Texture Zeros and WB Transformations in the Quark Sector of the Standard Model

Yithsbey Giraldo Úsuga

Universidad de Nariño

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Abstract

Stimulated by the recent attention given to the texture zeros found in the quark mass matrices sector of the Standard Model, an analytical method for identifying (or to exclude) texture zeros models will be implemented here, starting from arbitrary quark mass matrices and making a suitable weak basis (WB) transformation, we are be able to find equivalent quark mass matrix. It is shown that the number of non-equivalent quark mass matrix representations is finite. We give exact numerical results for parallel and non-parallel four-texture zeros models. We find that some five-texture zeros Ansätze are in agreement with all present experimental data. And we confirm definitely that six-texture zeros of Hermitian quark mass matrices are not viable models anymore.

Introduction

Although the gauge sector of the Standard Model (SM) with the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry is very successful, the Yukawa sector of the SM is still poorly understood. The origin of the fermion masses, the mixing angles and the CP violation remain as open problems in particle physics. There have been a lot of studies of possible fundamental symmetries in the Yukawa coupling matrices of the SM. In the absence of a more fundamental theory of interactions, an independent phenomenological model approach to search for possible textures or symmetries in the fermion mass matrices is still playing an important role.

In the SM, the mass term is given by

$$-\mathcal{L}_M = \bar{u}_R M_u u_L + \bar{d}_R M_d d_L + h.c.$$

Hermitian quark mass matrices: the mass matrices M_u and M_d are three-dimensional complex matrices. We can consider quark mass matrices to be hermitian as the unitary matrix can be absorbed in the right handed quark fields.

WB Transformations

The most general WB transformation [1], that leaves the physical content invariable and the mass matrices Hermitian, is

$$\begin{aligned} M_u &\longrightarrow M_u' = U^\dagger M_u U, \\ M_d &\longrightarrow M_d' = U^\dagger M_d U, \end{aligned} \qquad (1)$$

where U is an arbitrary unitary matrix.

1. The WB transformation generates all mass matrix representations

"In the SM, any two pairs of Hermitian quark mass matrices, given by (M_u, M_d) and (M_u', M_d'), with identical eigenvalues and flavor mixing parameters, to a specific scale energy, are related through a WB transformation."

2. The preliminary matrix representation

The u-diagonal representation:

$$M_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix},$$

$$M_d = V D_d V^\dagger, \quad \text{where} \quad V = U_u^\dagger U_d.$$

The d-diagonal representation.

3. A unique negative eigenvalue

"Each one of quark mass matrices M_u and M_d contains exactly one negative eigenvalue."

Numerical Five-Texture Zeros

There are a wide variety of four and five-texture zeros representations. Using a specific approach, some non-parallel texture are easy to obtain. But more laborious methods are required in parallel cases.

4. Five-texture zeros

The corresponding five-texture zeros representation obtained is

$$M_u' = \begin{pmatrix} 0 & 0 & -92.3618 + 157.694i \\ 0 & 5748.17 & 28555.1 + 5911.83i \\ -92.3618 - 157.694i & 28555.1 - 5911.83i & 166988 \end{pmatrix},$$

$$M_d' = \begin{pmatrix} 0 & 13.9899 & 0 \\ 13.9899 & 0 & 424.808 \\ 0 & 424.808 & 2796.9 \end{pmatrix}.$$

Analytical Five-Texture Zeros and the ckm Matrix

The five-texture zeros matrix derived above was not considered or ruled out in literature:

$$M_u = P^\dagger \begin{pmatrix} 0 & 0 & |C_u| \\ 0 & A_u & |B_u| \\ |C_u| & |B_u| & \tilde{B}_u \end{pmatrix} P, \quad M_d = \begin{pmatrix} 0 & |C_d| & 0 \\ |C_d| & 0 & |B_d| \\ 0 & |B_d| & A_d \end{pmatrix},$$

where $P = \mathrm{diag}(e^{-i\phi_{cu}}, e^{-i\phi_{bu}}, 1)$. Considering $\lambda_{2u} = -m_c$, we have:

5. Analytical Five-Texture Zeros

$$|V_{ud}| \approx |V_{cs}| \approx |V_{tb}| \approx 1,$$

$$|V_{us}| \approx |V_{cd}| \approx \left| \sqrt{\frac{A_u + m_c}{A_u}} \sqrt{\frac{m_u}{m_c}} + e^{\pm i(\phi_{bu} - \phi_{cu})} \sqrt{\frac{m_d}{m_s}} \right|,$$

$$|V_{cb}| \approx |V_{ts}| \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{\pm i\phi_{bu}} \sqrt{\frac{A_u + m_c}{m_t}} \right|,$$

$$\frac{|V_{ub}|}{|V_{cb}|} \approx \sqrt{\frac{m_u}{m_c}} \left| \frac{\sqrt{\frac{A_u}{m_t}} - e^{-i\phi_{bu}} \sqrt{\frac{A_u + m_c}{A_u}} \sqrt{\frac{m_s}{m_b}}}{\sqrt{\frac{A_u + m_c}{m_t}} - e^{-i\phi_{bu}} \sqrt{\frac{m_s}{m_b}}} \right|,$$

$$\frac{|V_{td}|}{|V_{ts}|} \approx \sqrt{\frac{m_d}{m_s}},$$

where we assume that $m_u \ll A_u \ll m_t$.

Conclusions

The main conclusions of this work are:

- Two quark mass matrices representations giving the same physical quantities must be related through a WB transformation.
- Using the definition of WB transformation, it was shown that the number of nonequivalent representations for the quark mass matrices is finite.
- Through WB transformations, it was relatively easy to find nonparallel four- texture zeros mass matrices. More difficult, but feasible, was the case for parallel four-texture zeros mass matrices, which were found in an exact way.
- Significant was the consistent five-texture zeros quark mass matrix found by us.
- We have determined the impossibility of finding quark mass matrices having a total of six-texture zeros.

References

[1] G.C. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe, Phys.Lett.B477, 2000 [hep-ph/9911418].

[2] K. Nakamura et al. (Particle Data Group), JP G 37, 075021 (2010) and 2011 partial update for the 2012 edition (URL: http://pdg.lbl.gov).