Phenomenology of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ model with exotic charged leptons

Juan C. Salazar,[1,2] William A. Ponce,[1] and Diego A. Gutiíerrez[1]

[1]*Instituto de Física, Universidad de Antioquia, A.A. 1226, Medellín, Colombia*

[2]*Depto. de Física, Universidad de Nariño, A.A. 1175, Pasto, Colombia*

(Dated: 20 February 2007)

A phenomenological analysis of the three-family model based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ with exotic charged leptons, is carried out. Instead of using the minimal scalar sector able to break the symmetry in a proper way, we introduce an alternative set of four Higgs scalar triplets, which combined with an anomaly-free discrete symmetry, produce quark and charged lepton mass spectrum without hierarchies in the Yukawa coupling constants. We also embed the structure into a simple gauge group and show some conditions to achieve a low energy gauge coupling unification, avoiding possible conflict with proton decay bounds. By using experimental results from the CERN-LEP, SLAC linear collider, and atomic parity violation data, we update constraints on several parameters of the model.

PACS numbers: 12.10.Dm, 12.15.Ff, 12.60.Cn

I. INTRODUCTION

The Standard Model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ [1], with all its successes, is in the unaesthetic position of having no explanation of several issues such as: hierarchical charged fermion masses, fermion mixing angles, charge quantization, strong CP violation, replication of families, neutrino masses and oscillations [2], etc.. All this make us think that we must call for extensions of the model.

Doing physics beyond the SM may imply to introduce a variety of new ingredients such as extra fermion fields (adding a right-handed neutrino field to each family constitute its simplest extension and has several consequences, as the implementation of the see-saw mechanism for the neutrinos, and the enlarging of the possible number of local Abelian symmetries that can be gauged simultaneously). Also one may include standard and nonstandard new scalar field representations with and without Vacuum Expectation Values (VEV), and extra gauge bosons which imply an enlarging of the local gauge group. Discrete symmetries and supersymmetry (SUSY) are also common extensions of the SM [3].

Interesting extensions of the SM are based on the local gauge group [4, 5, 6, 7, 8, 9] $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (called hereafter 3-3-1 for short). The several possible structures enlarge the SM in its gauge, scalar, and fermion sectors. Let us mention some outstanding features of 3-3-1 models: they are free of gauge anomalies if and only if the number of families is a multiple of three [4, 5, 6]; a Peccei-Quinn chiral symmetry can be implemented easily [10, 11]; the fact that one quark family has different quantum numbers than the other two may be used to explain the heavy top quark mass [12, 13]; the scalar sector includes several good candidates for dark matter [14], the lepton content is suitable for explaining some neutrino properties [15, 16], and last but not least, the hierarchy in the Yukawa coupling constants can be avoided by implementing several universal see-saw mechanisms [13, 17, 18].

So far, there are in the literature studies of five different 3-3-1 lepton flavor structures for three families, belonging to two different electric charge embedding into $SU(3)_L \otimes U(1)_X$, being the most popular one the original Pisano-Pleitez-Frampton model [4] (called the minimall model)in which the three left-handed lepton components for each family in the SM are associated to three $SU(3)_L$ triplets as $(\nu_l, l^-, l^+)_L$, where $l = e,\ \mu,\ \tau$ is a family index for the lepton sector, ν_l stands for the neutrino related to the flavor l, and l^+_L is the right-handed isospin singlet of the charged lepton l^-_L.

In a different embedding of the electric charge operator, the three left-handed lepton triplets are of the form $(\nu_l, l^-, \nu_l^c)_L$, $l = e,\ \mu,\ \tau$; where ν_l^c is related to the right-handed component of the neutrino field (a model with "right-handed neutrinos" [5]), with l^+_L becoming three $SU(3)_L$ singlets. For the same charge embedding, an almost unknown alternative of a 3-3-1 fermion structure is provided in Ref. [6], in which the three $SU(3)_L$ lepton triplets are of the form $(\nu_l, l^-, E_l^-)_L$, $l = e,\ \mu,\ \tau$; where E_l^- stands for an exotic charged lepton per family, with l^+_L and E^+_{lL} being six $SU(3)_L$ singlets (a model with "exotic charged leptons").

Contrary to the former three structures in which each lepton generation is treated identically, two more new models are analyzed in Ref. [7], which are characterized by each lepton generation having a different representation under the gauge group. Even further, more possible 3-3-1 fermion structures can be found in Refs. [8, 9], where also a classification of all the models without exotic electric charges is presented (if exotic electric charges are allowed the number of models run to infinity).

The aim of this paper is to find, for the version of the model that includes "exotic charged leptons"[6], the minimal set of ingredients able to implement universal see-saw mechanisms in the three charged fermion sectors, with the analysis done in a similar way that the one presented in Refs. [13, 18], where a related calculation was carried through for the model with " right-handed neutrinos" (model that, contrary to the present one, does not contain exotic electrons, becoming thus unable by itself to generate see-saw masses for charged leptons [18]). It will be shown in what follows that a convenient set of scalar fields, combined with a discrete symmetry, produces an appealing fermion mass spectrum without hierarchies for the Yukawa coupling constants. Besides, we are also going to study the embedding and unification of this structure into $SU(6)$, and set updated constraints on several parameters of the model.

This paper is organized as follows: in Sec. II we review the model, introduce the new scalar sector, embed the structure into a covering group and calculate the charged and neutral electroweak currents; in Sec. III we study the charged fermion mass spectrum; in Sec. IV we do the renormalization group equation analysis and show the conditions for the gauge coupling unification; in Sec. V we constraint several parameters of the model by fixing new bounds on the mixing angle between the two flavor diagonal neutral currents present in the model, and finally, in Sec. VI, we present our conclusions.

II. THE MODEL

The model we are about to study here was introduced in the literature for the first time in Ref. [6]. Some of the formulas quoted in subsections II A and II F of this section, are taken from Refs.[6] and[9]. Corrections to some minor printing mistakes in the original papers are included.

A. The Gauge Group

As it was stated above, the model we are interested in, is based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ which has 17 gauge bosons: one gauge field B^μ associated with $U(1)_X$, the 8 gluon fields G^μ associated with $SU(3)_c$ which remain massless after spontaneous breaking of the electroweak symmetry, and another 8 gauge fields associated with $SU(3)_L$ that we write for convenience as [9]

$$\sum_{\alpha=1}^{8} \lambda^\alpha A^\mu_\alpha = \sqrt{2}\begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} \\ W^{-\mu} & D^\mu_2 & K^{0\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D^\mu_3 \end{pmatrix}, \qquad (1)$$

where $D^\mu_1 = A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, $D^\mu_2 = -A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, and $D^\mu_3 = -2A^\mu_8/\sqrt{6}$. λ_α, $\alpha = 1, 2, ..., 8$, are the eight Gell-Mann matrices normalized as $Tr(\lambda^\alpha \lambda^\beta) = 2\delta_{\alpha\beta}$.

The charge operator associated with the unbroken gauge symmetry $U(1)_Q$ is given by:

$$Q = \frac{\lambda_{3L}}{2} + \frac{\lambda_{8L}}{2\sqrt{3}} + XI_3 \qquad (2)$$

where $I_3 = Diag.(1, 1, 1)$ is the diagonal 3×3 unit matrix, and the X values are related to the $U(1)_X$ hypercharge and are fixed by anomaly cancellation. The sine square of the electroweak mixing angle is given by

$$S^2_W = 3g^2_1/(3g^2_3 + 4g^2_1) \qquad (3)$$

where g_1 and g_3 are the coupling constants of $U(1)_X$ and $SU(3)_L$ respectively, and the photon field is given by [6, 9]

$$A^\mu_0 = S_W A^\mu_3 + C_W \left[\frac{T_W}{\sqrt{3}} A^\mu_8 + \sqrt{(1 - T^2_W/3)} B^\mu\right], \qquad (4)$$

where C_W and T_W are the cosine and tangent of the electroweak mixing angle, respectively.

There are two weak neutral currents in the model associated with the two flavor diagonal neutral gauge weak bosons

$$\begin{aligned} Z^\mu_0 &= C_W A^\mu_3 - S_W \left[\frac{T_W}{\sqrt{3}} A^\mu_8 + \sqrt{(1 - T^2_W/3)} B^\mu\right], \\ Z'^\mu_0 &= -\sqrt{(1 - T^2_W/3)} A^\mu_8 + \frac{T_W}{\sqrt{3}} B^\mu, \end{aligned} \qquad (5)$$

and another electrically neutral current associated with the gauge boson $K^{0\mu}$. In the former expressions Z^μ_0 coincides with the weak neutral current of the SM[6, 9]. Using Eqs. (4) and (5) we can read the gauge boson Y^μ associated with the $U(1)_Y$ hypercharge of the SM

$$Y^\mu = \left[\frac{T_W}{\sqrt{3}} A^\mu_8 + \sqrt{(1 - T^2_W/3)} B^\mu\right]. \qquad (6)$$

Equations (1-6) presented here are common to all the 3-3-1 gauge structures without exotic electric charges [5, 6, 7] as it is analyzed in Refs. [8, 9].

B. The spin 1/2 particle content

The quark content for the three families is the following [6]: $Q^i_L = (d^i, u^i, U^i)^T_L \sim (3, 3^*, 1/3)$, $i = 1, 2$, for two families, where U^i_L are two exotic quarks of electric charge 2/3 (the numbers inside the parenthesis stand for the $[SU(3)_c, SU(3)_L, U(1)_X]$ quantum numbers in that order); $Q^3_L = (u^3, d^3, D)^T_L \sim (3, 3, 0)$, where D_L is an exotic quark of electric charge $-1/3$. The right handed quarks are $u^{ac}_L \sim (3^*, 1, -2/3)$, $d^{ac}_L \sim (3^*, 1, 1/3)$ with $a = 1, 2, 3$, a family index, $U^{ic}_L \sim (3^*, 1, -2/3)$, $i = 1, 2$, and $D^c_L \sim (3^*, 1, 1/3)$.

The lepton content is given by [6] three $SU(3)_L$ triplets $L_{lL} = (\nu^0_l, l^-, E^-_l)^T_L \sim (1, 3, -2/3)$, for $l = e, \mu, \tau$ a lepton family index, and ν^0_l the neutrino field associated to the flavor l. The six lepton singlets are $l^+_L \sim (1, 1, 1)$, and $E^+_{lL} \sim (1, 1, 1)$. Notice in this model the presence of three exotic electrons E^-_l of electric charge -1 (used in what follows to implement the universal see-saw mechanism in the charged lepton sector), and the fact that it does not contain right-handed neutrinos. For this model, universality for the known leptons of the three families is present at tree level in the weak basis.

With the former quantum numbers it is just a matter of counting to check that the model is free of the following gauge anomalies[8]: $[SU(3)_c]^3$; (as in the SM $SU(3)_c$ is vectorlike); $[SU(3)_L]^3$ (six triplets and six anti-triplets), $[SU(3)_c]^2U(1)_X$; $[SU(3)_L]^2U(1)_X$; $[grav]^2U(1)_X$ and $[U(1)_X]^3$, where $[grav]^2U(1)_X$ stands for the gravitational anomaly as described in Ref. [19].

C. The new scalar sector

Instead of using the set of three triplets of Higgs scalars introduced in the original paper [6], or the most economical set of two triplets introduced in Ref. [9] (none of them able to produce a realistic mass spectrum), we propose here to start working with the following set of four Higgs scalar fields, and VEV:

$$\begin{aligned}
\langle\phi_1^T\rangle &= \langle(\phi_1^+,\phi_1^0,\phi_1^{'0})\rangle = \langle(0,0,v_1)\rangle \sim (1,3,1/3)\\
\langle\phi_2^T\rangle &= \langle(\phi_2^+,\phi_2^0,\phi_2^{'0})\rangle = \langle(0,v_2,0)\rangle \sim (1,3,1/3)\\
\langle\phi_3^T\rangle &= \langle(\phi_3^0,\phi_3^-,\phi_3^{'-})\rangle = \langle(v_3,0,0)\rangle \sim (1,3,-2/3)\\
\langle\phi_4^T\rangle &= \langle(\phi_4^+,\phi_4^0,\phi_4^{'0})\rangle = \langle(0,0,V)\rangle \sim (1,3,1/3),
\end{aligned}\tag{7}$$

with the hierarchy $v_1 \sim v_2 \sim v_3 \sim v \sim 10^2$ GeV $<< V$. Notice that the vacuum has been aligned arbitrarily such that $\langle\phi_1^0\rangle = \langle\phi_2^{\prime 0}\rangle = \langle\phi_4^0\rangle = 0$, in order to accomplish for the following facts:

- To have at the electroweak scale v an effective theory with properties resembling the two Higgs doublet extension of the SM.
- To properly implement several universal see-saw mechanisms [17].
- To avoid unnecessary mixing in the electroweak gauge boson sector [9].

The alternative of minimizing the scalar potential is a complicated and fruitless task at this stage of development of this particular model.

The set of scalars and VEV in Eq. (7) break the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ symmetry in two steps,

$$\begin{aligned}
&SU(3)_c \otimes SU(3)_L \otimes U(1)_X \stackrel{(V+v_1)}{\longrightarrow}\\
&SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \stackrel{(v_2+v_3)}{\longrightarrow} SU(3)_c \otimes U(1)_Q,
\end{aligned}$$

which allows for the matching conditions $g_2 = g_3$ and

$$\frac{1}{g_Y^2} = \frac{1}{g_1^2} + \frac{1}{3g_2^2}, \tag{8}$$

where g_2 and g_Y are the gauge coupling constants of the $SU(2)_L$ and $U(1)_Y$ gauge groups in the SM, respectively.

We will see in what follows that this scalar structure properly breaks the symmetry, provides with masses for the gauge bosons and, combined with a discrete symmetry, it is enough to produce a consistent mass spectrum for the charged fermion sectors (quarks and leptons). The mass spectrum for the neutral lepton sector requires new ingredients as it is going to be analyzed in Sec. III D.

D. $SU(6) \supset SU(5)$ as a covering group

The Lie algebra of $SU(3) \otimes SU(3) \otimes U(1)$ is a maximal subalgebra of the simple algebra of $SU(6)$. The five fundamental irreducible representations (irreps) of $SU(6)$ are: $\{6\}, \{6^*\}, \{15\}, \{15^*\}$ and the $\{20\}$ which is real. The branching rules for these fundamental irreps into $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ are [20]:

$$\begin{aligned}
\{6\} &\to (3,1,-1/3) \oplus (1,3,1/3),\\
\{15\} &\to (3^*,1,-2/3) \oplus (1,3^*,2/3) \oplus (3,3,0),\\
\{20\} &\to (1,1,1) \oplus (1,1,-1) \oplus (3,3^*,1/3)\\
&\quad \oplus (3^*,3,-1/3),
\end{aligned}$$

where we have normalized the $U(1)_X$ hypercharge according to our convenience.

From these branching rules and from the fermion structure presented in Sec. II B, it is clear that all the particles in the 3-3-1 model with exotic electrons, can be included in the following $SU(6)$ reducible representation

$$4\{6^*\} + 6\{20\} + 5\{15\} + 3\{15^*\}, \tag{9}$$

which, besides the particles in the representations already stated in the previous section, includes new exotic particles, as for example

$$\begin{aligned}
(N^0, E^+, E'^+)_L &\sim (1,3^*,2/3) \subset \{15\},\\
E_L^- &\sim (1,1,-1) \subset \{20\},\\
(D'^c, U'^c, U''^c)_L &\sim (3^*,3,-1/3) \subset \{20\}.
\end{aligned}$$

The analysis reveals that the reducible representation in (9) is free of anomalies (irrep $\{20\}$ is real and anomaly-free, and the anomaly of one $\{15\}$ is twice the anomaly of a $\{6\}$ [20]).

It is clear from the following decomposition of irrep $\{6^*\}$ of $SU(6)$ into $SU(5) \otimes U(1)$

$$\{6^*\} = \{d^c, N_E^0, E^-, N_E^{0c}\}_L \longrightarrow \{d^c, N_E^0, E^-\}_L \oplus N_{EL}^{0c}, \tag{10}$$

that for $N_{EL}^0 = \nu_{eL}$ and $E_L^- = e_L^-$, we obtain the known $SU(5)$ model of Georgi and Glashow [21]; so in some sense, the $SU(6)$ here is an extension of one of the first Grand Unified Theories (GUT) studied in the literature.

E. The gauge boson sector

After breaking the symmetry with $\langle\phi_i\rangle,\ i = 1,\ldots,4$, and using the covariant derivative for triplets $D^\mu = \partial^\mu - ig_3\lambda_L^\alpha A_\alpha^\mu/2 - ig_1XB_\mu I_3$, we get the following mass terms in the gauge boson sector.

1. Spectrum in the charged gauge boson sector

A straightforward calculation shows that the charged gauge bosons $K^{\pm}_{\mu}$ and $W^{\pm}_{\mu}$ do not mix with each other and get the following masses: $M^2_{K^\pm} = g_3^2(V^2+v_1^2+v_3^2)/2$ and $M_W^2 = g_3^2(v_2^2+v_3^2)/2$, which for $g_3 = g_2$ and using the experimental value $M_W = 80.423 \pm 0.039$ GeV (experimental values throughout the paper are taken from Ref. [22]) implies $\sqrt{v_2^2+v_3^2} \simeq 175$ GeV. In the same way $K^{0\mu}$ (and its antiparticle $\bar{K}^{0\mu}$) does not mix with the other two electrically neutral gauge bosons and gets a bare mass $M^2_{K^0} = g_3^2(V^2+v_1^2+v_2^2)/2 \approx M^2_{K^\pm}$. Notice that v_1 does not contribute to the $W^\pm$ mass because it is associated with an $SU(2)_L$ singlet Higgs scalar.

2. Spectrum in the neutral gauge boson sector

The algebra now shows that in this sector the photon field A_0^μ in Eq. (4) decouples from Z_0^μ and $Z_0^{\prime\mu}$ and remains massless. Then in the basis $(Z_0^\mu, Z_0^{\prime\mu})$ we obtain the following 2×2 mass matrix

$$\frac{\eta^2 g_3^2}{4C_W^2}\begin{pmatrix} \frac{v_2^2+v_3^2}{\eta^2} & \frac{v_2^2 C_{2W}-v_3^2}{\eta} \\ \frac{v_2^2 C_{2W}-v_3^2}{\eta} & v_2^2 C_{2W}^2 + v_3^2 + 4(V^2+v_1^2)C_W^4 \end{pmatrix}, \tag{11}$$

where $C_{2W} = C_W^2 - S_W^2$ and $\eta^{-2} = (3-4S_W^2)$. This matrix provides with a mixing between Z_0^μ and $Z_0^{\prime\mu}$ of the form

$$\tan(2\theta) = \frac{2\sqrt{(3-4S_W^2)}(v_2^2 C_{2W}-v_3^2)}{4C_W^4(V^2+v_1^2)-2v_3^2 C_{2W} - v_2^2(3-4S_W^2-C_{2W}^2)} \overset{V\to\infty}{\longrightarrow} 0. \tag{12}$$

The physical fields are then

$$\begin{aligned} Z_1^\mu &= Z_0^\mu \cos\theta - Z_0^{\prime\mu}\sin\theta\,, \\ Z_2^\mu &= Z_0^\mu \sin\theta + Z_0^{\prime\mu}\cos\theta. \end{aligned}$$

An updated bound on the mixing angle θ is going to be calculated in Sec. V using experimental results.

F. Currents

1. Charged currents

The Hamiltonian for the currents charged under the generators of $SU(3)_L$ is :

$$H^{CC} = g_3(W^+_\mu J^\mu_{W^+} + K^+_\mu J^\mu_{K^+} + K^0_\mu J^\mu_{K^0})/\sqrt{2} + H.c.,$$

where

$$\begin{aligned} J^\mu_{W^+} &= \left[\bar{u}^3_L\gamma^\mu d^3_L - (\sum_{i=1}^{2}\bar{u}^i_L\gamma^\mu d^i_L) + \sum_{l=e,\mu,\tau}\bar{\nu}_{lL}\gamma^\mu l^-_L\right] \\ J^\mu_{K^+} &= \left[(\sum_{i=1}^{2}\bar{U}^i_L\gamma^\mu d^i_L) - \bar{u}^3_L\gamma^\mu D_L + \sum_{l=e,\mu,\tau}\bar{\nu}^0_{lL}\gamma^\mu E^-_{lL}\right] \\ J^\mu_{K^0} &= \left[(\sum_{i=1}^{2}\bar{u}^i_L\gamma^\mu U^i_L) - \bar{D}_L\gamma^\mu d^3_L + \sum_{l=e,\mu,\tau}\bar{E}^-_{lL}\gamma^\mu l^-_L\right], \end{aligned}$$

where K^0_μ is an electrically neutral gauge boson, but it carries a kind of weak V-isospin charge, besides it is flavor nondiagonal.

2. Neutral currents

The neutral currents $J_\mu(EM)$, $J_\mu(Z)$ and $J_\mu(Z')$ associated with the Hamiltonian $H^0 = eA^\mu J_\mu(EM) + (g_3/C_W)Z^\mu J_\mu(Z) + (g_1/\sqrt{3})Z'^\mu J_\mu(Z')$ are

$$\begin{aligned} J_\mu(EM) &= \frac{2}{3}\left[\sum_{a=1}^{3}\bar{u}_a\gamma_\mu u_a + \sum_{i=1}^{2}\bar{U}^i\gamma_\mu U^i\right] \\ &- \frac{1}{3}\left[\sum_{a=1}^{3}\bar{d}^a\gamma_\mu d^a + \bar{D}\gamma_\mu D\right] \\ &- \sum_{l=e,\mu,\tau}(\bar{l}^-\gamma_\mu l^- + \bar{E}^-_l\gamma_\mu E^-_l) \\ &= \sum_f q_f \bar{f}\gamma_\mu f, \\ J_\mu(Z) &= J_{\mu,L}(Z) - S_W^2 J_\mu(EM), \\ J_\mu(Z') &= J_{\mu,L}(Z') + T_W J_\mu(EM), \end{aligned}$$

where $e = g_3 S_W = g_1 C_W\sqrt{(1-T_W^2/3)} > 0$ is the unit of electric charge, q_f is the electric charge of the fermion f in units of e, and $J_\mu(EM)$ is the electromagnetic current.

The left-handed currents are

$$\begin{aligned} J_{\mu,L}(Z) &= \frac{1}{2}[\sum_{a=1}^{3}(\bar{u}^a_L\gamma_\mu u^a_L - \bar{d}^a_L\gamma_\mu d^a_L) \\ &+ \sum_{l=e,\mu,\tau}(\bar{\nu}_{lL}\gamma_\mu\nu_{lL} - \bar{l}^-_L\gamma_\mu l^-_{lL})] \\ &= \sum_F \bar{F}_L T_{3f}\gamma_\mu F_L, \end{aligned} \tag{13}$$

$$\begin{aligned} J_{\mu,L}(Z') &= S_{2W}^{-1}[(\bar{u}_{1L}\gamma_\mu u_{1L} + \bar{u}_{2L}\gamma_\mu u_{2L} - \bar{d}_{3L}\gamma_\mu d_{3L} \\ &- \sum_l(\bar{\nu}_{lL}\gamma_\mu\nu_{lL})] \\ &+ T_{2W}^{-1}[(\bar{d}_{1L}\gamma_\mu d_{1L} + \bar{d}_{2L}\gamma_\mu d_{2L} - \bar{u}_{3L}\gamma_\mu u_{3L} \\ &- \sum_l(\bar{l}^-_L\gamma_\mu l^-_L)] \\ &- T_W^{-1}[(\bar{U}_{1L}\gamma_\mu U_{1L} + \bar{U}_{2L}\gamma_\mu U_{2L} - \bar{D}_L\gamma_\mu D_L \\ &- \sum_l(\bar{E}^-_{lL}\gamma_\mu E^-_{lL})] = \sum_F \bar{F}_L T'_{3f}\gamma_\mu F_L, \end{aligned} \tag{14}$$

where $S_{2W} = 2S_W C_W$, $T_{2W} = S_{2W}/C_{2W}$, $T_{3f} = Dg(1/2, -1/2, 0)$ is the third component of the weak isospin, $T'_{3f} = Dg(S_{2W}^{-1}, T_{2W}^{-1}, -T_W^{-1})$ is a convenient 3×3 diagonal matrix, acting both of them on the representation 3 of $SU(3)_L$ (the negative value when acting on the representation 3^*, which is also true for the matrix T_{3f}) and F is a generic symbol for the representations 3 and 3^* of $SU(3)_L$. Notice that $J_\mu(Z)$ is just the generalization of the neutral current present in the SM. This allows us to identify Z_μ as the neutral gauge boson of the SM, which is consistent with Eqs.(5) and (6).

The couplings of the flavor diagonal mass eigenstates Z_1^μ and Z_2^μ are given by:

$$\begin{aligned} H^{NC} &= \frac{g_3}{2C_W}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu[a_{iL}(f)(1-\gamma_5) \\ &\quad + a_{iR}(f)(1+\gamma_5)]f\} \\ &= \frac{g_3}{2C_W}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu[g(f)_{iV} - g(f)_{iA}\gamma_5]f\}, \end{aligned}$$

where

$$\begin{aligned} a_{1L}(f) &= \cos\theta(T_{3f} - q_f S_W^2) + \Theta\sin\theta(T'_{3f} - q_f T_W), \\ a_{1R}(f) &= -q_f\left(\cos\theta S_W^2 + \Theta\sin\theta T_W\right), \\ a_{2L}(f) &= \sin\theta(T_{3f} - q_f S_W^2) - \Theta\cos\theta(T'_{3f} - q_f T_W), \\ a_{2R}(f) &= -q_f\left(\sin\theta S_W^2 - \Theta\cos\theta T_W\right), \end{aligned} \tag{15}$$

where $\Theta = S_W C_W/\sqrt{(3-4S_W^2)}$. From this coefficients we can read

$$\begin{aligned} g(f)_{1V} &= \cos\theta(T_{3f} - 2q_f S_W^2) + \Theta\sin\theta(T'_{3f} - 2q_f T_W), \\ g(f)_{2V} &= \sin\theta(T_{3f} - 2q_f S_W^2) - \Theta\cos\theta(T'_{3f} - 2q_f T_W), \\ g(f)_{1A} &= \cos\theta T_{3f} + \Theta\sin\theta T'_{3f}, \\ g(f)_{2A} &= \sin\theta T_{3f} - \Theta\cos\theta T'_{3f}. \end{aligned} \tag{16}$$

The values of $g_{iV},\ g_{iA}$ with $i = 1, 2$ are listed in Tables I and II.

As can be seen, in the limit $\theta = 0$ the couplings of Z_1^μ to the ordinary leptons and quarks are the same as in the SM; due to this we can test the new physics beyond the SM predicted by this particular model.

III. FERMION MASSES AND MIXING

The Higgs scalars introduced in Sec. II C break the symmetry in a proper way and, at the same time, produce mass terms for the fermion fields via Yukawa interactions.

In order to restrict the number of Yukawa couplings, and produce an appealing mass spectrum, we introduce an anomaly-free discrete Z_2 symmetry [23] with the following assignments of charges:

$$\begin{aligned} Z_2(Q_L^a, \phi_2, \phi_3, \phi_4, u_L^{ic}, d_L^{ac}, E_{lL}^+) &= 1 \\ Z_2(\phi_1, u_L^{3c}, U_L^{ic}, D_L^c, L_{lL}, l_L^+) &= 0, \end{aligned} \tag{17}$$

where $a = 1, 2, 3,\ i = 1, 2$ and $l = e, \mu, \tau$ are family indices as above.

Before entering into details let us mention that in some cases we may use a negative mass entry or find a negative mass eigenvalue, which are not troublesome, because we can always exchange the sign of the quark mass either by a change of phase, or either by a transformation $\psi \longrightarrow \gamma_5\psi$ in the Weyl spinor ψ.

A. The up quark sector

The most general invariant Yukawa Lagrangean for the Up quark sector, without using the Z_2 symmetry, is given by

$$\begin{aligned} \mathcal{L}_Y^u &= \sum_{i=1}^{2}[\sum_{\alpha=1,2,4} Q_L^i \phi_\alpha C(\sum_a h_{ia}^{u\alpha} u_L^{ac} + \sum_{j=1}^{2} h_{ij}^{U\alpha} U_L^{jc})] \\ &\quad + Q_L^3\phi_3^* C(\sum_{i=1}^{2} h_i^U U_L^{ic} + \sum_{a=1}^{3} h_a^u u_L^{ac}) + h.c., \end{aligned} \tag{18}$$

where the $h's$ are Yukawa couplings and C is the charge conjugation operator. Then, in the basis $(u^1, u^2, u^3, U^1, U^2)$, and with the Z_2 symmetry enforced, we get from Eqs.(17,18), the following tree-level Up quark mass matrix:

$$M_u = \begin{pmatrix} 0 & 0 & h_{13}^{u2}v_2 & h_{11}^{U2}v_2 & h_{12}^{U2}v_2 \\ 0 & 0 & h_{23}^{u2}v_2 & h_{21}^{U2}v_2 & h_{22}^{U2}v_2 \\ 0 & 0 & h_3^u v_3 & h_1^U v_3 & h_2^U v_3 \\ h_{11}^{u1}v_1 & h_{12}^{u1}v_1 & h_{13}^{u4}V & h_{11}^{U4}V & h_{12}^{U4}V \\ h_{21}^{u1}v_1 & h_{22}^{u1}v_1 & h_{23}^{u4}V & h_{21}^{U4}V & h_{22}^{U4}V \end{pmatrix}, \tag{19}$$

which is a see-saw type mass matrix. As a matter of fact, the analysis shows that the matrix $M_u M_u^\dagger$, for all the Yukawa coupling constants of order one but different from each other, and $v_1 \approx v_2 \approx v_3 << V$, has the following set of eigenvalues: two of order V^2 associated with the two heavy exotic Up quarks, one of order v_3^2 associated with the top quark, and two see-saw eigenvalues of order $(v_i v_j/V)^2$ for $i, j = 1, 2, 3$, related somehow to the quarks u and c in the first two families.

Also notice from matrix (19) that the permutation symmetry $u^1 \leftrightarrow u^2$ imposed on the quarks of the first two families (which implies among other things that $h_{11}^{u1} = h_{12}^{u1} \equiv h_1^u$ and $h_{21}^{u1} = h_{22}^{u1} \equiv h_2^u$) conduces to a rank four see-saw type mass matrix, with the zero eigenvalue associated to the eigenstate $(1, -1, 0, 0, 0)/\sqrt{2}$. The $u^1 \leftrightarrow u^2$ symmetry is thus related, in the context of this model, with a massless up type quark, that we identify with the u quark in the first family.

In what follows, and without loss of generality, we are going to impose the condition $v_1 = v_2 = v_3 \equiv v << V$, with the value for v fixed by the mass of the charged weak gauge boson $M_{W^\pm}^2 = g_3(v_2^2 + v_3^2)/2 = g_3 v^2$ which implies $v \approx 123$ GeV. Also, and in order to avoid proliferation of

TABLE I: The $Z_1^\mu \longrightarrow \bar{f}f$ couplings.

f	$g(f)_{1V}$	$g(f)_{1A}$
$u^{1,2}$	$(\frac{1}{2}-\frac{4S_W^2}{3})\cos\theta - \Theta(T_{2W}^{-1}+\frac{4T_W}{3})\sin\theta$	$\frac{1}{2}\cos\theta - \Theta T_{2W}^{-1}\sin\theta$
u^3	$(\frac{1}{2}-\frac{4S_W^2}{3})\cos\theta + \Theta(s_{2W}^{-1}-\frac{4T_W}{3})\sin\theta$	$\frac{1}{2}\cos\theta + \Theta S_{2W}^{-1}\sin\theta$
$d^{1,2}$	$(-\frac{1}{2}+\frac{2S_W^2}{3})\cos\theta - \Theta(S_{2W}^{-1}-\frac{2T_W}{3})\sin\theta$	$-\frac{1}{2}\cos\theta - \Theta S_{2W}^{-1}\sin\theta$
d^3	$(-\frac{1}{2}+\frac{2S_W^2}{3})\cos\theta + \Theta(T_{2W}^{-1}+\frac{2T_W}{3})\sin\theta$	$-\frac{1}{2}\cos\theta + \Theta T_{2W}^{-1}\sin\theta$
$U^{1,2}$	$-\frac{4S_W^2}{3}\cos\theta + \Theta(T_W^{-1}-\frac{4T_W}{3})\sin\theta$	$\Theta T_W^{-1}\sin\theta$
D	$\frac{2S_W^2}{3}\cos\theta - \Theta(T_W^{-1}-\frac{2T_W}{3})\sin\theta$	$-\Theta T_W^{-1}\sin\theta$
$e^-,\ \mu^-,\ \tau^-$	$(-\frac{1}{2}+2S_W^2)\cos\theta + \Theta(T_{2W}^{-1}+2T_W)\sin\theta$	$-\frac{1}{2}\cos\theta + \Theta T_{2W}^{-1}\sin\theta$
$\nu_e,\ \nu_\mu,\ \nu_\tau$	$\frac{1}{2}\cos\theta + \Theta S_{2W}^{-1}\sin\theta$	$\frac{1}{2}\cos\theta + \Theta S_{2W}^{-1}\sin\theta$
$E_e^-,\ E_\mu^-,\ E_\tau^-$	$2S_W^2\cos\theta - \Theta(T_W^{-1}-2T_W)\sin\theta$	$-\Theta T_W^{-1}\sin\theta$

TABLE II: The $Z_2^\mu \longrightarrow \bar{f}f$ couplings.

f	$g(f)_{2V}$	$g(f)_{2A}$
$u^{1,2}$	$(\frac{1}{2}-\frac{4S_W^2}{3})\sin\theta + \Theta(T_{2W}^{-1}+\frac{4T_W}{3})\cos\theta$	$\frac{1}{2}\sin\theta + \Theta T_{2W}^{-1}\cos\theta$
u^3	$(\frac{1}{2}-\frac{4S_W^2}{3})\sin\theta - \Theta(S_{2W}^{-1}-\frac{4T_W}{3})\cos\theta$	$\frac{1}{2}\sin\theta - \Theta S_{2W}^{-1}\cos\theta$
$d^{1,2}$	$(-\frac{1}{2}+\frac{2S_W^2}{3})\sin\theta + \Theta(S_{2W}^{-1}-\frac{2T_W}{3})\cos\theta$	$-\frac{1}{2}\sin\theta + \Theta S_{2W}^{-1}\cos\theta$
d^3	$(-\frac{1}{2}+\frac{2S_W^2}{3})\sin\theta - \Theta(T_{2W}^{-1}+\frac{2T_W}{3})\cos\theta$	$-\frac{1}{2}\sin\theta - \Theta T_{2W}^{-1}\cos\theta$
$U^{1,2}$	$\frac{-4S_W^2}{3}\sin\theta - \Theta(T_W^{-1}-\frac{4T_W}{3})\cos\theta$	$-\Theta T_W^{-1}\cos\theta$
D	$\frac{2S_W^2}{3}\sin\theta + \Theta(T_W^{-1}-\frac{2T_W}{3})\cos\theta$	$\Theta T_W^{-1}\cos\theta$
$e^-,\ \mu^-,\ \tau^-$	$(-\frac{1}{2}+2S_W^2)\sin\theta - \Theta(T_{2W}^{-1}+2T_W)\cos\theta$	$-\frac{1}{2}\sin\theta - \Theta T_{2W}^{-1}\cos\theta$
$\nu_e,\ \nu_\mu,\ \nu_\tau$	$\frac{1}{2}\sin\theta - \Theta S_{2W}^{-1}\cos\theta$	$\frac{1}{2}\sin\theta - \Theta S_{2W}^{-1}\cos\theta$
$E_e^-,\ E_\mu^-,\ E_\tau^-$	$2S_W^2\sin\theta + \Theta(T_W^{-1}-2T_W)\cos\theta$	$\Theta T_W^{-1}\cos\theta$

unnecessary parameters at this stage of the analysis, we propose to start with the following simple mass matrix

$$M'_u = h_c v \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & h/h_c & 1 & 1 \\ 1 & 1 & \delta^{-1} & h\delta^{-1}/h_c & \delta^{-1} \\ 1 & 1 & \delta^{-1} & \delta^{-1} & \delta^{-1} \end{pmatrix}, \tag{20}$$

where $\delta = v/V$ is a perturbation expansion parameter and all the Yukawa coupling constants have been set equal to a common value h_c, except $h_3^u \equiv h$ which controls the top quark mass and $h_{11}^{U4} = h$ which simplifies the analysis and avoids democracy in the heavy sector.

The eigenvalues of $M'_u M'^{\dagger}_u$, neglecting terms of order δ^3 and higher are: a zero eigenvalue associated to the eigenvector $(1,-1,0,0,0)/\sqrt{2}$ that we identify with the up quark u in the first family, a see-saw eigenvalue $4h_c^2v^2\delta^2$ related to the eigenvector

$$[0,\eta,0,-(h-h_c)^2\delta,(h-h_c)^2\delta]/N + \mathcal{O}(\delta^2),$$

where $\eta = 1 + 2\delta(h+h_c)^2/(h-h_c)^2$ and N is a normalization factor, both values associated with the charm quark c in the second family; a tree-level value $(h-h_c)^2v^2/2 + \mathcal{O}(\delta^2)$ related to the eigenvector $[0,0,2\sqrt{2},-(h+h_c)\delta,(h+3h_c)\delta]/N'$ that we identify with the top quark t in the third family. There are also two heavy values $(h-h_c)^2V^2$ and $(2h_ch+4h^2)V^2+\mathcal{O}(\delta^2)$ associated with the two heavy states.

Using for for the top quark mass $m_t \approx 175$ GeV [22] we get $(h-h_c) \approx 2$, and using for the charm quark mass $m_c \approx 1.25$ GeV [22] we set the following bounds for the 3-3-1 mass scale: 2.5 TeV $\leq V \leq$100 TeV, for $0.1 \leq h_c \leq 4$, a Yukawa coupling constant in the perturbative regime.

The consistency of the model requires to find a mechanism able to produce a mass for the up quark u in the first family, mass which is protected by the symmetry $u^1 \leftrightarrow u^2$ between the quarks of the first two families. For this purpose the radiative mechanism [24] can be implemented by using the rich scalar sector of the model. As a matter of fact, the two radiative diagrams depicted in Fig. 1 (one for U^1 and another for U^2) can be extracted from the Lagrangean, where the mixing in the Higgs sector in the diagram comes from a term in the scalar potential of the form $f\phi_1\phi_2\phi_3$.

The contribution from the two diagrams in Fig. 1 is finite and it is

$$(M_u)_{ik} = fv_3N_{ik}[M^2m_1^2\ln(M^2/m_1^2) - \tag{21}$$

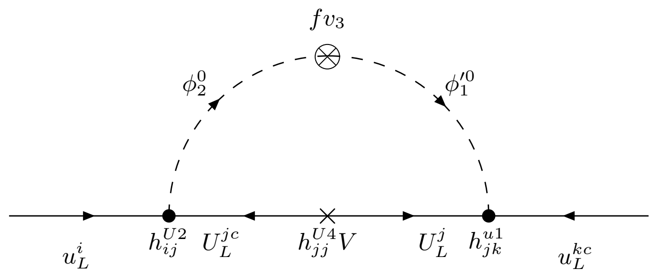

FIG. 1: One loop diagram contributing to the radiative generation of the up quark mass. $i, j, k = 1, 2$ are indexes for the first two families.

$$M^2 m_2^2 \ln(M^2/m_2^2) + m_1^2 m_2^2 \ln(m_1^2/m_2^2)],$$

where

$$N_{ik} = \frac{M \sum_j h_{ij}^{U2} h_{jj}^{U4} h_{jk}^{u1}}{[16\pi^2(m_2^2 - m_1^2)(M^2 - m_1^2)(M^2 - m_2^2)},$$

with $M \approx V$ the mass of the exotic Up quark U^j in the diagram, and m_1 and m_2 are the masses of $\phi_1'^0$ and ϕ_2^0 respectively. To estimate the contribution given by this diagram we assume the validity of the "extended survival hypothesis" (ESH) [25] which in our case means $m_1 \approx m_2 \approx v << V$, producing a value

$$(M_u)_{ik} \approx -\frac{f\delta \ln \delta}{8\pi^2} \sum_{j=1}^{2} h_{ij}^{U2} h_{jj}^{U4} h_{jk}^{u1},$$

which for the symmetry $u^1 \leftrightarrow u^2$ mentioned above implies a democratic type mass submatrix in the upper left 2×2 mass matrix M_u. So, in order to produce a mass different from zero for the up quark in the first family, this symmetry must be slightly broken. The simplest way found to accomplish this breaking is to set $h_{21}^{u1} = 1 + k_u$ and $h_{12}^{U2} = 1 - k_u$, with k_u a small parameter, and all the other Yukawa coupling constants as in matrix (20) (this is what we mean by "slightly broken"), with k_u related to the u quark mass m_u which thus becomes

$$m_u \approx -[\frac{(h+1)}{2}]^2 k_u^2 f\delta \ln \delta/(8\pi^2),$$

a positive value ($\ln \delta < 0$), which for $h \approx 1$, $V \approx 25$ TeV, and $v \simeq 123$ GeV implies $m_u \approx 0.3 k_u^2 f 10^{-3}$. So, a value of $f \approx v$ (as implied by the ESH [25])and $k_u = 0.2$ implies $m_u \approx 1.5$ MeV, without introducing a new mass scale, neither a hierarchy in the Yukawa coupling constants of the Up quark sector of this particular model.

B. The down quark sector

The most general Yukawa terms for the Down quark sector, using the four Higgs scalars introduced in Sec.(II B), are

$$\mathcal{L}_Y^d = \sum_{i=1}^{2} Q_L^i \phi_3 C(\sum_{a=1}^{3} h_{ia}^d d_L^{ac} + h_i'^D D_L^c) + Q_L^3 \sum_{\alpha=1,2,4} \phi_\alpha^* (h_\alpha^D D_L^c + \sum_{a=1}^{3} h_{a\alpha}^d d_L^{ac}) + h.c. \quad (22)$$

In the basis (d^1, d^2, d^3, D) and using the discrete symmetry Z_2, the former expression produces a 4×4 mass matrix with two zero eigenvalues, one see-saw eigenvalue associated with the bottom quark b in the third family, and a heavy eigenvalue of order V related with the exotic quark D. Unfortunately, the Z_2 symmetry used, allows to the right-handed ordinary Down quarks d_L^{ac} to couple only to ϕ_1 in a vertex where only Q_L^3 is present; as a consequence, any set of radiative diagrams able to provide mass terms to the Down quarks in the first two families, ends up in democratic type mass submatrices in the (d^1, d^2, d^3) subspace, and the rank of the mass matrix can not be changed.

The simplest way found to provide with masses for the down d and strange s quarks in the context of this model, is to add new ingredients. We proposse to add first an extra Down exotic quark D', with quantum numbers $D_L' \sim (3, 1, -1/3)$, $D_L'^c \sim (3^*, 1, 1/3)$ (which by the way do not affect the anomaly cancellation in the model because it belongs to a vectorlike representation). Also, and in order to implement the see-saw mechanism for this new exotic quark, we introduce a neutral scalar field $\phi_5^0 \sim (1, 1, 0)$ with VEV $< \phi_5^0 >= v_5 \approx v$ (which does not contribute to the $W^\pm$ mass). The Z_2 charges of the new fields are all zero.

With the new fields, and in the basis (d^1, d^2, d^3, D, D'), the following 5×5 mass matrix is obtained:

$$M_d = \begin{pmatrix} 0 & 0 & 0 & h_1'^D v_3 & h_{13}^{D'} v_3 \\ 0 & 0 & 0 & h_2'^D v_3 & h_{23}^{D'} v_3 \\ 0 & 0 & 0 & h_2^D v_2 & h_{32}^{D'} v_2 \\ h_{11}^d v_1 & h_{21}^d v_1 & h_{31}^d v_1 & h_4^D V & h_4^{D'} V \\ h_5^1 v_5 & h_5^2 v_5 & h_5^3 v_5 & h' M' & hM \end{pmatrix}, \quad (23)$$

where $M' \approx M$ are bare masses introduced by hand, that we set of the order of V.

The matrix M_d is again a see-saw type mass matrix, with the product $M_d M_d^\dagger$ having rank one. As the algebra shows, for the particular case $v_2 = v_3$, the eigenvector related to the zero eigenvalue is proportional to

$$[(h_2'^D h_{32}^{D'} - h_{23}^{D'} h_2^D), (h_{13}^{D'} h_2^D - h_1'^D h_{32}^{D'}), (h_1'^D h_{23}^{D'} - h_{13}^{D'} h_2'^D)].$$

In what follows and in order to simplify matters we are going to set again $v_1 = v_2 = v_3 = v_5 \equiv v$, start with all the Yukawa coupling constants equal to a common value h_b and, in order to avoid democracy in the heavy sector, we are going to assume conservation of the heavy flavor in the (D, D') basis, which means $h_4^{D'} = h' = 0$. With this assumptions we get an hermitian Down quark mass matrix with two zero eigenvalues related to the eigenvectors $(1,-1,0,0,0)/\sqrt{2}$ and $(1,1,-2,0,0)/\sqrt{6}$, that we identify with the down d and strange s quarks of the first two families. There is also for such a matrix a see-saw eigenvalue $6h_bv\delta$ associated with the eigenvector $(1,1,1,-3\delta,-3\delta)/\sqrt{3+18\delta^2}$ that we identify with the bottom quark b in the third family. The other two eigenvalues of the matrix are of order V.

Notice from this analysis that $m_b/m_c \approx 3h_b/h_c$ without a hierarchy between h_b and h_c.

The matrix M_d, with the constraints discussed in the previous section, can not either generate radiative masses for the quarks in the first two families, due to the flavor symmetry $d^1 \leftrightarrow d^2 \leftrightarrow d^3$ present. To generate masses for them such a symmetry must be broken. Working in this direction, let us partially break the symmetry, keeping at this stage the $d^1 \leftrightarrow d^2$ symmetry. This is achieved by putting all the Yukawa coupling constants equal to a common value h_b, except $h_{32}^{D'} = h_5^3 \equiv h_s = h_d(1+k_s)$, where k_s is a number smaller than one, related to the strange quark mass (when $k_s = 0$, $m_s = 0$).

The new orthogonal mass matrix generated in this way is a see-saw rank four mass matrix, with the zero eigenvalue related to the eigenvector $(1,-1,0,0,0)/\sqrt{2}$ associated with the down quark d in the first family. The two see-saw eigenvalues are

$$h_bv\delta(6+2k_s+k_s^2 \pm \sqrt{36+24k_s+8k_s^2+4k_s^3+k_s^4})/2,$$

producing $m_b \approx h_bv\delta(6+2k_s+k_s^2/3)$ and $m_s \approx 2h_bv\delta k_s^2/3$, which implies $k_s \approx 3\sqrt{m_s/m_b} \approx 0.47 \approx (h_s/h_b - 1)$, where $m_s \approx 120$ GeV and $m_b \approx 4.8$ GeV were used [22]. From the former analysis we get $h_s \approx 1.47h_b$ without a hierarchy between h_s and h_b.

Finally, radiative diagrams able to produce nonzero mass for the quark d in the first family, must be found. For this purpose the two diagrams in Fig. 2 can be extracted from the most general Lagrangean, where the scalar mixing are coming from terms in the scalar potential of the form $\lambda_{13}(\phi_1\phi_1^*)(\phi_3\phi_3^*)$ for the upper diagram and $\lambda_{35}(\phi_3\phi_3^*)(\phi_5\phi_5^*)$ for the lower one (two more diagrams using the u_L^3 mass entries $h_3^u v_3$ in the fermion propagator are of the same order of the two diagrams depicted in Fig 2, because the charged Higgs scalars mixing are proportional to $\lambda v_1 V$).

In order to avoid hierarchies in the coupling constants $\lambda_{13} \approx \lambda_{35} \approx 1$ is going to be used. Again, democracy in the first two families is avoided by breaking the $d^1 \leftrightarrow d^2$ symmetry which is achieved by using $h_1'^D \approx 1 - k_d$ and $h_{11}^d \approx 1 + k_d$, with k_d a small number of order 10^{-1}, related to the d quark mass by the relation:

$$m_d \approx -h_b^2k_d^2v\delta\ln(\delta)/4\pi^2 \approx 2\frac{h_b^2k_d^2}{h_U^2k_u^2}m_u,$$

which for $m_u \approx 3$ MeV and $m_d \approx 6$ MeV [22] implies $k_d \approx \sqrt{1.5}k_u$.

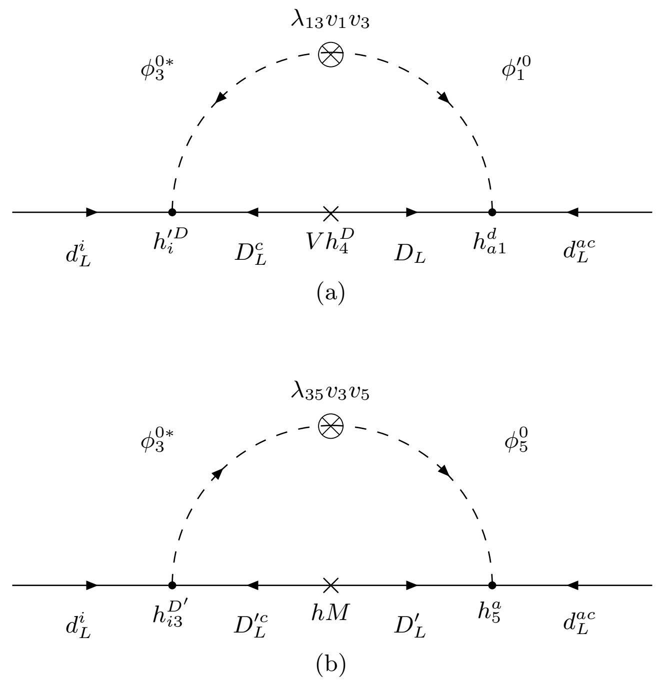

FIG. 2: One loop diagram contributing to the radiative generation of the down quark mass. As in the main text, $i = 1,2$ and $a = 1,2,3$ are family indexes.

C. The quark mixing matrix

For a model like the one studied here, the ordinary quark mixing matrix V_{mix} becomes the upper left 3×3 submatrix of the unitary 5×5 matrix $V = V_L^u V_L^{d\dagger}$, where V_L^u and V_L^d are unitary matrices that diagonalize $M_uM_u^\dagger$ and $M_dM_d^\dagger$ respectively. As a consequence, V_{mix} fails to be unitary, and special attention must be paid to the constraints coming from the experimental results which imply minimal mixing for the known quarks.

From the experimental side, the known results show that the 3×3 quark mixing matrix, parametrized as

$$V_{mix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \quad (24)$$

is almost diagonal, with measured values given by [22]

$$\begin{pmatrix} 0.9728 \pm 0.0030 & 0.2257 \pm 0.0021 & (36.7 \pm 4.7)\times 10^{-4} \\ 0.230 \pm 0.011 & 0.957 \pm 0.095 & (41.6 \pm 0.6)\times 10^{-3} \\ (1.0 \pm 0.1)\times 10^{-2} & (41.0 \pm 3.0)\times 10^{-3} & > 0.78 \end{pmatrix}$$

(notice that we are quoting the most uncertain direct measured values and not the values constrained by the unitary of V_{CKM}, the Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix [26]).

One more complication in the frame of this model comes from the fact that our mass matrices may be flavor democratic in the Down quark sector but not in the Up quark sector, due to the three non zero tree-level top quark mass entries present in (19). By fortune, the model is full of free parameters and this last inconvenience can be circumvented by letting $h_{11}^{U4}, h_{12}^{U4}, h_{21}^{U4}$ and h_{22}^{U4} to become free parameters in the interval $0.1 \leq |h_{ij}^{U4}| \leq 4,\ i,j=1,2$. The numerical analysis shows that $h_{12}^{U4} = h_{21}^{U4} = 0$ instead of one, is a more appropriate set of values in order to reproduce the experimental values of V_{mix}; unfortunately, the analytical results are not quite so neat for this last set of values, as compared with the previous quoted results.

The analysis also shows that violation of unitary in this model is proportional to δ^2 and so, a large 3-3-1 scale, should reproduce fairly well the measured experimental results. The numerical analysis shows that, for $h_{12}^{U4} = h_{21}^{U4} = 0,\ h_{11}^{U4} = h_{22}^{U4} = 1$ and $V \approx 100$ TeV, with the other parameters as in the two previous sections, reproduce not only the experimental quoted values for V_{mix}, but also all the unitary constrained values of the V_{CKM} mixing matrix. Lowering down the 3-3-1 scale to 60 TeV, keeping all the other parameters as above, reproduces also all the experimental unitary constraints values of V_{CKM}, except V_{td} which turns out to be $(1.3 \pm 0.3) \times 10^{-2}$ (statistical error) instead of the unitary value of $(8.14{+}0.32{-}0.64)\times 10^{-3}$ quoted in Ref. [22], which predicts a β angle 1.2 larger in the $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ unitary triangle, result reflected already indirectly in the large $B^0 - \bar{B}^0$ mixing measured at the B-factories [22]. But such a large 3-3-1 scale is a price too high to be payed, and renders at the end with a model unable to be tested in the upcoming generation of accelerators.

What we proposse at this point is to perform a numerical analysis using $h_{12}^{U4},\ h_{21}^{U4},\ h_{11}^{U4},\ h_{22}^{U4},\ h_b,\ k_u,\ k_s$ and k_d as aleatory variables, with all the other Yukawa coupling constants equal to one, except h which fixes the top quark mass, h_c which fixes the 3-3-1 mass scale and h_b which fixes the bottom quark mass. The analysis is constrained by the six quark mass values and the experimental measured values of V_{mix}, but not by the values obtained by imposing unitary in the V_{CKM} mixing matrix. Then we look for the predictions of the model.

The ramdon numerical analysis using Mathematica Monte Carlo subroutines showed that, at the 3-3-1 scale of 10 TeV, the following set of parameters $h_{12}^{U4} = h_{21}^{U4} = 0.26,\ h_{11}^{U4} = h_{22}^{U4} = -0.96,\ k_u = -0.15,\ k_s = 0.38$ and $k_d = 0.17$, reproduces the values of the V_{CKM} with unitary constraints, except for three of them: $V_{td} = (1.1 \pm 0.2) \times 10^{-2}$, $V_{ub} = (45.8 \pm 5) \times 10^{-4}$ and $V_{cb} = (40.2 \pm 0.8) \times 10^{-3}$ (all the errors are statistical), which implies a large $B^0 - \bar{B}^0$ mixing coming from V_{td} and a depletion of the branching decay $b \to s\gamma$ coming from V_{cb}; decay described by the magnetic dipole transition which is proportional to[27] $M_{b\to s\gamma} \sim V_{cb}V_{cs}^*$, with a value of $(42.21 + 0.10 - 0.80) \times 10^{-3}$ quoted for V_{cb} in Ref. [22].

D. The charged lepton sector

The most general Yukawa terms for the charged lepton sector, without using the Z_2 symmetry, is

$$\mathcal{L}_Y^l = \sum_{\alpha=1,2,4}\sum_{l,l'=e,\mu,\tau} L_{lL}\phi_\alpha^* C(h_{ll'}^{E\alpha}E_{l'}^+ + h_{ll'}^{e\alpha}l'^+)_L + h.c., \tag{25}$$

which in the basis $(e,\ \mu,\ \tau,\ E_e,\ E_\mu,\ E_\tau)$ and with the discrete symmetry in Eq. (17) enforced, produces the following 6×6 mass matrix

$$M_e = \begin{pmatrix} 0 & 0 & 0 & h_{ee}^{E2}v_2 & h_{e\mu}^{E2}v_2 & h_{e\tau}^{E2}v_2 \\ 0 & 0 & 0 & h_{\mu e}^{E2}v_2 & h_{\mu\mu}^{E2}v_2 & h_{\mu\tau}^{E2}v_2 \\ 0 & 0 & 0 & h_{\tau e}^{E2}v_2 & h_{\tau\mu}^{E2}v_2 & h_{\tau\tau}^{E2}v_2 \\ h_{ee}^{e1}v_1 & h_{e\mu}^{e1}v_1 & h_{e\tau}^{e1}v_1 & h_{ee}^{E4}V & h_{3e\mu}^{E4}V & h_{e\tau}^{E4} \\ h_{\mu e}^{e1}v_1 & h_{\mu\mu}^{e1}v_1 & h_{\mu\tau}^{e1}v_1 & h_{\mu e}^{E4}V & h_{\mu\mu}^{E4}V & h_{\mu\tau}^{E4}V \\ h_{\tau e}^{e1}v_1 & h_{\tau\mu}^{e1}v_1 & h_{\tau\tau}^{e1}v_1 & h_{\tau e}^{E4}V & h_{\tau\mu}^{E4}V & h_{\tau\tau}^{E4}V \end{pmatrix} \tag{26}$$

where again $v_1 = v_2 = v_3 = v << V$ is going to be used. Assuming for simplicity conservation of the family lepton number in the exotic sector ($h_{ll'}^{E4} = h_l\delta_{ll'}$ which does not affect at all the main results), the matrix (26) still remains with 21 Yukawa coupling constants and it is full of physical possibilities. For example, if all the 21 Yukawa coupling constants are different to each other (but of order one), we have that $M_eM_e^\dagger$ is a rank zero mass matrix, with three eigenvalues of order V^2 and three see-saw eigenvalues of order $v^2\delta^2$.

To start the analysis let us imposes the symmetry $e \leftrightarrow \mu \leftrightarrow \tau$, make all the Yukawa coupling constants equal to a common value h_τ and use conservation of the family lepton number in the exotic sector. With these assumptions the following orthogonal mass matrix is obtained:

$$M'_e = h_\tau v \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & \delta^{-1} & 0 & 0 \\ 1 & 1 & 1 & 0 & \delta^{-1} & 0 \\ 1 & 1 & 1 & 0 & 0 & \delta^{-1} \end{pmatrix}, \tag{27}$$

which is a symmetric rank four see-saw mass matrix, with the six eigenvalues given by

$$h_l v[0, 0, (\delta^{-1} \pm \sqrt{36 + \delta^{-2}})/2, V, V], \tag{28}$$

with the two zero eigenvalues related to the null subspace $(1,-1,0,0,0,0)/\sqrt{2}$ and $(1,1,-2,0,0,0)/\sqrt{6}$ that we identify in first approximation with the electron and the muon states (resembling the Down quark sector). Equations (27) and (28) implies that the τ lepton may be identify approximately with the vector $(1,1,1,0,0,0)/\sqrt{3}$, up to mixing with the heavy exotic leptons [the exact eigenvector is $(1,1,1,-3\delta,-3\delta,-3\delta)/\sqrt{3+27\delta^2}$], with a

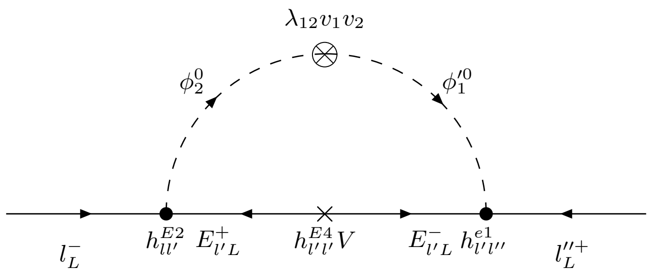

FIG. 3: One loop diagram contributing to the radiative generation of the electron mass.

mass value $-m_\tau \approx 9h_\tau v\delta = 4.5h_\tau m_c/h_u$, which for $m_\tau \approx 1.777$ GeV implies the relationship $h_u \approx 3h_\tau$.

The next step is to break the $e \leftrightarrow \mu \leftrightarrow \tau$ symmetry but just in the τ sector, keeping for a while the $e \leftrightarrow \mu$ symmetry. This is simple done by letting $h^{E2}_{\tau\tau} = h^{e1}_{\tau\tau} \equiv h_\mu \neq 1$ but of order one, with all the other Yukawa coupling consants as in Eq. (27). We thus get a rank five orthogonal mass matrix, with two see-saw eigenvalues, and a zero mass eigenstate related to the eigenvector $(1,-1,0,0,0,0)/\sqrt{2}$ that we identify with the electron state. The two see-saw eigenvalues, neglecting terms of $\mathcal{O}(\delta^2)$, are given by

$$\frac{h_\tau v\delta}{2}[8+(\frac{h_\mu}{h_\tau})^2 \pm (2+\frac{h_\mu}{h_\tau}\sqrt{12-4(h_\mu/h_\tau)+(h_\mu/h_\tau)^2}]. \tag{29}$$

Using for $m_\tau \approx 1777$Gev and $m_\mu \approx 107.7$ Mev [22] we get $h_\mu \approx 2.87h_\tau$, which in turn implies $m_\tau \approx 15.3h_\tau v\delta \approx 7.6h_\tau m_c/h_u$.

Again, radiative corrections able to generate masses to the electron must be found. For that purpose the three diagrams in Fig. 3 can be extracted from the Lagrangian (one for each exotic charged lepton), where the mixing in the Higgs sector is coming from a term in the scalar potential of the form $\lambda_{12}(\phi_1\phi_1^*)(\phi_2\phi_2^*)$. There are two more diagrams coming from the terms $f\phi_1\phi_2\phi_3$ and $f'\phi_1\phi_3\phi_4$ which are proportional to $v_3(f'-f)$ that can be neglected under the assumption $f' = f \approx v$.

The contribution given by this diagram, again under the assumption of validity of the ESH [25] is

$$(M_e')_{ll'} \approx \frac{\lambda_{12}\delta\ln\delta}{8\pi^2}\sum_{l'} h^{E2}_{ll'}h^{e1}_{ll''},$$

that for the particular values of the Yukawa coupling constants in matrix (27) generate a democratic mass submatrix in the 2×2 upper left corner of M_e'. Again, the alternative we have at hand is to softly break the $e \leftrightarrow \mu$ symmetry present in the mass matrix (27). This is achieved by letting $h^{E2}_{ee} \approx 1-k_e$ and $h^{e1}_{ee} \approx 1+k_e$, with $k_e \sim 10^{-1}$ as before.

The evaluation of the diagram in Fig. 3 gives

$$m_e \approx -\frac{\lambda_{12}v\delta\ln\delta k_e^2}{4\pi^2},$$

which for $m_e = 0.51$ MeV [22], $\lambda_{12} \approx 1$, $V \approx 25$ TeV, and $v = 124$ GeV, produces a value of $k_e \approx 0.08$, in agreement with our original assumption.

E. The neutral lepton sector

With the particle content introduced so far there are not tree-level mass terms for the neutrinos. Masses for the neutral lepton sector are obtained only by enlarging the model with extra fields, which may implement one or several of the following mechanisms:

1. *Tree-Level masses*

In the context of the model studied here, tree-level masses for neutrinos can be generated only by introducing scalar fields belonging to irrep $\{6^*\}$ of $SU(3)_L$. These scalars can be written as the 3×3 symmetric tensor

$$\chi_{\{\alpha,\beta\}} = \begin{pmatrix} \chi_{11}^{-4/3+X} & \chi_{12}^{-1/3+X} & \chi_{13}^{-1/3+X} \\ & \chi_{22}^{2/3+X} & \chi_{23}^{2/3+X} \\ & & \chi_{33}^{2/3+X} \end{pmatrix} \sim (1,6^*,X), \tag{30}$$

where the upper symbol stands for the electric charge. Clearly, a VEV of the form $\langle\chi_{11}(1,6^*,4/3)^0\rangle \sim \omega$ is able to produce the following Majorana mass terms: $h^\nu_{l'l}\omega\nu^0_{l'}\nu^0_{lL}$. If so, $h^\nu_{l'l}$ must become very small numbers, or either ω must be a new very small mass scale in order to cope with the experimental constraints [2], implying for the model a hierarchy in the Yukawa coupling constants, or either the introduction of a new mass scale for the model.

2. *See-Saw masses*

The see-saw mechanism can be implemented in the model by adding a singlet, electrically neutral Weyl spinor $N^0_L \sim (1,1,0)$ with Z_2 charge 1, which picks up a tree-level mass value $V'N^0_LN^0_L$ with V' an undetermined mass scale. Then, a Yukawa Lagrangian of the form

$$\sum_l h_l L_{lL}\phi_3^* N_L^0,$$

will produce a see-saw type mass matrix

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & h_ev_3 \\ 0 & 0 & 0 & h_\mu v_3 \\ 0 & 0 & 0 & h_\tau v_3 \\ h_ev_3 & h_\mu v_3 & h_\tau v_3 & V' \end{pmatrix}, \tag{31}$$

which has two nonzero tree-level mass eigenvalue $V' \pm \sqrt{V'^2+4(h_e^2+h_\mu^2+h_\tau^2)v_3^2}$, one of them of the see-saw type and proportional to v_3^2/V' which for a convenient

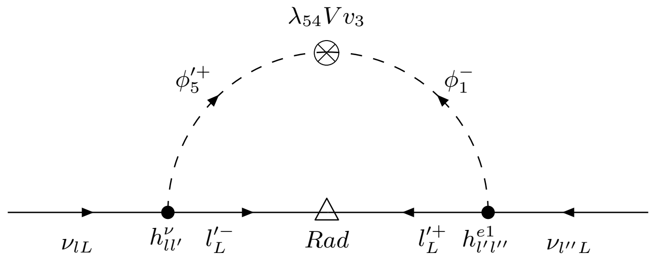

FIG. 4: Loop diagrams contributing to the radiative generation of Majorana masses for the neutrinos.

larger value of V' (again a new mass scale) produces a small neutrino mass. Of course, this mechanism alone is not enough to explain the spectrum because two neutrinos will remain massless, something which is ruled out by experimental results [2].

3. *Radiative masses*

Radiative Majorana masses for the neutrinos are generated when a new scalar triplet $\phi_5 = (\phi_5^{++}, \phi_5^+, \phi_5'^+) \sim (1, 3, 4/3)$ is introduced, with a Z_2 charge equal to zero (notice that $\langle\phi_5\rangle \equiv 0$). This new scalar triplet couple to the spin 1/2 leptons via a term in the Lagrangian of the form:

$$\mathcal{L} = \sum_{ll'} h^\nu_{ll'} L_{lL} L_{l'L} \phi_5 = \sum_{ll'} h^\nu_{ll'} [\phi_5^{++} (l_L^- E^-_{l'L} - l_L'^- E^-_{lL}) + \phi_5^+ (E^-_{lL} \nu_{l'L} - E^-_{l'L} \nu_{lL}) + \phi_5'^+ (\nu_{lL} l_L'^- - \nu_{l'L} l_L^-)],$$

for $l \neq l' = e, \mu, \tau$.

Using ϕ_5, the following terms in the scalar potential Lagrangian are allowed by the Z_2 discrete symmetry

$$\lambda_{51}(\phi_5.\phi_1^*)(\phi_3.\phi_2^*);\ \lambda_{52}(\phi_5.\phi_1^*)(\phi_3.\phi_4^*);$$

$$\lambda_{53}(\phi_5.\phi_2^*)(\phi_3.\phi_1^*);\ \lambda_{54}(\phi_5.\phi_4^*)(\phi_3.\phi_1^*).$$

The former expressions allow to draw the radiative diagram depicted in Fig. 4, which is the only diagram available for the radiative mechanism in the neutral lepton sector.

Notice by the way that this diagram is already a second order radiative diagram because its charged lepton mass insertion is already a first order radiative correction (see the diagram in Fig. 3) and its value is smaller than the value produced by any other radiative diagram already studied in this paper. Attempts to draw a diagram with the exotic heavy leptons in the fermion propagator became fruitless, due to the Z_2 symmetry introduced in the analysis [a term like $(\phi_5.\phi_2^*)(\phi_3.\phi_4^*)$ is not Z_2 allowed!].

4. *The Zee-Babu mechanism*

Introducing a new $SU(3)_L$ singlet, electrically charged scalar, as it is done for example in Ref. [15], the two loop diagram of the Zee-Babu mechanism [28] can be included in the context of this model.

Without going into further details, let us say that the neutrino mass spectrum is outside the scope of the present analysis.

IV. GAUGE COUPLING UNIFICATION

In a field theory, the coupling constants are defined as effective values, which are energy scale dependent according to the renormalization group equation. In the modified minimal substraction scheme [29], which we adopt in what follows, the one loop renormalization group equation (RGE) for $\alpha = g^2/4\pi$ is given by

$$\mu \frac{d\,\alpha}{d\,\mu} \simeq -b\alpha^2, \tag{32}$$

where μ is the energy at which the coupling constant α is evaluated. The constant value b, called the beta function, is completely determined by the particle content of the model by

$$2\pi b = \frac{11}{6} C(\text{vectors}) - \frac{2}{6} C(\text{fermions}) - \frac{1}{6} C(\text{scalars}),$$

where $C(\ldots)$ is the group theoretical index of the representation inside the parentheses (we are assuming Weyl fermions and complex scalar fields [20]).

For the energy interval $m_Z < \mu < M_G$, the one loop solutions to the RGE (32) for the three SM gauge coupling constants are

$$\alpha_i^{-1}(m_Z) = \frac{\alpha_i^{-1}(M_G)}{c_i} - b_i(F, H) \ln\left(\frac{M_G}{m_Z}\right), \tag{33}$$

where $i = Y, 2, c$ refers to the coupling constants of $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ respectively, with the beta functions given by

$$2\pi \begin{pmatrix} b_Y \\ b_2 \\ b_c \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{22}{3} \\ 11 \end{pmatrix} - \begin{pmatrix} \frac{20}{9} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} F - \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix} H, \tag{34}$$

where F is the number of families contributing to the beta functions and H is the number of low energy $SU(2)_L$ scalar field doublets ($H = 1$ for the SM). In Eq. (33) the constants c_i are group theoretical factors which depend upon the embedding of the SM factors into a covering group, and warrant the same normalization for the covering group G and for the three group factors in the SM. For example, if the covering group is $SU(5)$, then $(c_Y, c_2, c_c) = (3/5, 1, 1)$, but they are different for other covering groups (see for example the Table in Ref. [30]).

The three running coupling constants α_i in Eq. (33), may or may not converge into a single energy GUT scale M_G; if they do, then $\alpha_i(M_G) = \alpha$ is a constant independent of the index i. Now, for a given embedding into a fixed covering group, the c_i values are fix, and if we use for $F = 3$ (an experimental fact) and $H = 1$ as in the SM, then Eqs. (33) constitute a set of three equations with two unknowns, α and M_G which may or may not have a consistent solution (more equations than unknowns).

The inputs to be used in Eq. (33) for $\alpha_i^{-1}(m_Z)$ are calculated from the experimental results [22]

$$
\begin{aligned}
\alpha_{em}^{-1}(m_Z) &= \alpha_Y^{-1}(m_Z) + \alpha_2^{-1}(m_Z) \\
&= 127.918 \pm 0.018 \\
\sin^2\theta_W(m_Z) &= 1 - \alpha_Y^{-1}(m_Z)\alpha_{em}(m_Z) \\
&= 0.23120 \pm 0.00015 \\
\alpha_c(m_Z) &= 0.1213 \pm 0.0018,
\end{aligned}
$$

which imply $\alpha_Y^{-1}(m_Z) = 98.343 \pm 0.036$, $\alpha_2^{-1}(m_Z) = 29.575 \pm 0.054$, and $\alpha_c^{-1}(m_Z) = 8.244 \pm 0.122$.

It is a well known fact that the model based on the non supersymmetric $SU(5)$ group of Georgi and Glashow [21] lacks of gauge coupling unification because M_G calculated from the RGE is not unique in the range 10^{14} GeV $\leq M_G \leq 10^{16}$ GeV, predicting for the proton lifetime τ_p a value between 2.5×10^{28} years and 1.6×10^{30} years, values that are ruled out by experimental measurements [31]. If we introduce one more free parameter in the solutions to the RGE as for example letting H to become a free integer number, then we have now three unknowns with three equations that always have mathematical solution (not necessarily with physical meaning). Doing that in Eqs. (33) we find that for $H = 7$ (seven Higgs doublets) we get the unique solution $M_G = 10^{13}$ GeV $>> m_Z$ which, altough a physical solution, it is ruled out by the proton lifetime. So, if we still want unification, new physics at an intermediate mass scale M_V such that $m_Z < M_V < M_G$ must exists, being supersymmetry (SUSY) a popular candidate for that purpose [31].

The question now is if the 3-3-1 model under consideration in this paper, introduces an intermediate mass scale M_V such that it achieves proper gauge coupling unification, being an alternative for SUSY. To answer this question using $SU(6)$ as the covering group as presented in Section II D, we must solve the following set of seven equations:

$$
\begin{aligned}
\alpha_i^{-1}(m_Z) &= \frac{\alpha_i^{-1}(M_V)}{c_i} - b_i(F,H)\ln\left(\frac{M_V}{m_Z}\right) \\
\alpha_j^{-1}(M_V) &= \frac{\alpha^{-1}}{c'_j} - b'_j\ln\left(\frac{M_G}{M_V}\right) \\
\alpha_Y^{-1}(M_V) &= \alpha_1^{-1}(M_V) + \alpha_3^{-1}(M_V)/3,
\end{aligned} \tag{35}
$$

where the last equation is just the matching conditions in Eq.(8), and $i = Y, 2, c$ and $j = 1, 3, c$ for the SM and the 3-3-1 model, respectively. The constants c_i are $(c_Y, c_2, c_3) = (3/5, 1, 1)$ as before, and $(c'_1, c'_3, c'_c) = (3/4, 1, 1)$, with the value $c'_1 = 3/4$ calculated from the electroweak mixing angle in Eq. (3). b'_j stand for the beta functions for the 3-3-1 model under study here.

Eqs. (35) constitute a set of seven equations with seven unknowns α, $\alpha_j(M_V)$, M_V, M_G and $\alpha_Y(M_V)$ $[\alpha_2(M_V) = \alpha_3(M_V)$ according to the matching conditions]. There is always mathematical solution to this set of equations, but we want only physical solutions, that is solutions such that $m_Z < M_V < M_G$.

The new beta functions calculated with the particle content introduced in Sections II A, II B, II C and III B (it includes the new exotic Down quark D') are:

$$
2\pi\begin{pmatrix} b'_1 \\ b'_3 \\ b'_c \end{pmatrix} = \begin{pmatrix} 0 - 12 - 11/9 \\ 11 - 4 - 4/6 \\ 11 - 20/3 - 0 \end{pmatrix} = \begin{pmatrix} -119/9 \\ 19/3 \\ 13/3 \end{pmatrix}, \tag{36}
$$

where in the middle term we have separated the contributions coming from the gauge bosons, the fermion fields and the scalar fields in that order. When we introduce these values in Eq. (35) we do not obtain a physical solution in the sense that we get $m_Z < M_G < M_V$.

Of course, if there are more particles at the 3-3-1 mass scale then the beta functions given in Eqs. (36) are not the full story. In particular we know from Sec. III D that at least new Higgs scalars are needed in order to generate a consistent lepton mass spectrum, so let us allow the presence in our model of the following Higgs scalar multiplets at the 3-3-1 mass scale: $N_X^{(1)}$ $SU(3)_L$ singlets (with $U(1)_X$ hypercharge equal to X), $N_X^{(3)}$ triplets (color singlets), $\tilde{N}_X^{(3)}$ leptoquark triplets (color triplets) and $N_X^{(6)}$ sextuplets (color singlets). These new particles contribute to the beta functions b'_j with the extra values:

$$
2\pi\begin{pmatrix} b'_1 \\ b'_3 \\ b'_c \end{pmatrix} = -\begin{pmatrix} \sum_X X^2 f(N_X^{(6)}, \tilde{N}_X^{(3)}, N_X^{(3)}, N_X^{(0)}) \\ \frac{1}{6}\sum_X (N_X^{(3)} + 3\tilde{N}_X^{(3)} + 5N_X^{(6)}) \\ \sum_X \tilde{N}_X^{(3)}/2 \end{pmatrix}, \tag{37}
$$

where the function $f(\ldots)$ is $f(N_X^{(6)}, \tilde{N}_X^{(3)}, N_X^{(3)}, N_X^{(0)}) = (2N_X^{(6)} + 3\tilde{N}_X^{(3)} + N_X^{(3)} + N_X^{(0)}/3)$; with this new $SU(3)_L$ multiplets contributing or not to the beta functions b_i of the SM factor groups, in agreement with the extended survival hypothesis [25] (for example, a sextuplet with a VEV $\langle\chi_{11}(1, 6^*, 4/3)\rangle \sim \omega$ contributes as an $SU(2)_L$ doublet in b_Y and b_2, etc.).

The calculation shows that for the following set of extra scalar fields which do not develop VEV: $N_X^{(0)} = 0$, $N_{1/3}^{(3)} = 1$, $N_{-2/3}^{(3)} = 1$, $\tilde{N}_X^{(3)} = 0$ $N_0^{(3)} = 8$ and $N_0^{(6)} = 15$, the set of equations in (37) has the physical solution

$$
M_V \approx 2.0\text{TeV} < M_G \approx 3.0 \times 10^7\text{GeV}, \tag{38}
$$

which provides with a convenient 3-3-1 mass scale, and a low unification GUT mass scale, as it is shown in Fig. 3.

But, is this low GUT scale in conflict with proton decay? The answer is not, because due to the Z_2 symmetry our unifying group is $SU(6) \times Z_2$. Then we must assign to each irrep of $SU(6)$ in Eq. (9) a given Z_2 value in

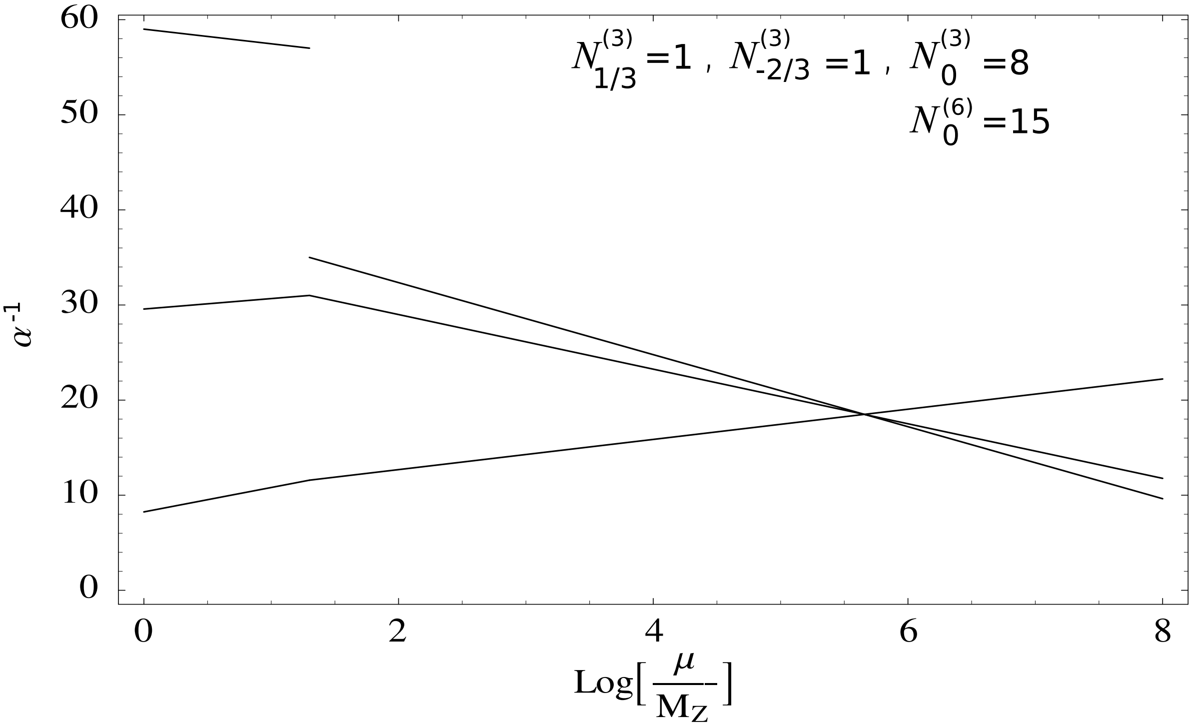

FIG. 5: Solutions to the RGE for the 3-3-1 model. For the meaning of $N_X^{(r)}$ see the main text.

accord with the Z_2 value assigned to the 3-3-1 states in Eq. (17). For example, if we assign to one of the four $\{6^*\} = \{D^c, -N_E^0, E^-, N_E^{0c}\}_L$ states in (9) a Z_2 value equal to 1, then we can perfectly identify D_L^c with one of the ordinary down quarks $(d^c, s^c, b^c)_L$, but then $(-N_E^0, E^-, N_E^{0c})_L$ can not correspond to $(-\nu_l^0, l^-, \nu_l^{0c})_L$ because all of them have a Z_2 value equal to zero; and the same for the other way around. As a consequence, the down quark d_L^c can not live together with $(\nu_e, e^-)_L$ in the same $SU(6)\times Z_2$ irrep, and the proton can not decay to light states belonging to the weak basis. The decay can of course occur via the mixing of ordinary 3-3-1 states with the extra new states in $SU(6)$, but such a mixing is of the order of $(M_V/M_G)^2$ which is a very small value. Of course, this argument is valid as far as we can find a mechanism able to produce GUT scale masses for all the extra states, but such analysis is outside the present work.

V. CONSTRAINTS ON THE PARAMETERS

In this section we are going to set bounds on the mass of the new neutral gauge boson Z_2^μ, and its mixing angle with the ordinary neutral gauge boson Z_1^μ. We also are going to set constraints coming from unitary violation of the quark mixing matrix and the possible existence of FCNC effects.

A. Bounds on M_{Z_2} and θ.

The diagonalizing of the quark mass matrices presented in sections (III A) and (III B) allow us to identify the mass eigenstates as a function of the flavor states. This information is going to be used next, in order to set proper bounds for $\sin\theta$, the mixing angle between the two neutral currents, and M_{Z_2}, the mass of the new neutral gauge boson. In the analysis we are going to include the c and b quark couplings to Z_1^μ, values measured with good accuracy at the Z pole from CERN e^+e^- collider (LEP) [22]. Experimental measurements from the SLAC Linear Collider (SLC), and atomic parity violation are also going to be taken into account. The set of experimental constraints used are presented in Table III.

The expression for the partial decay width for $Z_1^\mu \to f\bar{f}$, including only the electroweak and QCD virtual corrections is

$$\begin{aligned}\Gamma(Z_1^\mu \to f\bar{f}) &= \frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}}\rho\Big\{\frac{3\beta-\beta^3}{2}[g(f)_{1V}]^2 \\ &+ \beta^3[g(f)_{1A}]^2\Big\}(1+\delta_f)R_{EW}R_{QCD}, \quad (39)\end{aligned}$$

where f is an ordinary SM fermion, Z_1^μ is the physical gauge boson observed at LEP, $N_C = 1$ for leptons while for quarks $N_C = 3(1+\alpha_s/\pi + 1.405\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3)$, where the 3 is due to color and the factor in parenthesis represents the universal part of the QCD corrections for massless quarks (for fermion mass effects and further QCD corrections which are different for vector and axial-vector partial widths see Ref. [32]); R_{EW} are the electroweak corrections which include the leading order QED corrections given by $R_{QED} = 1 + 3\alpha/(4\pi)$. R_{QCD} are further QCD corrections (for a comprehensive review see Ref. [33] and references therein), and $\beta = \sqrt{1 - 4m_f^2/M_{Z_1}^2}$ is a kinematic factor which can be taken equal to 1 for all the SM fermions except for the bottom quark. The factor δ_f contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark for which the contribution coming from the top quark at the one loop vertex radiative correction is parametrized as $\delta_b \approx 10^{-2}[1/5 - m_t^2/(2M_{Z_1}^2)]$ [34].

The ρ parameter can be expanded as $\rho = 1 + \delta\rho_0 + \delta\rho_V$ where the oblique correction $\delta\rho_0$ is given by $\delta\rho_0 \approx 3G_F m_t^2/(8\pi^2\sqrt{2})$, and $\delta\rho_V$ is the tree level contribution due to the $(Z_\mu - Z'_\mu)$ mixing which can be parametrized as $\delta\rho_V \approx (M_{Z_2}^2/M_{Z_1}^2 - 1)\sin^2\theta$. Finally, $g(f)_{1V}$ and $g(f)_{1A}$ are the coupling constants of the physical Z_1^μ field with ordinary fermions which for this model are listed in Table I.

Notice that in our expression for $\Gamma(Z_1^\mu \longrightarrow f\bar{f})$ in Eq. (39), the 3-3-1 contributions are kept at tree-level, which as a first approximation is correct due to the fact that $\delta\rho_0(331) \approx 0$, since only $SU(2)_L$ Higgs scalar singlets and doublets develop VEV [35].

In what follows we are going to use the experimental values [22]: $M_{Z_1} = 91.188$ GeV, $m_t = 174.3$ GeV, $\alpha_s(m_Z) = 0.1192$, $\alpha(m_Z)^{-1} = 127.938$, and $\sin\theta_W^2 = 0.2333$. The experimental values are introduced using the definitions $R_\eta \equiv \Gamma_Z(\eta\eta)/\Gamma_Z(hadrons)$ for $\eta = e, \mu, \tau, b, c, s, u, d$.

As a first result notice from Table I that our model predicts $R_e = R_\mu = R_\tau$, in agreement with the experimental results in Table III, independent of any flavor mixing at tree-level.

The effective weak charge in atomic parity violation, Q_W, can be expressed as a function of the number of protons (Z) and the number of neutrons (N) in the atomic nucleus in the form

$$Q_W = -2\left[(2Z+N)c_{1u} + (Z+2N)c_{1d}\right], \qquad (40)$$

where $c_{1q} = 2g(e)_{1A}g(q)_{1V}$. The theoretical value for Q_W for the Cesium atom is given by [36] $Q_W(^{133}_{55}Cs) = -73.09 \pm 0.04 + \Delta Q_W$, where the contribution of new physics is included in ΔQ_W which can be written as [37]

$$\Delta Q_W = \left[\left(1 + 4\frac{S_W^4}{1-2S_W^2}\right)Z - N\right]\delta\rho_V + \Delta Q'_W. \quad (41)$$

The term $\Delta Q'_W$ is model dependent and it can be obtained for our model by using $g(e)_{iA}$ and $g(q)_{iV}$, $i = 1, 2$, from Tables I and II. The value we obtain is

TABLE III: Experimental data and SM values for some parameters related with neutral currents.

	Experimental results	SM
Γ_Z(GeV)	2.4952 ± 0.0023	2.4966 ± 0.0016
$\Gamma(had)$ (GeV)	1.7444 ± 0.0020	1.7429 ± 0.0015
$\Gamma(l^+l^-)$ (MeV)	83.984 ± 0.086	84.019 ± 0.027
$\Gamma(inv)(MeV)$	499.0 ± 1.5	501.81 ± 0.13
$\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to Xe\nu)}$	$3.39^{+0.62}_{-0.54} \times 10^{-3}$	$(3.23 \pm 0.09) \times 10^{-3}$
R_e	20.804 ± 0.050	20.744 ± 0.018
R_μ	20.785 ± 0.033	20.744 ± 0.018
R_τ	20.764 ± 0.045	20.790 ± 0.018
R_b	0.21638 ± 0.00066	0.21569 ± 0.00016
R_c	0.1720 ± 0.0030	0.17230 ± 0.00007
$Q_W(Cs)$	$-72.65 \pm 0.28 \pm 0.34$	-73.10 ± 0.03
$Q_W(Tl)$	-116.6 ± 3.7	-116.81 ± 0.04
M_{Z_1}(GeV)	91.1872 ± 0.0021	91.1870 ± 0.0021

$$-\Delta Q'_W = (9.16Z + 4.94N)\sin\theta + (4.63Z + 3.74N)\frac{M_{Z_1}^2}{M_{Z_2}^2}. \qquad (42)$$

The discrepancy between the SM and the experimental data for ΔQ_W is given by [38]

$$\Delta Q_W = Q_W^{exp} - Q_W^{SM} = 0.45 \pm 0.48, \qquad (43)$$

which is 1.1 σ away from the SM predictions.

Introducing the expressions for Z pole observable in Eq.(39), with ΔQ_W in terms of new physics in Eq.(41) and using experimental data from LEP, SLC and atomic parity violation (see Table III), we do a χ^2 fit and we find the best allowed region in the $(\theta - M_{Z_2})$ plane at 95% confidence level (C.L.). In Fig. 6 we display this region which gives us the constraints

$$-0.0026 \leq \theta \leq -0.0006, \quad 2\ \mathrm{TeV} \leq M_{Z_2} \leq 100\ \mathrm{TeV}, \qquad (44)$$

with a central value at about 20 TeV.

As we can see the mass of the new neutral gauge boson is compatible with the bound obtained in $p\bar{p}$ collisions at the Fermilab Tevatron [39]. From our analysis we can see that M_{Z_2} peaks at a finite value larger than 100 TeV when for $|\theta| \to 0$, which still copes with the experimental constraints on the ρ parameter.

B. Bounds from unitary violation of the quark mixing matrix

The see-saw mass mixing matrices for quarks and leptons presented in Eqs. (19), (23) and (26) are not a consequence of the particular discrete Z_2 symmetry introduced in Eq. (17); a stright forward calculation shows

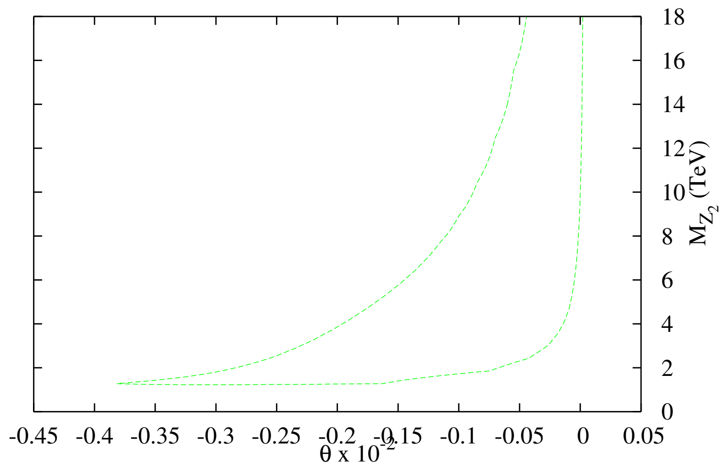

FIG. 6: Contour plot displaying the allowed region for θ Vs. M_{Z_2} at 95% C.L..

that any discrete symmetry will reproduce the same mass matrices as far as we impose the following constraints:

- To have pure see-saw mass matrices in the Down, and Charged Lepton sectors.
- To have a tree-level mass entry for the top quark mass in the third family, plus see-saw entries for the other two families in the Up quark sector.
- To work with the non-minimal set of five Higgs scalars as introduced in the main text.

As a consequence of the mixing between ordinary and exotic quarks, violation of unitary in the quark mixing matrix appears as discussed already in Sec. III C. This violation must be compatible with the experimental constraints of the mixing parameters as discussed in section 11 of Ref. [22].

For the model discussed here, the structure of the quark mass matrices implies a mixing proportional to $\cos\delta$ (with $\delta = v/V$ as before) for the known quarks of each sector, which, when combined in the V_{mix} entries, gives a mixing of the form $\cos^2\delta = 1 - \sin^2\delta \approx 1 - \delta^2$, being δ^2 proportional to the violation of unitary in the model. Taking for $V \approx M_{Z_2} \approx 2$ TeV [the lower bound in Eq. (44)], we obtain $\delta^2 \approx 3.2 \times 10^{-3}$, which is above the limit of the allowed unitary violation of V_{mix} [22]. As discussed in Sec. III C, a value of $V \approx 10$ TeV for the 3-3-1 mass scale is safe as far as the present violation of unitary in V_{mix} is concerned.

C. FCNC processes

In a model like this, with four scalar triplets and mixing of ordinary with exotic fermion fields, we should worry about possible FCNC effects which may come either from the scalar sector, from the gauge boson sector, or from the unitary violation of V_{mix}.

First, notice that due to our Z_2 symmetry, FCNC coming from the scalar sector are not present at tree-level because each flavor couples only to a single scalar triplet. But FCNC effects can occur in $J_{\mu,L}(Z)$ and $J_{\mu,L}(Z')$ in Eqs. (13) and (14), respectively, due to the mixing of ordinary and heavy exotic fermion fields (notice from Eq. (13) that $J_{\mu,L}(Z)$ only includes as active quarks the three ordinary up and down-type quarks).

The stringest constraints in FCNC in the quark sector came from the transition $d \leftrightarrow s$, and the best place to look for them is in the $(K_L^0 - K_S^0)$ mass difference, which may get contributions from the exchange of Z_1^μ and Z_2^μ. The contribution from Z_1^μ is proportional to $|V_{us}^* V_{ud}|^2 \approx |V_{us}^* V_{ud}|^2_{CKM} + 4\delta^4$ (where $|V_{us}^* V_{ud}|^2_{CKM}$ refers to the CKM). Then, the mixing of light and heavy quarks implies extra FCNC effects proportional to $4\delta^4$, which for $V \approx 2$ TeV implies a contribution to new FCNC effects above the allowed limits. So again, the 3-3-1 mass scale V must be raised. Taking $V \approx 10$ TeV as discussed before, FCNC effects are now of the order of 10^{-7}, value to be compared with the experimental bound $m(K_L) - m(K_s) \approx 3.48 \pm 0.006 \times 10^{-12}$ MeV [22], given that $4\delta^4 < 0.006/3.48$, which means that in the context of this model there is room in the experimental uncertainties to include new FCNC effects, coming from the mixing between ordinary and exotic quarks.

Now, the FCNC contributions from Z_2^μ are safe, because they are not only constrained by the δ parameter, but also by the mixing angle $-0.0026 \leq \theta \leq -0.0006$ as given in Eq. (44).

VI. CONCLUSIONS

During the last decade several 3-3-1 models for one [40] and three families have been analyzed in the literature, the most popular one being the original Pisano-Pleitez-Frampton model [4]. Other four different three-family 3-3-1 models are presented in Refs. [5, 6, 7], one of them being the subject of study of this paper. The systematic analysis presented in Refs. [8, 9] shows that there are in fact an infinite number of models based on the 3-3-1 local gauge structure, most of them including particles with exotic electric charges. But the number of models with particles without exotic electric charges are just a few [7, 9].

In this paper we have carried out a systematic study of a 3-3-1 model that we have called a model with "exotic charged leptons". In concrete, we have calculated for the first time its charged and neutral currents (see Tables I and II), we have embedded the structure into $SU(6)$ as a covering group, looked for unification possibilities, studied the gauge boson and fermion mass spectrum, and finally, by using a variety of experimental results, we have set constraints in several parameters of the model.

In our analysis we have done a detailed study of the conditions that produce a consistent charged fermion mass spectrum, a subject not even touched in the original paper [6], except for a brief discussion of the neutrino sector done in Ref. [15]. First we have shown that a set of four Higgs scalars is enough to properly break the symmetry producing a consistent mass spectrum in the gauge boson sector. Then, the introduction of an appropriate anomaly-free discrete Z_2 symmetry plus an extra exotic down quark and a singlet scalar field, allow the construction of an appealing mass spectrum in the electrically charged fermion sector, without hierarchies in the Yukawa coupling constants. In particular we have carried a program for the quark sector in which: the four exotic quarks get heavy masses at the TeV scale, the top quark gets a tree-level mass at the electroweak scale, then the bottom, charm and strange quarks get see-saw masses and finally, the two quarks in the first family get radiative masses; the former without introducing strong hierarchies in the Yukawa coupling constants, neither new mass scales in the model.

The Higgs sector used in order to break the symmetry and to provide with masses to the charged fermions, plus whatever extra scalar fields could be needed to explain the masses and oscillations of the neutral lepton sector, renders the model with a quite complicated scalar potential, with several trilinear couplings possible (like for example $f\phi_1\phi_2\phi_3$ already used to give mass to the u quark in the first family). This couplings are able to generate VEV for all the fields that feel them [41]. As a consequence, the pattern of spontaneous symmetry breaking becomes unstable and the minimization of the scalar potential may become a hopeless task. But this subject is far beyond the purpose of the present analysis.

We have also embedded the model into the covering group $SU(6) \supset SU(5)$ and studied the conditions for gauge coupling unification at a scale $M_G \approx 3 \times 10^7$ GeV. The analysis has shown that a physical ($m_Z < M_V < M_G$) one loop solution to the RGE can be achieved at the expense of introducing extra scalar fields at the intermediate energy scale M_V.

The fact that the RGE produces the same 3-3-1 mass scale than the lower limit obtained in the phenomenological analysis presented in Sec.V [compare Eqs. (38) and (44)] is not accidental neither fortuitous. As a matter of fact, the extra scalar fields contributing to the beta functions in Eq.(37), were just introduced for doing this job. A different set of scalar fields will produce either a different 3-3-1 and GUT mass scales, not unification at all, or either unphysical solutions. Eventhough our analysis may look a little arbitrary, we emphasize that we took the decision to play only with the most obscure part of any local gauge theory: the Higgs and scalar sectors.

Without looking at the neutral lepton sector, we may say that there are in this model only two mass scales: the 3-3-1 scale $V \geq 2$ TeV, and the electroweak scale $v \approx 10^2$ GeV. Notice also that the discrete symmetry Z_2 introduced in the main text has the effect that each quark flavor gains a mass only from one Higgs field, which suppresses possible FCNC effects.

What is lacking in this paper is a detailed analysis of the neutrino masses and oscillations. We could said that the study presented in Ref. [15] covers this part of the analysis; but unfortunately this is not the case. Comparing: the authors in Ref. [15] use a different set of scalar fields, with a total set of just four scalar triplets, one of them being ϕ_5 in Sec. (III E 3) which generates one-loop radiative Majorana masses, using the exotic heavy leptons as the seed; including also one electrically double charged Higgs scalar, singlet under $SU(3)_L$, which turns on the Zee-Babu mechanism. In their analysis they do not use a discrete symmetry, and they work under the assumption that the ordinary leptons (e, μ and τ) has tree-level diagonal mass terms. Clearly, most of their assumptions do not fit in our picture. So, a detailed study of the neutral lepton sector must be done in the context of the model presented in this paper. Neutrino physics in this model is very rich and it deserves further attention.

Similar studies to the one presented here but for the model with "right-handed neutrinos" [5], have been done in Refs. [13, 18]. Contrary to what is obtained here, the paper in Ref. [13] shows that for the model with "right-handed neutrinos", the see-saw mechanism for the Up and Down quark sectors can be implemented without including extra quark fields. But the model here does not need extra exotic electrons, which is the case for the model with "right-handed neutrinos" [18]. Besides, the two models are embedded into $SU(6)$ as the common covering group, with extra scalar fields added in such a way that unification of the three gauge coupling constants is achieved at a relatively low energy scale, without conflict with proton decay bounds. Also, similar results for the bounds of the 3-3-1 mass scale V and mixing angle θ [18] were found.

We have presented in this paper, original results compared with previous analysis [6, 15]. First and most important, our Higgs sector and VEV are different to the ones introduced in the original paper [6]. They imply different mass matrices for gauge bosons and fermion fields, with quite a different phenomenology. The most important fact about our Higgs sector is that it allows for a consistent charged fermion mass spectrum, without a strong hierarchy between the Yukawa coupling constants. Besides, it allows for the first time in the context of the model, the identification of the quark mass eigenstates, as a function of the weak states. Using that information, a consistent phenomenologycal analysis which sets reliable bounds on new physics coming from heavy neutral currents can be done.

As far as the particle spectrum is concerned, let us say that in the scalar sector, and according to the ESH [25], at least one more $SU(2)_L$ neutral singlet and a second Higgs doublet should show up at the electroweak scale, with all the other Higgs scalars getting a mass at the TeV scale (the neutral singlet does not couple to the SM fermions at the tree level). For the charged fermions, the four exotic quarks (two Up and two Down) and the three exotic electrons should get masses at the 3-3-1 scale (2 TeV $\leq V \leq$10 TeV). Some of these particles should show up at the forthcoming LHC facilities.

Acknowledgments

We acknowledge partial financial support from COLCIENCIAS in Colombia, and from CODI in the "Universidad de Antioquia"

Note added in proof: During the time period of revision of the original manuscript, the paper: "Lepton masses and mixing without Yukawa Hierarchies" by W.A.Ponce and O.Zapata, appears [42]. That paper addresses, in full detail, the lepton sector of the model studied here; in particular, a set of constraints were impposed in the parameters of the lepton sector such that lepton FCNC and neutrinoless double beta decay, became suppressed below the experimental bounds.

[1] For an excellent compendium of the SM see: J.F. Donoghue, E. Golowich, and B. Holstein, *Dynamics of the Standard Model*, (Cambridge University Press, Cambridge, England, 1992).

[2] For a recent review see: R.N.Mohapatra and Y. Smirnov, Ann. Rev. Nucl. Part. Sci., Vol. **56**, 569 (2006).

[3] For discussions and review see: R. N. Mohapatra, *Unification and Supersymmetry* (Springer, New York, 1986); P. Langacker, Phys. Rep. C **72**, 185 (1981); H.E. Haber and G. L. Kane, *ibid.* C **117**, 75 (1985).

[4] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); P.H. Frampton, Phys. Rev. Lett. **69**, 2887 (1992); V. Pleitez and M.D. Tonasse, Phys. Rev. D **48**, 2353 (1993); *ibid* 5274 (1993); D. Ng, Phys. Rev. D **49**, 4805 (1994); L. Epele, H. Fanchiotti, C. García Canal and D. Gómez Dumm, Phys. Lett. B **343** 291 (1995); M. Özer, Phys. Rev. D **54**, 4561 (1996).

[5] J.C. Montero, F.Pisano and V. Pleitez, Phys. Rev. D **47**, 2918 (1993); R. Foot, H.N. Long and T.A. Tran, Phys. Rev. D **50**, R34 (1994); H.N. Long, Phys. Rev. D **53**, 437 (1996); *ibid* **54**, 4691 (1996); V. Pleitez, Phys. Rev. D **53**, 514 (1996).

[6] M. Özer, Phys. Rev. D **54**, 1143, (1996).

[7] D.L.Anderson and M. Sher, Phys. Rev. D **72** 095014 (2005).

[8] W.A. Ponce, J.B. Flórez, and L.A. Sánchez, Int. J. Mod. Phys. A**17**, 643 (2002); W.A. Ponce, Y. Giraldo and L.A. Sánchez, "*Systematic study of 3-3-1 models*", in proceedings of the VIII Mexican Workshop of Particles and Fields, Zacatecas, Mexico, 2001. Edited by J.L. Díaz-Cruz *et al.* (AIP Conf. Proceed. Vol. **623**, N.Y., 2002), pp. 341-346 [hep-ph/0201133].

[9] W.A. Ponce, Y. Giraldo and L.A. Sánchez, Phys. Rev. D**67**, 075001 (2003).

[10] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[11] P.B. Pal, Phys. Rev. D **52**, 1659 (1995); A.G. Dias, C.A. de S. Pires, and P.S. Rodrigues da Silva, Phys. Rev. D**68**, 115009 (2003).

[12] P.H. Frampton, in *Proceedingd of the International Symposium on Particle Theory and Phenomenology, Ames, IA*, 1995 [Particle Phys. 229 (1995)].

[13] H.Fanchiotti, C. García-Canal, and W.A.Ponce, Europhys. Lett. **72**, 733 (2005).

[14] D. Fregolente, and M.D. Tonasse, Phys. Lett B**555**, 7 (2003); H.N. Long, and N.Q. Lan, Europhys. Lett **64**, 571 (2003); S. Filippi, W.A. Ponce, and L.A. Sánchez, Europhys. Lett. **73**, 142 (2006).

[15] T.Kitabayashi and M.Yasue, Phys. Rev. D**64**, 057301 (2001); D.Chang and H.N.Long, Phys. Rev. D **73**, 053006 (2006).

[16] Y.Okamoto and M.Yasue, Phys. Lett. B**466**, 267 (1999), T.Kitabayashi and M.Yasue, Phys. Rev. D**63**, 095002 (2001); *ibid*, D**63**, 095006 (2001); *ibid*, D**67**, 015006 (2003).

[17] A.Davidson and K.C.Waly, Phys. Rev. Lett., **59**, 393 (1987); S.Rajpoot, Phys. Rev. D**36**, 1479 (1989); D.Chang and R.N.Mohapatra, Phys. Rev. Lett., **58**, 1600 (1987).

[18] D.A.Gutiérrez, W.A.Ponce and L.A.Sánchez, Eur. Phys. J. C **46**, 497 (2006).

[19] R.Delbourgo, Phys. Lett. **40B**, 381 (1972); L.Alvarez-Gaume, Nucl. Phys. **B234**, 262 (1984).

[20] R.Slansky, Phys. Rep. **79**, 1 (1981).
[21] H.Georgi and S.L.Glashow, Phys. Rev. Lett. **32**, 438 (1974); H. Georgi, H.R.Quinn and S. Weinberg, *ibid*, **33**, 451 (1974).
[22] Particle Data Group: W. -M. Yao *et al*, Journal of Physics G **33**, 1 (2006).
[23] L.M. Kraus and F. Wilczek, Phys. Rev. Lett. **62**, 1221 (1989); L.E.Ibañez and G.G. Ross, Phys. Lett. B **260**, 291 (1991).
[24] K.S.Babu and E.Ma, Mod. Phys. lett. A **4**, 1975 (1994).
[25] F. del Aguila and L. Ibañez, Nucl. Phys. B**177**, 60 (1981).
[26] N.Cabibbo, Phys. Rev. Lett. **10**, 531,(1963); M.Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[27] See Equation (6.1) in Ref. [1].
[28] A.Zee, Phys. Lett. B**93**, 389 (1980); *ibid* B**161**, 141 (1985); K.S.Babu, Phys. Lett. B**203**, 132 (1988). For a recent discussion see D.Chang and A.Zee, Phys. Rev. D**61**, 071303 (2000).
[29] W.A.Bardeen, A.Buras, D.Duke ant T.Muta, Phys. Rev. D**18**, 3998 (1978).
[30] A.Pérez Lorenzana, W.A.Ponce and A.Zepeda, Phys. Rev. D**59**, 116004 (1999).
[31] U.Amaldi, *et al*, Phys. Rev. D**36**, 1385 (1987); U.Amaldi, W. de Boer and H.Fürstenau, Phys. Lett. B**260**, 447 (1991); P.Langacker and M.X.Luo, Phys. Rev. D**44**, 817 (1991); J.Ellis, S.Kelly and D.V.Nanopoulos, Phys. Lett. B**260**, 131 (1991).
[32] K.G. Chetyrkin and J.H. Kün, Phys. Lett. B**406**, 102 (1997).
[33] A.Leike, Phys. Rep. **317**, 143 (1999).
[34] J. Bernabeu, A. Pich and A. Santamaria, Nucl. Phys. B**363**, 326 (1991).
[35] M.Czakon, J.Gluza, F.Jegerlehner and M.Zralek, Eur. Phys. J. C**13**, 275 (2000).
[36] W.J. Marciano and A. Sirlin, Phys. Rev D**29**, 75 (1984); W.J. Marciano and J. Rosner, Phys. Rev. Lett. **65**, 2963 (1990).
[37] L. Durkin and P. Langacker, Phys. Lett. **166**B, 436 (1986).
[38] J. Guéna, M.Lintz and M.A.Bouchiat, Mod. Phys. Lett **A20**, 375 (2005); J.S.M. Ginges and V.V. Flambaum, Phys. Rep. **397**, 63 (2004); J.L.Rosner, Phys. Rev. D**65**, 073026 (2002); A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000).
[39] F. Abe *et al.*, Phys. Rev. Lett. **84**, 212 (2000).
[40] L.A.Sánchez, W.A.Ponce and R.Martínez, Phys. Rev. D**64**, 075013 (2001); *ibid*, D**65**, 055013 (2002).
[41] P.H.Frampton, P.I.Krastev, and J.T.Liu, Mod. Phys. Lett. A **9**, 761 (1994).
[42] W.A. Ponce and O. Zapata, Phys. Rev. D**74**, 003007 (2006).