psi"_L = (\nu_e, e^-, E^-)_L^T \sim (1,3,-2/3)$. The multiplet structure of this new anomaly-free three family model is given by:

$$\begin{array}{rcl}
\psi"^a_L & = & (\nu^a, e^a, E^a)_L^T \sim (1,3,-2/3) \\
e^{ca} & \sim & (1,1,1), \;\; E^{ca} \sim (1,1,1) \\
q"^1_L & = & (u^1, d^1, D)_L^T \sim (3,3,0)
\end{array}$$

$$
\begin{aligned}
q"^{i}_{L} &= (d^i, u^i, U^i)^T_L \sim (3, 3^*, 1/3)\\
u^{ca}_L &\sim (3, 1, -2/3),\;\; d^{ca}_L \sim (3, 1, 1/3)\\
D^c_L &\sim (3, 1, 1/3),\;\; U^{ci}_L \sim (3, 1, 2/3).
\end{aligned}
$$

This model has been analyzed in the literature in Ref. [4]. We call this **Model B**.

1.3 Other models

Now we want to consider other possible 331 models without exotic electric charges ($b = 1/2$). Let us start first defining the following closed set of fermions (closed in the sense that they include the antiparticles of the charged particles):
$S_1 = [(\nu_\alpha, \alpha^-, E^-_\alpha); \alpha^+; E^+_\alpha]$ with quantum numbers $[(1, 3, -2/3); (1, 1, 1); (1, 1, 1)]$.
$S_2 = [(\alpha^-, \nu_\alpha, N^0_\alpha); \alpha^+]$ with quantum numbers $[(1, 3^*, -1/3); (1, 1, 1)]$.
$S_3 = [(d, u, U); u^c; d^c; U^c]$ with quantum numbers $(3, 3^*, 1/3); (3^*, 1, -2/3); (3^*, 1, 1/3)$ and $(3^*, 1, -2/3)$ respectively.
$S_4 = [(u, d, D); d^c; u^c; D^c]$ with quantum numbers $(3, 3, 0); (3^*, 1, 1/3); (3^*, 1, -2/3)$ and $(3^*, 1, 1/3)$ respectively.
$S_5 = [(e^-, \nu_e, N^0_1); (E^-, N^0_2, N^0_3); (N^0_4, E^+, e^+)]$ with quantum numbers $(1, 3^*, -1/3)$; $(1, 3^*, -1/3)$ and $(1, 3^*, 2/3)$ respectively.
$S_6 = [(\nu_e, e^-, E^-); (E^+_2, N^0_1, N^0_2); (N^0_3, E^-_2, E^-_3); e^+; E^+_1; E^+_3]$ with quantum numbers $[(1, 3, -2/3);\ (1, 3, 1/3);\ (1, 3, -2/3);\ (1, 1, 1);\ (1, 1, 1);\ (1, 1, 1)]$

The anomalies for the former sets are presented in the following Table.

TABLE I. Anomalies for S_i.

Anomalies	S_1	S_2	S_3	S_4	S_5	S_6
$[SU(3)_c]^2U(1)_X$	0	0	0	0	0	0
$[SU(3)_L]^2U(1)_X$	$-2/3$	$-1/3$	1	0	0	-1
$[grav]^2U(1)_X$	0	0	0	0	0	0
$[U(1)_X]^3$	10/9	8/9	$-12/9$	$-6/9$	6/9	12/9
$[SU(3)_L]^3$	1	-1	-3	3	-3	3

Notice from the Table that **Model A** is given by $(3S_2 + S_3 + 2S_4)$ and **Model B** by $(3S_1 + 2S_3 + S_4)$. But they are not the only anomaly-free structures we may build. Let us see:

1.3.1 One family models

There are two anomaly-free one family structures that can be extracted from the Table. They are:
Model C: $(S_4 + S_5)$. This model is associated with an E_6 subgroup and has been analyzed in Ref. [5].
Model D: $(S_3 + S_6)$. This model is associated with an $SU(6)_L \otimes U(1)_X$ subgroup and has been analyzed in Ref. [6].

The former two models can become realistic models (for 3 families) just by carbon copy each family as in the SM, that is, taking $3(S_4 + S_5)$ and $3(S_3 + S_6)$.

1.3.2 Two family models

There are three two family models. They are given by: $(S_1+S_2+S_3+S_4)$, $2(S_4+S_5)$ and $2(S_3 + S_6)$. These three models are not realistic.

1.3.3 Three family models

Besides models **A** and **B** we have two more. They are: **Model E**: $(S_1 + S_2 + S_3 + 2S_4 + S_5)$ and **Model F**: $(S_1 + S_2 + 2S_3 + S_4 + S_6)$.

The main feature of these last two models is that, contrary to all the other models, each one of the three families is treated in a different way. As far as we know, these two models have not been studied in the literature so far.

We may construct now four, five, etc. family models (a four family model is given for example by: $2(S_1 + S_2 + S_3 + S_4)$), but as for the two family case, they are not realistic.

2 The scalar sector

Even though the representation content for the fermion fields may vary significantly from model to model, all $SU(3)_L \otimes U(1)_X$ models presented have a gauge boson sector which depends only on the b parameter. In what follows we are going to reffer only to models for which $b = 1/2$ (models **A-F**). For that particular value of b there are only two Higgs scalars which may develop a nonzero Vacuum Expectation Value (VEV), they are $\phi_1(1, 3^*, -1/3) = (\phi_1^-, \phi_1^0, \phi_1^{'0})$, with (VEV) $\langle\phi_1\rangle = (0, v, V)^T$ and $\phi_2(1, 3^*, 2/3) = (\phi_2^0, \phi_2^+ \phi_2^{'+})$ with VEV $\langle\phi_2\rangle = (v', 0, 0)^T$, with the hierarchy $V > v \sim v' \sim 250$ GeV, the electroweak mass scale.

Our aim is to break the symmetry in the way

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \longrightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(3)_c \otimes U(1)_Q,$$

and produce mass terms for the fermion fields at the same time.

In some models it is more convenient to work with a different set of Higgs fields. For example, in the one family model in Ref. [5] the following three scalars were used: $\phi_1(1,3^*,-1/3)$ with $\langle\phi_1\rangle = (0,0,V)^T$, $\phi_2(1,3^*,-1/3)$ with $\langle\phi_2\rangle = (0,v/\sqrt{2},0)^T$ and $\phi_3(1,3^*,2/3)$ with $\langle\phi_3\rangle = (v'/\sqrt{2},0,0)^T$. For that particular case we got the following mass terms for the charged gauge bosons in the electroweak sector: $M^2_{W^\pm} = (g^2/4)(v^2+v'^2)$, $M^2_{K^\pm} = (g^2/4)(2V^2+v'^2)$ and $M^2_{K^0(\bar{K}^0)} = (g^2/4)(2V^2+v^2)$. For the neutral gauge bosons we got a mass term of the form:

$$M = V^2(\frac{g'B^\mu}{3} - \frac{gA_8^\mu}{\sqrt{3}})^2 + \frac{v^2}{8}(\frac{2g'B^\mu}{3} - gA_3^\mu + \frac{gA_8^\mu}{\sqrt{3}})^2 + \frac{v'^2}{8}(gA_3^\mu - \frac{4g'B^\mu}{3} + \frac{gA_8^\mu}{\sqrt{3}})^2.$$

Diagonalizing M and defining

$$\begin{aligned}
Z_1^\mu &= Z_\mu\cos\theta + Z'_\mu\sin\theta,\\
Z_2^\mu &= -Z_\mu\sin\theta + Z'_\mu\cos\theta,\\
-\tan(2\theta) &= \frac{\sqrt{12}C_W(1-T_W^2/3)^{1/2}[v'^2(1+T_W^2) - v^2(1-T_W^2)]}{3(1-T_W^2/3)(v^2+v'^2) - C_W^2[8V^2 + v^2(1-T_W^2)^2 + v'^2(1+T_W^2)^2]},
\end{aligned}$$

we get that the photon field A^μ and the neutral fields Z_μ and Z'_μ are given by

$$\begin{aligned}
A^\mu &= S_W A_3^\mu + C_W\left[\frac{T_W}{\sqrt{3}}A_8^\mu + (1-T_W^2/3)^{1/2}B^\mu\right],\\
Z^\mu &= C_W A_3^\mu - S_W\left[\frac{T_W}{\sqrt{3}}A_8^\mu + (1-T_W^2/3)^{1/2}B^\mu\right],\\
Z'^\mu &= -(1-T_W^2/3)^{1/2}A_8^\mu + \frac{T_W}{\sqrt{3}}B^\mu.
\end{aligned}$$

S_W and C_W are, respectively, the sine and cosine of the electroweak mixing angle ($T_W = S_W/C_W$) defined by $S_W = \sqrt{3}g'/\sqrt{3g^2+4g'^2}$. Also we can identify the Y hypercharge associated with the SM gauge boson as:

$$Y^\mu = \left[\frac{T_W}{\sqrt{3}}A_8^\mu + (1-T_W^2/3)^{1/2}B^\mu\right].$$

In the limit $\theta \longrightarrow 0$, $M_Z = M_{W^\pm}/C_W$, and $Z_1^\mu = Z^\mu$ is the gauge boson of the SM. This limit is obtained either by demanding $V \longrightarrow \infty$ or $v'^2 = v^2(C_W^2 - S_W^2)$. In general θ may be different from zero although it takes a very small value, determined from phenomenology for each particular model.

3 Conclusions

In this paper we have studied the theory of $SU(3)c \otimes SU(3)_L \otimes U(1)_X$ in detail. By restricting the fermion field representations to particles without exotic electric charges we end up with six different realistic models, two one family models and four models for three families which are relatively new in the literature, with two of them (models **E** and **F**) introduced here for the first time, as far as we know.

If we allow for particles with exotic electric charges an infinite number of models can be constructed, where the model in Ref. [2] is just one of them.

The low energy predictions of the six models are not the same. All of them have in common a new neutral current which mixes with the SM neutral current which is also included as part of each model (see Refs. [3]-[6]).

The most remarkable result of our analysis is that, contrary to what is stated in Ref. [2], the 331 theory can be used to construct either one family models or multi-family models, with the number of families being a free number. Conspicuously enough are the existence of models **E** and **F** for three families, where the three families are treated different.

Acknowledgments

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