Mixing angles from five texture zeros of the quark mass matrices

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Noviembre 8 de 2019

Abstract

In the Standard Model, we will deduce a configuration with five texture zeros for the quark mass matrices that it is not of the Fritzsch type. It is valid and generates all the physical quantities of interest: that includes the quark masses, the inner angles of the Cabibbo-Kobayashi-Maskawa unitary triangle, and the phase responsible for the violation of the charge-parity symmetry. To achieve this, we must include non physical phases in the unitary matrices that diagonalize the quark mass matrices to bring the Cabibbo-Kobayashi-Maskawa mixing matrix to its standard form.

Introduction

- Models like the Standard Model (SM) or its extensions, where the right fields are $SU(2)$ singlets, it is always possible to choose a suitable basis for the right quarks using the unitary matrix coming from the *polar decomposition theorem* of linear algebra, in such a way that the up and down quark mass matrices become hermitian matrices, i. e.,

$$M_u^\dagger = M_u, \quad \text{y} \quad M_d^\dagger = M_d.$$

- In the SM, left and right quarks can be transformed unitarily, so that the gauge currents remain invariant, and as a result, the quark mass matrices are transformed into new equivalent matrices. This process basically consists of a common unitary transformation applied over M_u and M_d which is known as a “Weak Basis” (WB) Transformation [1], as follows:

$$M_u \to M_u' = U^\dagger M_u U, \quad M_d \to M_d' = U^\dagger M_d U,$$

where U is an arbitrary unitary matrix that preserves the hermiticity of quark mass matrices.

- Any physically viable quark mass matrix can be derived from specific quark mass matrices by making a WB transformation.

$$U_u^\dagger M_u U_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix},$$

$$U_d^\dagger M_d U_d = D_d = \begin{pmatrix} \lambda_{1d} & 0 & 0 \\ 0 & \lambda_{2d} & 0 \\ 0 & 0 & \lambda_{3d} \end{pmatrix},$$

$$\text{CKM mixing matrix} = V = U_u^\dagger U_d,$$

where

$$|\lambda_{1u}| = m_u, |\lambda_{2u}| = m_c, |\lambda_{3u}| = m_t,$$
$$|\lambda_{1d}| = m_d, |\lambda_{2d}| = m_s, |\lambda_{3d}| = m_b.$$

and

$$\lambda_{1q} < 0 \oplus \lambda_{2q} < 0 \oplus \lambda_{3q} < 0.$$

for $q = u, d$.

Quark masses and CKM mixing matrix

The mass of quarks and the observed parameters of the CKM matrix $|V_{ij}|$ are given in the SM scheme at a renormalization scale of $\mu = m_Z$ [2]:

$$m_u = 1.38^{+0.42}_{-0.41}\,,\ m_c = 638^{+43}_{-84},\ m_t = 172100 \pm 1200\,,$$
$$m_d = 2.82 \pm 0.48\,,\ m_s = 57^{+18}_{-12}\,,\ m_b = 2860^{+160}_{-60}.$$

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix},$$

1. The basic quark mass matrices

The diagonal representation u [3, 4]:

$$M_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix},$$
$$M_d = V D_d V^\dagger.$$

The diagonal representation d.

$$M_u = V^\dagger D_u V,$$
$$M_d = D_d = \begin{pmatrix} \lambda_{1d} & 0 & 0 \\ 0 & \lambda_{2d} & 0 \\ 0 & 0 & \lambda_{3d} \end{pmatrix},$$

Five numerical texture zeros

2. Pattern with one and two diagonal zeros

Permutation matrices	Two diagonal zero patterns $(p_i\, M_q\, p_i^T)$	One diagonal zero patterns $(p_i\, M_q\, p_i^T)$
$p_1 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & 0 \\ \|\xi_q\| & 0 & \|\beta_q\| \\ 0 & \|\beta_q\| & \alpha_q \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & 0 \\ \|\xi_q\| & \gamma_q & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}$
$p_2 = \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \|\xi_q\| \\ 0 & \alpha_q & \|\beta_q\| \\ \|\xi_q\| & \|\beta_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \|\xi_q\| \\ 0 & \alpha_q & 0 \\ \|\xi_q\| & 0 & \gamma_q \end{pmatrix}$
$p_3 = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} \alpha_q & \|\beta_q\| & 0 \\ \|\beta_q\| & 0 & \|\xi_q\| \\ 0 & \|\xi_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & 0 \\ 0 & \gamma_q & \|\xi_q\| \\ 0 & \|\xi_q\| & 0 \end{pmatrix}$
$p_4 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & \|\beta_q\| \\ \|\xi_q\| & 0 & 0 \\ \|\beta_q\| & 0 & \alpha_q \end{pmatrix}$	$\begin{pmatrix} \|\gamma_q\| & \|\xi_q\| & 0 \\ \|\xi_q\| & 0 & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}$
$p_5 = \begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & \|\beta_q\| \\ 0 & 0 & \|\xi_q\| \\ \|\beta_q\| & \|\xi_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & 0 \\ 0 & 0 & \|\xi_q\| \\ 0 & \|\xi_q\| & \gamma_q \end{pmatrix}$
$p_6 = \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} 0 & \|\beta_q\| & \|\xi_q\| \\ \|\beta_q\| & \alpha_q & 0 \\ \|\xi_q\| & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \gamma_q & 0 & \|\xi_q\| \\ 0 & \alpha_q & 0 \\ \|\xi_q\| & 0 & 0 \end{pmatrix}$

3. Numerical quark mass matrices (in MeV units)

$$M_u' = \begin{pmatrix} 0 & 0 & -79.32 + 154.72i \\ 0 & 5539.2 & 28125.9 + 6112.8i \\ -79.323 - 154.72i & 28125.9 - 6112.8i & 167126.0 \end{pmatrix},$$

$$M_d' = \begin{pmatrix} 0 & 13.891097 & 0 \\ 13.891097 & 0 & 421.41405 \\ 0 & 421.41405 & 2797.9042 \end{pmatrix}.$$

Five analytical texture zeros and the CKM mixing matrix

The texture matrix of five zeros previously obtained has the following standard structure:

$$M_u = P^\dagger \begin{pmatrix} 0 & 0 & |\xi_u| \\ 0 & \alpha_u & |\beta_u| \\ |\xi_u| & |\beta_u| & \gamma_u \end{pmatrix} P, \quad M_d = \begin{pmatrix} 0 & |\xi_d| & 0 \\ |\xi_d| & 0 & |\beta_d| \\ 0 & |\beta_d| & \alpha_d \end{pmatrix},$$

where $P = \operatorname{diag}(e^{-i\phi_{\xi_u}}, e^{-i\phi_{\beta_u}}, 1)$ (with $\phi_{\beta_u} \equiv \arg(\beta_u)$ and $\phi_{\xi_u} \equiv \arg(\xi_u)$). We have nine free parameters to reproduce ten physical magnitudes: six (6) quark masses, three (3) mixing angles and one (1) CP violation phase in the CKM matrix, which implies physical relations between the quark masses and mixings.

4. The mixings

$$|V_{ud}| \approx |V_{cs}| \approx |V_{tb}| \approx 1,$$

$$|V_{us}| \approx \left|\sqrt{\frac{\alpha_u - m_c}{\alpha_u}}\sqrt{\frac{m_u}{m_c}} - e^{i(\phi_{\beta_u} - \phi_{\xi_u})}\sqrt{\frac{m_d}{m_s}}\right|,$$

$$|V_{cd}| \approx \left|\sqrt{\frac{\alpha_u - m_c}{\alpha_u}}\sqrt{\frac{m_u}{m_c}} - e^{i(\phi_{\xi_u} - \phi_{\beta_u})}\sqrt{\frac{m_d}{m_s}}\right|,$$

$$|V_{cb}| \approx \left|\sqrt{\frac{m_s}{m_b}} - e^{i\phi_{\beta_u}}\sqrt{\frac{\alpha_u - m_c}{m_t}}\right|,$$

$$|V_{ts}| \approx \left|\sqrt{\frac{m_s}{m_b}} - e^{-i\phi_{\beta_u}}\sqrt{\frac{\alpha_u - m_c}{m_t}}\right|,$$

$$\frac{|V_{ub}|}{|V_{cb}|} \approx \sqrt{\frac{m_u}{m_c}}\left|\frac{\sqrt{\frac{\alpha_u}{m_t}} - e^{-i\phi_{\beta_u}}\sqrt{\frac{\alpha_u - m_c}{\alpha_u}}\sqrt{\frac{m_s}{m_b}}}{\sqrt{\frac{\alpha_u - m_c}{m_t}} - e^{-i\phi_{\beta_u}}\sqrt{\frac{m_s}{m_b}}}\right|,$$

$$\frac{|V_{td}|}{|V_{ts}|} \approx \sqrt{\frac{m_d}{m_s}},$$

where it is considered $\alpha_u \ll m_t$. Let's consider $\alpha_u \gtrsim m_c$ to adjust the experimental data, which gives $(\phi_{\beta_u} - \phi_{\xi_u}) \sim -\pi/2$, which is an important contribution term for the CP violation.

Conclusions

The main conclusions of this work are:

- We found only two different numerical texture patterns of five zeros.
- We have nine free parameters to reproduce ten physical magnitudes: six (6) quark masses, three (3) mixing angles and one (1) CP-violation phase of the CKM matrix, which implies physical relations between the quark masses and mixings.
- The Gatto-Sartori-Tonin (GST) relationship is maintained, and an important contribution of the CP violation is still shown in the context of the model.

References

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