Parametrization of Minimal Non-universal EW Extensions of the Standard Model

Y. Giraldo†, E. Rojas‡

†yithsbey@udenar.edu.co , ‡eduro4000@gmail.com

Universidad de Nariño

Abstract

In the Standard Model, we obtain a non-Fritzsch like configuration with five texture zeros for the quark mass matrices. This matrix generates the quark masses, the inner angles of the CKM unitary triangle, and the CP-violating phase in the quark sector. This work can be applied to the PMNS matrix in the lepton sector, by assuming Dirac masses for the neutrinos, where non-trivial predictions for the neutrino masses and mixing angles are expected.

Five texture zeros

For the quark family, by performing weak basis (WB) transformations on arbitrary quark mass matrices, it is always possible to obtain either of the following two arrangements [5]:

$$M_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix}, M_d = V D_d V^\dagger, \text{or}, M_u = V^\dagger D_u V, M_d = D_d = \begin{pmatrix} \lambda_{1d} & 0 & 0 \\ 0 & \lambda_{2d} & 0 \\ 0 & 0 & \lambda_{3d} \end{pmatrix}. \quad (1)$$

Therefore, because of their simplicity, considering them to be the general initial bases for the quark mass matrices [1, 3, 4, 5] —where V is the CKM mixing matrix, and the eigenvalues $|\lambda_{iq}|$ $(i = 1, 2, 3)$ are the up- $(q = u)$ and down- $(q = d)$ quark masses— is convenient.

$$|\lambda_{1u}| = m_u, |\lambda_{2u}| = m_c, |\lambda_{3u}| = m_t, |\lambda_{1d}| = m_d,$$
$$|\lambda_{2d}| = m_s, |\lambda_{3d}| = m_b, \text{ where } |\lambda_{1q}| \ll |\lambda_{2q}| \ll |\lambda_{3q}| \quad (2)$$

The properties of the WB transformations allow us to use the bases (1) as the initial matrices to generate any physical structure in the quark mass matrix sector. If there are texture zeros, this transformation can find them. Since some texture zeros are in the diagonal elements of the hermitian mass matrices, it implies that at least one and at most two of their eigenvalues are negative [1]. Also, in the case of two negative eigenvalues, these mass matrices can be reduced to having one by adding a negative sign in the bases (1), as follows: $M_u = -(-M_u)$ or/and $M_d = -(-M_d)$, and implementing WB transformations for the terms in parentheses. Therefore, without losing generality, the texture zeros in the models can be deduced by assuming that each quark mass matrix, M_u and M_d, contains precisely one single negative eigenvalue [4], i.e.,
λ_{iq} is negative for one value of i and positive for the others.
Each *realistic* quark mass matrix can contain, at most, three texture zeros. Also, there are only two possible patterns according to the distribution of the three zeros in the elements of the mass matrix. In the first case, the mass matrix has a single zero in the diagonal elements, while in the other case, it has only two zeros in the diagonal entries.
The two respective basic patterns are as follows:

$$M_{1q} = \begin{pmatrix} 0 & |\xi_q| & 0 \\ |\xi_q| & \gamma_q & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}, \quad M_{2q} = \begin{pmatrix} 0 & |\xi_q| & 0 \\ |\xi_q| & 0 & |\beta_q| \\ 0 & |\beta_q| & \alpha_q \end{pmatrix}, \quad (3)$$

where $(q = u$ or $d)$, and we can observe that by making WB transformations of the form $p_i\, M_{1,2q}\, p_i^T$ and considering all permutation matrices p_i $\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}, \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}, \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}, \begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}, \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix},$
we get as many viable cases as possible for each pattern considered. All viable three-zero textures for quark mass matrices are summarized here. These patterns are general, and including phases is not necessary, as they can be absorbed by the other mass matrix (u or d) through a WB transformation.
Let us start with the standard representation of the pattern of two zeros in the diagonal entries, M_{2q}, expression (3), for which we have:

$$\alpha_q = \lambda_{1q} + \lambda_{2q} + \lambda_{3q}, |\xi_q| = \sqrt{\frac{-\lambda_{1q}\lambda_{2q}\lambda_{3q}}{\alpha_q}}, |\beta_q| = \sqrt{-\frac{(\lambda_{1q} + \lambda_{2q})(\lambda_{1q} + \lambda_{3q})(\lambda_{2q} + \lambda_{3q})}{\alpha_q}}. \quad (4)$$

The result (4) for $|\xi_q|$ must be a real number, and because only an eigenvalue λ_{iq} is assumed negative, Eq. (), we have that $\alpha_q > 0$, where, together with (4) for β_q, and hierarchy (2), only one possibility is allowed:

$$\lambda_{1q}, \lambda_{3q} > 0 \quad \text{and} \quad \lambda_{2q} < 0, \quad \text{with} \quad \alpha_q > 0. \quad (5)$$

Numerical solutions

The matrix that diagonalizes M_{2q} is

$$U_{2q} = \begin{pmatrix} e^{ix}\sqrt{\frac{\lambda_{2q}\lambda_{3q}(\alpha_q-\lambda_{1q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & -e^{iy}\sqrt{\frac{\lambda_{1q}\lambda_{3q}(\lambda_{2q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{1q}\lambda_{2q}(\alpha_q-\lambda_{3q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ e^{ix}\sqrt{\frac{\lambda_{1q}(\lambda_{1q}-\alpha_q)}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & e^{iy}\sqrt{\frac{\lambda_{2q}(\alpha_q-\lambda_{2q})}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\lambda_{3q}-\alpha_q)}{(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ -e^{ix}\sqrt{\frac{\lambda_{1q}(\alpha_q-\lambda_{2q})(\alpha_q-\lambda_{3q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & -e^{iy}\sqrt{\frac{\lambda_{2q}(\alpha_q-\lambda_{1q})(\lambda_{3q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\alpha_q-\lambda_{1q})(\alpha_q-\lambda_{2q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \end{pmatrix}, \quad (6)$$

where α_q is in Eq. (4).
Performing a WB transformation on the second base of (1), using, in this case, the unitary matrix given in (6) with $q = d$ (i.e., U_{2d}), we have

$$M'_d = U_{2d}(D_d)U_{2d}^\dagger = \begin{pmatrix} 0 & |\xi_d| & 0 \\ |\xi_d| & 0 & |\beta_d| \\ 0 & |\beta_d| & \alpha_d \end{pmatrix}, M'_u = U_{2d}\,(V^\dagger D_u V)\,U_{2d}^\dagger. \quad (7)$$

According to (5), in this case $\lambda_{1d}, \lambda_{3d} > 0$, $\lambda_{2d} < 0$, and $\alpha_d = \lambda_{1d} + \lambda_{2d} + \lambda_{3d} > 0$.
The calculations are simplified if we define the following variables for the phases introduced in (6):

$$e^{ix} = x_1 + ix_2, \quad e^{iy} = y_1 + iy_2, \quad (8)$$

where $x_1^2 + x_2^2 = 1$ and $y_1^2 + y_2^2 = 1$, and it is satisfied that $|x_1|, |x_2|, |y_1|, |y_2| \leq 1$.
The elements of the matrix M'_u, in (7) become *hypersurfaces* for the set of points (x_1, x_2, y_1, y_2) in $\mathbb{R}^4$ for each case considered: $\lambda_{1u} = -m_u$ or $\lambda_{2u} = -m_c$ or $\lambda_{3u} = -m_t$. Taking (8) into account, the analysis of these hypersurfaces shows that only elements (1,2) and (1,3) of M'_u can give solutions equal to zero (texture zeros).
Let's take the case $\lambda_{1u} = -m_u$ as an example; we obtain the best results considering the following masses for quarks (in MeV units): $m_u = 1.71604$, $m_d = 2.9042$, $m_s = 65$, $m_c = 567$, $m_b = 2860$, $m_t = 172100$, which are close to the central values and are within the one-σ errors. The solutions are: $x_1 = 0.684994$, $y_1 = -0.500433$, $x_2 = 0.728548$, $y_2 = -0.865775$, with the component $M'_u(1,1) = 0$. The corresponding numerical matrices obtained for the quark masses with five texture zeros are the following

$$M'_u = \begin{pmatrix} 0 & 0 & -79.32299208381 + 154.7195315i \\ 0 & 5539.23021 & 28125.9455 + 6112.7938593i \\ -79.32299208381 - 154.7195315i & 28125.9455 - 6112.7938593i & 167126.0537497 \end{pmatrix} \text{MeV}, \quad (9a)$$

$$M'_d = \begin{pmatrix} 0 & 13.891097 & 0 \\ 13.891097 & 0 & 421.41405 \\ 0 & 421.41405 & 2797.9042 \end{pmatrix} \text{MeV}, \quad (9b)$$

and their diagonalization matrices are respectively

$$U'_u = \begin{pmatrix} 0.6762634914995 + 0.734812367i & -0.050244372844 + 0.01389199i & -0.00044949942643 + 0.000888263847i \\ 0.02750465770 - 0.043183583035i & -0.4970428606 - 0.849312285i & 0.1665852443687 + 0.03528548775i \\ -0.00340858222 + 0.00924818236i & 0.1150663143 + 0.12513711i & 0.98538127634 - 0.00519998920457i \end{pmatrix},$$

$$U'_d = \begin{pmatrix} 0.67017852 + 0.71279034i & 0.10351850 + 0.17909241i & 0.00070804245 \\ 0.14011366 + 0.14902248i & -0.48438959 - 0.83801926i & 0.14577693 \\ -0.021125534 - 0.022468754i & 0.071301225 + 0.12335484i & 0.98931723 \end{pmatrix},$$

which gives the correct CKM mixing matrix with a precision level of 1σ [6], $V = U_u'^\dagger U_d'$, including the phase responsible for the CP violation. The case $\lambda_{2u} = -m_c$ has already been done in article [4]. We also note that the first diagonal base (1) and the pattern with a zero in the diagonal (3) do not give additional consistent solutions with five texture zeros.

Conclusions

We have made a complete study of the texture zeros in the quark sector of the SM, starting from general quark mass matrices, based on the WB transformation property [4], to generate any possible mass matrix configuration. This result allowed us to use specific bases, (1), to reproduce as many texture zeros as possible. In this way, we discovered a numerical texture pattern consisting of five zeros, including permutations, whose matrix representation is in (9); the pattern is not Fritzsch type [2] because of the way texture zeros are present. There are no additional representations of five texture zeros apart from the one given in article [4]. This is a work in progress. By assuming Dirac masses for the neutrinos, next step is to apply our results to the PMNS matrix in the lepton sector where nontrivial results are expected.

References

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