# Morfismos Irreducibles de Álgebras Repetitivas

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# Content



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# Section



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# Repetitive Algebras

- $A$ is a finite dimensional $k$-algebra, with $A$ basic and $k$ is a field algebraically closed.
- $A$-mod the category of finitely generated left $A$-modules.
- $D = Hom_k(-, k)$ the standard duality on $A$-mod.

The **repetitive algebra** $\widehat{A}$ of $A$ (Hughes and Waschbusch).

- The underlying vector space of repetitive algebra $\widehat{A}$ is given by

$$\widehat{A} = (\oplus_{i\in\mathbb{Z}} A) \oplus (\oplus_{i\in\mathbb{Z}} DA),$$

  $\widehat{a} = (a_i, \varphi_i)_{i\in\mathbb{Z}}$ with $a_i \in A$, $\varphi_i \in DA$ and almost all $a_i, \varphi_i$ being zero.

- The multiplication is defined by

$$\widehat{a} \cdot \widehat{b} = (a_i, \varphi_i)_{i\in\mathbb{Z}} \cdot (b_i, \psi_i)_{i\in\mathbb{Z}} = (a_i b_i, a_{i+1}\psi_i + \varphi_i b_i)_{i\in\mathbb{Z}}.$$

An interpretation is to consider $\widehat{A}$ as the doubly infinite matriz algebra, without identity

$$\begin{bmatrix} \ddots & & & 0 & \\ \ddots & A_{i-1} & & & \\ & (DA)_{i-1} & A_i & & \\ & & (DA)_i & A_{i+1} & \\ & 0 & & \ddots & \ddots \end{bmatrix},$$

in which matrices have only finite many non-zero entries, $A_i = A$ for all $i \in \mathbb{Z}$ is placed on the main diagonal, $(DA)_i = DA$ for all $i \in \mathbb{Z}$ and all the remaining entries are zero.

- A $\widehat{A}$-module. $M = (M_i, f_i)_{i\in\mathbb{Z}}$, where the $M_i$ are $A$-modules, all but finitely many being zero, the $f_i$ are $A$-homomorphims $f_i : DA \otimes_A M_i \longrightarrow M_{i+1}$, such that $f_{i+1}(1 \otimes f_i) = 0$ for all $i \in \mathbb{Z}$.
- A $\widehat{A}$-homomorphism $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ between $\widehat{A}$-modules is a sequence $h = (h_i)_{i\in\mathbb{Z}}$ of $A$-homomorphims

$$\begin{array}{ccc} DA \otimes_A M_i & \xrightarrow{f_i} & M_{i+1} \\ \downarrow{\scriptstyle 1\otimes h_i} & & \downarrow{\scriptstyle h_{i+1}} \\ DA \otimes_A N_i & \xrightarrow{g_i} & N_{i+1}. \end{array}$$

- A $\widehat{A}$-module. $M = (M_i, f_i)_{i\in\mathbb{Z}}$, where the $M_i$ are $A$-modules, all but finitely many being zero, the $f_i$ are $A$-homomorphims $f_i : DA \otimes_A M_i \longrightarrow M_{i+1}$, such that $f_{i+1}(1 \otimes f_i) = 0$ for all $i \in \mathbb{Z}$.
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$$\begin{array}{ccc} DA \otimes_A M_i & \xrightarrow{f_i} & M_{i+1} \\ \downarrow{\scriptstyle 1\otimes h_i} & & \downarrow{\scriptstyle h_{i+1}} \\ DA \otimes_A N_i & \xrightarrow{g_i} & N_{i+1}. \end{array}$$

Let $M = (M_i, f_i)_{i\in\mathbb{Z}}$ be an $\widehat{A}$-module. For each $i \in \mathbb{Z}$, we fix $\varphi \in DA$, so we can define an $A$-homomorphims $f_i^{\varphi} : M_i \longrightarrow M_{i+1}$ by $f_i^{\varphi}(m) := f_i(\varphi \otimes m)$, for all $m \in M_i$.

**Lemma**

*Let $M = (M_i, f_i)_{i\in\mathbb{Z}}$ be an $\widehat{A}$-module. $f_{i+1}(1 \otimes f_i) = 0$ for all $i \in \mathbb{Z}$ if and only $f_{i+1}^{\varphi'} f_i^{\varphi} = 0$ for all $i \in \mathbb{Z}$ and $\varphi', \varphi \in DA$.*

## Lemma

*Let* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *be* $\widehat{A}$*-homomorphism. The following diagram is conmmutative*

$$\begin{array}{ccc} DA \otimes_A M_i & \xrightarrow{f_i} & M_{i+1} \\ \downarrow{\scriptstyle 1\otimes h_i} & & \downarrow{\scriptstyle h_{i+1}} \\ DA \otimes_A N_i & \xrightarrow{g_i} & N_{i+1}, \end{array}$$

*for all* $i \in \mathbb{Z}$ *if and only if the following diagram is conmmutative*

$$\begin{array}{ccc} M_i & \xrightarrow{f_i^{\varphi}} & M_{i+1} \\ \downarrow{\scriptstyle h_i} & & \downarrow{\scriptstyle h_{i+1}} \\ N_i & \xrightarrow{g_i^{\varphi}} & N_{i+1}. \end{array}$$

*for all* $i \in \mathbb{Z}$ *and* $\varphi \in DA$.

**Definition**

*Let* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *be an* $\widehat{A}$*-homomorphism, then* $h$ *is called* ***smonic*** *(resp.* ***sepic****) if all its components* $h_i$ *are split mono (resp. split epi).*

**Proposition (Standard forms)**

*If* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *is a smonic, (up to isomorphism) we can and will assume that* $N_i = M_i \oplus N'{}_i$*, and that* $h_i = (1, 0)^t$*, and, if it is sepic we will write* $M_i = N_i \oplus M'{}_i$*, and* $h_i = (1, 0)$*. Forthermore, if* $h$ *is split mono (resp. split epi) the* $A$*-homomorphisms* $f_i$ *(resp.* $g_i$*) are of the form* $f_i = \begin{pmatrix} f_i & 0 \\ 0 & e_i \end{pmatrix}$ *(resp.* $g_i = \begin{pmatrix} g_i & 0 \\ 0 & \epsilon_i \end{pmatrix}$ *).*

**Proposition (Induced factorization)**

*Let* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *be a* $\widehat{A}$*-homomorphism,* $J = [a, b]$ *be a finite interval of* $\mathbb{Z}$ *and* $h_J$ *be the truncation of* $h$ *to* $J$*. Supose that* $h_J : M_J \to N_J$ *admits a factorization* $h_J = m_J n_J$*. (In other words, there is a truncated* $\widehat{A}$*-module* $L_J = (L_i, d_i)_{i\in J}$ *and truncated* $\widehat{A}$*-morphisms* $n_J : M_J \to L_J$*,* $m_J : L_J \to N_J$*, such that* $h_J = m_J n_J$*). Then, this factorization can be extended to* $h$*, that is, there is a* $\widehat{A}$*-module (whose truncation to* $J$ *is* $L_J$*) and* $\widehat{A}$*-homomorphism* $n$*,* $m$*, whose truncation to* $J$ *are* $n_J$*,* $m_J$*, respectively, such that* $h = mn$*.*

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**Corollary**

*Let* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *be an irreducible* $\widehat{A}$*-homomorphism and* $J = [a, b]$ *be a finite interval of* $\mathbb{Z}$*. If* $h_J$ *is not split mono, then* $h_{J_-}$ *is a split epi. If* $h_J$ *is not split epi, then* $h_{J_+}$ *split mono.*

**Lemma**

*Let* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *be a* $\widehat{A}$*-homomorphism and* $J$ *be an interval of* $\mathbb{Z}$*. Then* $h_J : M_J \to N_J$ *is irreducible if and only if* $h_J : M_J \to N_J$ *is not split.*

**Corollary**

*Let* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *be an irreducible* $\widehat{A}$*-homomorphism,* $J$ *be an interval of* $\mathbb{Z}$ *and* $h_J$ *be the truncation of* $h$ *to* $J$*. Supose that* $h_J : M_J \to N_J$ *is irreducible. If* $J \subset I$*, then* $h_I$ *is an irreducible* $\widehat{A}$*-homomorphism.*

**Theorem**

*Let* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *be an irreducible* $\widehat{A}$*-homomorphism. Then one of the next conditions hold:*

(I) $h_i$ *is a split monomorphism* $\forall i \in \mathbb{Z}$*, that is* $h$ *is smonic;*

(II) $h_i$ *is a split epimorphism* $\forall i \in \mathbb{Z}$*, that is* $h$ *is sepic;*

(III) *there exists* $k \in \mathbb{Z}$ *such that* $h_k$ *is not split. In this case,* $k$ *is unique and* $h_k$ *is irreducible* $A$*-homomorphism.*

joint Merklen (2009), [3]

**Theorem**

*Let* $h : M = (M_i, f_i)_{i\in\mathbb{Z}} \longrightarrow N = (N_i, g_i)_{i\in\mathbb{Z}}$ *be an irreducible* $\widehat{A}$*-homomorphism. Then one of the next conditions hold:*

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# Section

# Form pm

$$
\begin{array}{ccccc}
\cdots X^{-1} & \xrightarrow{d^{-1}} & X^{0} & \xrightarrow{d^{0}} & X^{1} \\
\downarrow{\scriptstyle 1} & & \downarrow{\scriptstyle (1,0)^t} & & \downarrow{\scriptstyle (1,0)^t} \\
\cdots X^{-1} & \xrightarrow[\partial^{-1}]{} & X^{0} \oplus Y'^{0} & \xrightarrow[\partial^{0}]{} & X^{1} \oplus Y'^{1}
\end{array}
\qquad
\begin{array}{ccc}
\cdots X^{b} & \xrightarrow{d^{b}} & X^{b+1} \cdots \\
\downarrow{\scriptstyle (1,0)^t} & & \downarrow{\scriptstyle (1,0)^t} \\
\cdots X^{b} \oplus Y'^{b} & \xrightarrow[\partial^{b}]{} & X^{b+1} \oplus Y'^{b+1}
\end{array}
$$

For $0 \leq i \leq b$, the differential maps $\partial^i$ are of the form:

$$\partial^i = \begin{pmatrix} d^i & a^i \\ 0 & e^i \end{pmatrix}, \text{with } a^i \neq 0 \text{ for some } 0 \leq i \leq b.$$

For $\forall i \geq b, i \in J$, the differential maps $\partial^i$ are of the form:

$$\partial^i = \begin{pmatrix} d^i & 0 \\ 0 & e^i \end{pmatrix}$$

# Forma pe

$$
\begin{array}{ccccc}
\cdots Y^{-1} \oplus X'^{-1} & \xrightarrow{d^{-1}} & Y^{0} \oplus X'^{0} & \xrightarrow{d^{0}} & Y^{1} \oplus X'^{1} \\
\downarrow{\scriptstyle (1,0)} & & \downarrow{\scriptstyle (1,0)} & & \downarrow{\scriptstyle (1,0)} \\
\cdots Y^{-1} & \xrightarrow[\partial^{-1}]{} & Y^{0} & \xrightarrow[\partial^{0}]{} & Y^{1}
\end{array}
\qquad
\begin{array}{ccc}
\cdots Y^{b} \oplus X'^{b} & \xrightarrow{d^{b}} & Y^{b+1} \\
\downarrow{\scriptstyle (1,0)} & & \downarrow{\scriptstyle 1} \\
\cdots Y^{b} & \xrightarrow[\partial^{b}]{} & Y^{b+1}
\end{array}
$$

For $0 \leq i \leq b$, the differential maps $d^i$ are of the form

$$d^i = \begin{pmatrix} \partial^i & 0 \\ c^i & e^i \end{pmatrix}, \text{with } c^i \neq 0 \text{ for some } 0 \leq i \leq b.$$

For $\forall i \leq -1, i \in J$, the differential maps $d^i$ are form:

$$d^i = \begin{pmatrix} \partial^i & 0 \\ 0 & e^i \end{pmatrix}$$

# Forma nsp

$$
\begin{array}{ccccccccc}
Y^{-2}\oplus X'^{-2} & \xrightarrow{d^{-2}} & Y^{-1}\oplus X'^{-1} & \xrightarrow{d^{-1}} & X^{0} & \xrightarrow{d^{0}} & X^{1} & \xrightarrow{d^{1}} & X^{2}\cdots \\
\downarrow{\scriptstyle (1,0)} & & \downarrow{\scriptstyle (1,0)} & & \downarrow{\scriptstyle f_0} & & \downarrow{\scriptstyle (1,0)^t} & & \downarrow{\scriptstyle (1,0)^t} \\
\cdots Y^{-2} & \xrightarrow[\partial^{-2}]{} & Y^{-1} & \xrightarrow[\partial^{-1}]{} & Y^{0} & \xrightarrow[\partial^{0}]{} & X^{1}\oplus Y'^{1} & \xrightarrow[\partial^{1}]{} & X^{2}\oplus Y'^{2}
\end{array}
$$

With $f_0$ irreducible.
For $i \leq -2$, the differential maps $d^i$ are of the form

$$d^i = \begin{pmatrix} \partial^i & 0 \\ 0 & e^i \end{pmatrix}$$

For $i \geq 1$, the differential maps $\partial^i$ are of the form:

$$\partial^i = \begin{pmatrix} d^i & 0 \\ 0 & e^i \end{pmatrix}$$

joint Marcos, [2]

**Definition**

*Let $f : X \to Y$ be an morphism of complexes.*

(a) *If for all $J$ a finite interval of $\mathbb{Z}$ the morphism $f_J$ splits, then $f$ is called* ***locally split****.*

(b) *$f$ is called* ***monster*** *if $f$ is locally split and $f$ does not split.*

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(b) *$f$ is called* ***monster*** *if $f$ is locally split and $f$ does not split.*

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**Lemma**

*Let $f : X \to Y$ be a smonic, irreducible morphism of complexes, which is not a monster then there is unique integer $l_m(f)$ such that $f^{l_m(f)}$ is not isomorphism and $f_{]-\infty, l_m(f)[}$ is an isomorphism.*

**Lemma**

*Let $f : X \to Y$ be a smonic morphism of complexes and $J = [a, b]$ a finite interval of $\mathbb{Z}$. If $f_J$ is irreducible morphism, then $f^a : X^a \to Y^a$ is an isomorphism if and only if $f_{[a+1,b]}$ is an irreducible morphism.*

**Corollary**

*Let $f : X \to Y$ be a smonic morphism of complexes and $J = [a, b]$ a finite interval of $\mathbb{Z}$. If $f_J$ is irreducible morphism and $f^a : X^a \to Y^a$ is not isomorphism, then $a = l_m(f)$.*

**Definition**

*Let $f : X \to Y$ be an not monster, smonic, irreducible morphism of complexes and $r_m(f)$ the smallest integer such that $f_{[l_m(f), r_m(f)]}$ is a irreducible morphism. $f_{[l_m(f), r_m(f)]}$ is called the* ***monic heart*** *of $f$.*

**Corollary**

*Let $f : X \to Y$ be a smonic morphism of complexes and $J = [a, b]$ a finite interval of $\mathbb{Z}$. If $f_J$ is irreducible morphism and $f^a : X^a \to Y^a$ is not isomorphism, then $a = l_m(f)$.*

**Definition**

*Let $f : X \to Y$ be an not monster, smonic, irreducible morphism of complexes and $r_m(f)$ the smallest integer such that $f_{[l_m(f), r_m(f)]}$ is a irreducible morphism. $f_{[l_m(f), r_m(f)]}$ is called the* ***monic heart*** *of $f$.*

**Theorem**

*Let $f : X \to Y$ be a smonic and irreducible morphism of complexes. Then $f$ does not have a monic heart if and only if $f$ is monster morphism.*

**Theorem**

*Let $f : X \to Y$ be a smonic, irreducible and not monster morphism of complexes. Then $f_{[a,b]}$ is an irreducible morphism if and only if $f_{[a,b]}$ contains the monic heart of $f$.*

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**Theorem**

*Let $f : X \to Y$ be a smonic and irreducible morphism of complexes. Then $f$ does not have a monic heart if and only if $f$ is monster morphism.*

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**Theorem**

*Let $f : X \to Y$ be a sepic and irreducible morphism of complexes. Then $f$ does not have epic heart if and only if $f$ is monster morphism.*

**Theorem**

*Let $f : X \to Y$ be an not monster, sepic and irreducible morphism of complexes. Then $f_{[a,b]}$ is an irreducible morphism if and only if $f_{[a,b]}$ contains the epic heart of $f$.*

# Examples

$\mathcal{A}$ =$\Lambda$-proj, with $\Lambda$ an Artin algebra over the commutative Artinian ring $k$. Let S be a simple $\Lambda$-module and let

$$\cdots \longrightarrow P^s \longrightarrow \cdots \longrightarrow P^{-1} \longrightarrow P^0 \longrightarrow S \longrightarrow 0 \longrightarrow \cdots ,$$

be a minimal projective resolution of the simple $S$.
The morphism $u : P_S \to J_{-1}(P^0)$ is an irreducible morphism (Bautista and Salorio [1]).

$$\begin{array}{ccccccccccccc}
\cdots & \longrightarrow & P^s & \longrightarrow & \cdots & \longrightarrow & P^{-2} & \longrightarrow & P^{-1} & \xrightarrow{d^{-1}} & P^0 & \longrightarrow & 0 & \longrightarrow & \cdots \\
 & & \downarrow & & & & \downarrow & & \downarrow {\scriptstyle d^{-1}} & & \downarrow {\scriptstyle 1} & & \downarrow & & \\
\cdots & \longrightarrow & 0 & \longrightarrow & \cdots & \longrightarrow & 0 & \longrightarrow & P^0 & \xrightarrow{1} & P^0 & \longrightarrow & 0 & \longrightarrow & \cdots
\end{array}$$

# Examples

$\mathcal{A}$ =$\Lambda$-proj, with $\Lambda$ an Artin algebra over the commutative Artinian ring $k$. Let S be a simple $\Lambda$-module and let

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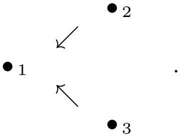
 & & \downarrow & & & & \downarrow & & \downarrow{\scriptstyle d^{-1}} & & \downarrow{\scriptstyle 1} & & \downarrow & & \\
\cdots & \longrightarrow & 0 & \longrightarrow & \cdots & \longrightarrow & 0 & \longrightarrow & P^0 & \xrightarrow{1} & P^0 & \longrightarrow & 0 & \longrightarrow & \cdots,
\end{array}$$

# Examples

We consider hereditary algebra $A$ defined by the quiver



# Examples

The morphism $f$ given by:

$$\begin{array}{ccccccccccc}
\cdots & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & P_2 & \longrightarrow & 0 & \longrightarrow & \cdots \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\cdots & \longrightarrow & 0 & \longrightarrow & P_1 & \longrightarrow & P_2 \oplus P_3 & \longrightarrow & 0 & \longrightarrow & \cdots ,
\end{array}$$

is an irreducible morphism.

$$\begin{array}{ccc}
0 & \longrightarrow & P_2 \\
\downarrow & & \downarrow \\
P_1 & \longrightarrow & P_2 \oplus P_3 ,
\end{array}$$

is the monic heart of $f$.

# Examples

The morphism $g$ given by:

$$\begin{array}{ccccccccccc}
\cdots & \longrightarrow & 0 & \longrightarrow & P_1 & \longrightarrow & P_2 \oplus P_3 & \longrightarrow & 0 & \longrightarrow & \cdots, \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\cdots & \longrightarrow & 0 & \longrightarrow & P_1 & \longrightarrow & P_2 & \longrightarrow & 0 & \longrightarrow & \cdots
\end{array}$$

is an irreducible morphism.

$$\begin{array}{ccc}
P_1 & \longrightarrow & P_2 \oplus P_3, \\
\downarrow & & \downarrow \\
P_1 & \longrightarrow & P_2
\end{array}$$

is the epic heart of $g$.

# Examples

We considere the hereditary algebra $A$ defined by the quiver

$$\bullet_1 \quad \leftarrow \quad \bullet_2 \quad \leftarrow \quad \bullet_3 \quad .$$

The morphism $f$ given by:

$$\begin{array}{ccccccccccc} \cdots & \longrightarrow & 0 & \longrightarrow & P_1 & \longrightarrow & P_2 & \longrightarrow & 0 & \longrightarrow & \cdots \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ \cdots & \longrightarrow & 0 & \longrightarrow & P_1 & \longrightarrow & P_3 & \longrightarrow & 0 & \longrightarrow & \cdots , \end{array}$$

is an irreducible morphism, so

$$\begin{array}{c} P_2 \\ \downarrow \\ P_3, \end{array}$$

is the line heart of $f$.

## Examples

The morphism $g$ given by:

$$\begin{array}{ccccccccccc}
\cdots & \longrightarrow & 0 & \longrightarrow & P_1 & \longrightarrow & P_3 & \longrightarrow & 0 & \longrightarrow & \cdots, \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\cdots & \longrightarrow & 0 & \longrightarrow & P_2 & \longrightarrow & P_3 & \longrightarrow & 0 & \longrightarrow & \cdots
\end{array}$$

is an irreducible morphism, so

$$\begin{array}{c}
P_1 \\
\downarrow \\
P_2,
\end{array}$$

is the line heart of $g$.

## Section

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