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Faddeev-Jackiw quantization of Proca Electrodynamics

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Abstract

The generalized symplectic formalism quantization method is employed to study the gauge invariance Proca electrodynamics theory. We show that the zero modes of the symplectic matrix are the generators of the gauge transformation. After fixing the gauge, the generalized brackets are calculated.

Keywords: Field theory; Faddeev-Jackiw method; Constrained systems; Proca electrodynamics.

1. Introduction

Faddeev and Jackiw [1] proposed a geometric method for the symplectic quantization of constrained systems. This method is based on Darboux's theorem [3] in which we do not need to introduce primary constraints as in the Dirac formalism [2]. Also, the classification of the constraints is not necessary, since all the constraints are held to the same standard [4]. Faddeev-Jackiw's method has been applied in several models [5].

The essential point of the symplectic quantization method is to make the system into a first order Lagrangian with some auxiliary fields, but the method does not depend on how the auxiliary fields are introduced to make the first order Lagrangian [1]. The first order Lagrangian, which consists of some symplectic variables and their generalized canonical momenta, gives the geometric structure of the manifold through the symplectic two form matrix. The classification of the system as constrained or unconstrained in the first order Faddeev-Jackiw formalism depends on the singular behavior of the symplectic two form matrix.

In this work we are going to study the symplectic quantization of gauge invariance Proca electrodynamics. It was shown that although Proca electrodynamics is not gauge invariant, it is possible to add certain nonlocal, nonpolynomial mass term with the help an auxiliary field to Lagrangian density and to transform it in a gauge theory [6]. The constraints and their algebra were constructed from a consistent Hamiltonian formulation using the Dirac formalism. Also, the problem of gauge fixing for the theory was studied and gauge conditions were introduced to calculate Dirac brackets for the dynamical variables. Here, we are going to derive the generalized symplectic brackets and show that they are equivalent to the Dirac brackets [7].

The work is organized as follow, in section 2 we analyzed the geometric structure of the Proca electrodynamics and finally in section 3 we present the conclusions.

2. Symplectic analysis for the Proca electrodynamics

The effective gauge invariant Lagrangian density which describes the Proca field is defined by:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}M^2\left[A_\mu + \frac{1}{e}\partial_\mu\theta\right]^2, \tag{1}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. The Lagrangian above is invariant under the following transformations,

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu\Lambda(x), \quad \theta(x) \rightarrow \theta(x) - e\Lambda(x). \tag{2}$$

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From (1) it is easy to write the Lagrangian in first order notation by introducing the canonical momenta π^μ and p_θ with respect to the fields A_μ and θ, respectively,

$$\pi^\mu = -F^{0\mu} \quad , \quad p_\theta = \frac{M^2}{e}\left[A_0 + \frac{1}{e}\partial_0\theta\right]. \tag{3}$$

The initial set of symplectic variables defining the extended space is given by the set $\xi^{(0)} = \left(A_k, \pi^k, \theta, p_\theta, A_0\right)$, and so, the starting Lagrangian density is written as follows:

$$\mathcal{L}^{(0)} = a_A^{(0)}(\xi)\,\xi^{(0)A} - \mathcal{H}^{(0)}(\xi), \tag{4}$$

where the zero iterated symplectic potential has the following form:

$$\begin{aligned}\mathcal{H}^{(0)} \equiv{}& \frac{1}{2}\left(\pi^k\right)^2 + \frac{1}{2}\frac{e^2}{M^2}p_\theta^2 + \pi^k\partial_k A_0 - eA_0 p_\theta \\ &+ \frac{1}{4}F_{kl}F_{kl} + \frac{1}{2}M^2\left[A_k + \frac{1}{e}\partial_k\theta\right]^2,\end{aligned} \tag{5}$$

and the canonical momenta $a^{(0)}(\xi)$ for the symplectic variables $\xi_k^{(0)}$ are:

$$\begin{aligned}& a_{A_k}^{(0)} = \pi^k \quad , \quad a_{\pi^k}^{(0)} = 0 \quad , \quad a_\theta^{(0)} = p_\theta, \\ & a_{p_\theta}^{(0)} = 0 \quad , \quad a_{A_0}^{(0)} = 0.\end{aligned} \tag{6}$$

Then, we obtain the zero iterated symplectic two-form matrix defined by

$$f_{AB}^{(0)}(\mathbf{x},\mathbf{y}) = \frac{\delta a_B^{(0)}(\mathbf{y})}{\delta\xi^{(0)A}(\mathbf{x})} - \frac{\delta a_A^{(0)}(\mathbf{x})}{\delta\xi^{(0)B}(\mathbf{y})}, \tag{7}$$

with the components

$$f_{AB}^{(0)}(\mathbf{x},\mathbf{y}) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\delta^3(\mathbf{x}-\mathbf{y}). \tag{8}$$

The symplectic matrix is singular and it has a zero mode

$$\tilde{v}^{A(0)} = \begin{pmatrix} 0 & 0 & 0 & 0 & v^{A_0}(\mathbf{x}) \end{pmatrix}, \tag{9}$$

where $v^{A_0}(\mathbf{x})$ is an arbitrary function. From this nontrivial zero-mode, we have the following constraint

$$\begin{aligned}\Omega^{(0)} &= \int d^3x v^{A_0}(\mathbf{x})\frac{\delta}{\delta\xi_{A_0}(\mathbf{x})}\int d^3y\mathcal{H}^{(0)}\left(\xi_k^{(0)}\right) \\ &= -\int d^3x v^{A_0}(\mathbf{x})\left[\partial_k^x\pi^k(\mathbf{x}) + ep_\theta(\mathbf{x})\right] \\ &= 0.\end{aligned} \tag{10}$$

With $v^{A_0}(\mathbf{x})$ arbitrary, the constraint is evaluated form (10) to be

$$\Omega^{(0)} \equiv \partial_k\pi^k + ep_\theta = 0. \tag{11}$$

According to the symplectic algorithm, the constraint (11) is introduced in the Lagrangian density by using Lagrangian multipliers $\lambda(\mathbf{x})$, thus, the first iterated Lagrangian density is written as

$$\mathcal{L}^{(1)} = a_A^{(1)}(\xi)\,\dot{\xi}^{A(1)} - \mathcal{H}^{(1)}(\xi), \tag{12}$$

with the first iterated symplectic potential

$$\begin{aligned}\mathcal{H}^{(1)} \equiv \mathcal{H}^{(0)}_{\Omega^{(0)}=0} ={}& \frac{1}{2}\left(\pi^k\right)^2 + \frac{1}{2}\frac{e^2}{M^2}p_\theta^2 + \\ &+ \frac{1}{4}F_{kl}F_{kl} + \frac{1}{2}M^2\left[A_k + \frac{1}{e}\partial_k\theta\right]^2.\end{aligned} \tag{13}$$

Here, we have enlarged the space with the first iterated set of symplectic variables defined by $\xi_k^{(1)} = \left(A_k, \pi^k, \theta, p_\theta, \lambda\right)$, with the new canonical one-form defined by,

$$\begin{aligned}& a_{A_k}^{(1)} \to \pi^k \quad , \quad a_{\pi^k}^{(1)} \to 0 \quad , \quad a_\theta^{(1)} \to p_\theta, \\ & a_{p_\theta}^{(1)} \to 0 \quad , \quad a_\lambda^{(1)} \to \partial_k\pi^k + ep_\theta.\end{aligned} \tag{14}$$

Thus, the first iterated symplectic matrix is written as

$$f_{AB}^{(1)}(\mathbf{x},\mathbf{y}) = \begin{pmatrix} 0 & -\delta_l^k & 0 & 0 & 0 \\ \delta_k^l & 0 & 0 & 0 & -\partial_k^x \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & e \\ 0 & -\partial_l^x & 0 & -e & 0 \end{pmatrix}\delta^3(\mathbf{x}-\mathbf{y}). \tag{15}$$

The modified symplectic matrix after the first iteration is again singular. As it can be seen, there is one new zero-mode associated to this matrix and it is written as:

$$\tilde{v}^{A(1)}(\mathbf{x}) = \begin{pmatrix} \partial_k^x\alpha(\mathbf{x}) & 0 & -e\alpha(\mathbf{x}) & 0 & \alpha(\mathbf{x}) \end{pmatrix}, \tag{16}$$

where $\alpha(\mathbf{x})$ is a new arbitrary quantity. A new constraint can be derived from (16), then, we have that

$$\begin{aligned}\Omega^{(1)} &= \int d^3x\tilde{v}^{A(1)}(\mathbf{x})\frac{\delta}{\delta\xi_A^{(1)}(\mathbf{x})}\int d^3y\mathcal{H}^{(1)}(\mathbf{y}) \\ &= \int d^3x\alpha(\mathbf{x})\left\{\partial_i^x\partial_k^x F_{ki}(\mathbf{x}) - M^2\partial_i^x\left[A_i(\mathbf{x})\right.\right. \\ &\left. + \frac{1}{e}\partial_i^x\theta(\mathbf{x})\right] + M^2\partial_k^x\left.\left[A_k(\mathbf{x}) + \frac{1}{e}\partial_k^x\theta(\mathbf{x})\right]\right\} \\ &= 0.\end{aligned} \tag{17}$$

Thus, $\Omega^{(1)}$ identically zero, therefore, the relation (17) indicates that there are no more constraints associated in the theory and as a result the symplectic matrix remains singular what characterizes the theory as a gauge theory.

In order to obtain a regular symplectic matrix a gauge fixing term must be added to the symplectic potential. We choose the Coulomb gauge $\Theta = \partial_k A_k + \frac{M^2}{e}\theta = 0$ and we obtain the second iterative Lagrangian, i.e.:

$$\mathcal{L}^{(2)} = a_A^{(2)}(\xi)\,\dot{\xi}^{A(2)} - \mathcal{H}^{(2)}(\xi), \tag{18}$$

associated with the symplectic variable $\xi_k^{(2)} = \left(A_k, \pi^k, \theta, p_\theta, \lambda, \eta\right)$, where $\eta(\mathbf{x})$ is the Lagrange multiplier corresponding to gauge fixing term. The canonical momenta $a_A^{(2)}(\xi)$ is:

$$a_{A_k}^{(2)} \to \pi^k \quad , \quad a_{\pi^k}^{(2)} \to 0 \quad , \quad a_\theta^{(2)} \to p_\theta \quad , \quad a_{p_\theta}^{(2)} \to 0,$$
$$a_\lambda^{(2)} \to \partial_k \pi^k + e p_\theta \quad , \quad a_\eta^{(2)} \to \partial_k A_k + \frac{M^2}{e}\theta. \tag{19}$$

Now, the second iterated symplectic potential is:

$$\mathcal{H}^{(2)} \equiv \mathcal{H}^{(1)}_{\Theta=0} = \frac{1}{2}\left(\pi^k\right)^2 + \frac{1}{2}\frac{e^2}{M^2}p_\theta^2 + \frac{1}{4}F_{kl}F_{kl} + \frac{1}{2}M^2\left[A_k + \frac{1}{e}\partial_k\theta\right]^2. \tag{20}$$

From (19) we obtain the second-iterated symplectic two-form matrix

$$f_{AB}^{(2)}(\mathbf{x},\mathbf{y}) \equiv \frac{\delta f_B^{(2)}(\mathbf{y})}{\delta\xi^{A(2)}(\mathbf{x})} - \frac{\delta f_A^{(2)}(\mathbf{x})}{\delta\xi^{B(2)}(\mathbf{y})} = \begin{pmatrix} 0 & -\delta_l^k & 0 & 0 & 0 & -\partial_k^x \\ \delta_k^l & 0 & 0 & 0 & -\partial_k^x & 0 \\ 0 & 0 & 0 & -1 & 0 & \frac{M^2}{e} \\ 0 & 0 & 1 & 0 & e & 0 \\ 0 & -\partial_l^x & 0 & -e & 0 & 0 \\ -\partial_l^x & 0 & -\frac{M^2}{e} & 0 & 0 & 0 \end{pmatrix} \delta^3(\mathbf{x}-\mathbf{y}). \tag{21}$$

Since this matrix is not singular the inverse of (21) can be determined, from which it is possible to identify the following generalized brackets:

$$\left\{A_i(\mathbf{x}), \pi^j(\mathbf{y})\right\} = \left(\delta_{ij} - \frac{\partial_i^x\partial_j^x}{\partial_i^x\partial_i^x}\right)\delta^3(\mathbf{x}-\mathbf{y}),$$
$$\left\{\theta(\mathbf{x}), p_\theta(\mathbf{y})\right\} = \left(1 + \frac{M^2}{D_x}\right)\delta^3(\mathbf{x}-\mathbf{y}). \tag{22}$$

3. Remarks and conclusions

In this paper we have studied Proca Electrodynamics gauge invariance with the symplectic quantization method. The results give us the Dirac brackets of the theory, which is an alternative to the orthodox Dirac method on constrained dynamics [2]. At the same time, we have shown that the symplectic approach is more intuitive in the sense that the constraints are related to the generalized canonical momenta and the Lagrange multipliers to the symplectic variables in the enlarged symplectic structure of the constrained manifold. For the Proca Electrodynamics we have shown that the number of the constraints is smaller and the structure of these constraints is very simple because we do not need to distinguish first or second class constraints, primary or secondary constraints, etc. We have easily obtained the Dirac brackets by reading directly from the inverse matrix $\left[f^{AB(2)}\right]^{-1}$ of the symplectic two form matrix. Finally, we can observe that the potential symplectic obtained at the final stage of iterations is exactly the Hamiltonian which is obtained through several steps with the usual Dirac formulation of the constrained systems.

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References

[1] L. Faddeev and R. Jackiw, Phys. Rev. Lett., **60**, 1692 (1988).

[2] P. A. M. Dirac, *Lectures in Quantum Mechanics*, Benjamin, New York,1964.

[3] R. Jackiw, *Quantization Without Tears*, hep-th/9306075.

[4] J. Barcelos-Neto and C. Wotzasek, Mod. Phys. Lett., **A7**, 1737 (1992); J. Barcelos-Neto and C. Wotzasek, Mod. Phys. Lett., **A7**, 4981 (1992); H. Montani and C. Wotzasek, Mod. Phys. Lett., **A8**, 3387 (1993).

[5] C. Wotzasek, Ann. Phys., **243**, 76 (1995); H.S. Blas, B.M. Pimentel, Ann. Phys., **282**, 67 (2000); E.M.C. Abreu, A.C.R. Mendes, C. Neves, W. Oliveira, R.C.N. Silva, JHEP **1306**, 093 (2013); R. Bufalo, B.M. Pimentel, Eur. Phys. J. C., **74**, 2993, (2014).

[6] J. M. Cornwall, Phys. Rev. **D10**, 500, (1974).

[7] B.M. Pimentel and G. E. R. Zambrano, *Gauge Invariance of Proca Electrodynamics*, to appear.

[8] A. Foussats, C. Repetto, O. P. Zandron and O. S. Zandron, Int. J. Mod. Phys, **36**, 2923 (1997).