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Hamiltonian Formulation of the Yang–Mills field on the null–plane

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We have studied the null–plane hamiltonian structure of the free Yang–Mills fields. Following the Dirac's procedure for constrained systems we have performed a detailed analysis of the constraint structure of the model and we give the generalized Dirac brackets for the physical variables. Using the correspondence principle in the Dirac's brackets we obtain the same commutators present in the literature and new ones.

1. Introduction

To quantize the theory on the null–plane [1], initial conditions on the hyperplane $x^+ = cte$ and equal x^+-commutation relations must be given and the hamiltonian must describe the time evolution from an initial value surface to other parallel surface that intersects the x^+-axis at some later time. Inside the null–plane framework, the lagrangian which describes a given field theory is singular, thus, the Dirac's method [2] allows to build the null–plane hamiltonian and the canonical commutation relations in terms of the independent fields of the theory.

It is interesting to observe that the null–plane quantization of a non-abelian gauge theory using the null–plane gauge condition, $A_- = 0$, identified the transverse components of the gauge field as the degrees of freedom of the theory and, therefore, the ghost fields can be eliminated of the quantum action [3].

Tomboulis has quantized the massless Yang–Mills field in the null–plane gauge $A_-^a = 0$ and has derived the Feynman rules [4]. However, it was shown that the null–plane quantization of this theory leads a set of second–class constraints in addition to the usual first–class constraints, characteristics of the usual instant form quantization, which leads to the introduction of additional ghost fields in the effective lagrangian [5]. Moreover, the theory has been quantized in the framework of the standard perturbation approach and it was explained that the difficulties appearing in the null–plane gauge are overcome using the gauge $A_+^a = 0$, such gauge provides a generating functional for the renormalized Green's functions that takes to the Mandelstam–Leibbrandt's prescription for the free gluon propagator [6].

In this paper we will discuss the null–plane structure of the pure Yang–Mills fields following Dirac's formalism for constrained systems. The work is organized as follows: In the section 2, we study the free Yang–Mills field, its constrained structure being analysed in detail, thus, we classify the constraints of the theory. In the section 3 the appropriated equations of motion of the dynamical variables are determined by using the extended hamiltonian, and the null–plane gauge is imposed to transform the set of first class constraints into second–class ones. In the section 4 the Dirac's brackets (DB) among the independent fields are obtained by choosing appropriate boundary conditions on the fields. Finally, we give our conclusions and remarks.

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5. Remarks and conclusions

In this work we have studied the null–plane Hamiltonian structure of the free Yang–Mills field. Performing a careful analysis of the constraint structure of Yang–Mills field, we have determined in addition to the usual set of first–class constraints, a second–class one, which is a characteristic of the null–plane dynamics [7]. The imposition of appropriated boundary conditions on the fields fixes the hidden subset of first class constraints [9] and eliminates the ambiguity on the operator ∂_-, that allows to get a unique inverse for the second class constraint matrix [7]. The Dirac brackets of the theory are quantized via correspondence principle same as derived by Tomboulis [4].

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