Alternative 3-3-1 models with exotic electric charges

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We report the most general classification of 3-3-1 models with $\beta = \sqrt{3}$. We found several solutions where anomaly cancellation occurs among fermions of different families. These solutions are particularly interesting as they generate non-universal heavy neutral vector bosons. Non-universality in the SM fermion charges under an additional gauge group generates Charged Lepton Flavor Violation (CLFV) and Flavor Changing Neutral Currents (FCNC); we discuss under what conditions the new models can evade constraints coming from these processes. In addition, we also report the Large Hadron Collider (LHC) constraints.

I. INTRODUCTION

Models with exotic fermions based on the gauge group symmetry $SU(3) \otimes SU(3) \otimes U(1)$ (hereafter 3-3-1 models for short) have been proposed since the early 1970s [1–11]; however, many of these models lacked important properties of what is known nowadays as 3-3-1 models. For a model to be interesting from a modern perspective [12], it must be chiral, the triangle anomalies must be canceled out only with a number of generations multiple of 3, and most importantly, it must contain the Standard Model (SM).

In the 1990s, non-universal models without exotic leptons gained popularity as they were very convenient in addressing flavor problems [13, 14]. These models have also been helpful in explaining neutrino masses [15–24], dark matter [25–35], charge quantization [36], strong CP violation [37, 38], muon anomalous magnetic moment (g-2 muon anomaly) [39–41] and flavor anomalies [42–45].

Pleitez and Frampton proposed the non-universal 3-3-1 models [13, 14] as examples of electroweak extensions with lepton number violation, where the number of families is determined by anomaly cancellation. In the literature, there are many examples of models without exotic electric charges, these models have been appropriately classified, and their phenomenology is well known [46–48]. The original model of Pleitez and Framton has exotic electric charges in the quark sector and corresponds to what is known in the literature as $\beta = \sqrt{3}$ [12]. As far as we know, an exhaustive classification of models with this β does not exist in the literature, and therefore a work in this line is necessary. It is important to notice that there are solutions for arbitrary β [49]; however, this solution does not account for all the possible models for a given β. As we will see, the parameter β cannot be arbitrarily large, from the matching conditions $|\beta| \lessapprox \cot\theta_W \sim 1.8$. This condition constitutes a very important restriction regarding the possible realizations of the 3-3-1 symmetry at low energies as it limits the number of possible non-trivial cases to a countable set.

In section II, we review the basics of the 3-3-1 models. In section III, we propose sets of fermions corresponding to families of quarks and leptons with the left-handed triplets, anti-triplets, and singlets of $SU(3)_L$. In section IV, we show the anomaly-free sets (AFSs) that constitute the basis for model building. This section lists all possible 3-3-1 models with $\beta = \sqrt{3}$ modulo lepton vector arrays. Finally, in section V, we show the collider constraints and the conditions the models must satisfy to avoid FCNC and CLFV restrictions.

II. 3-3-1 MODELS

In the subsequent discussion, we work the electroweak gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, expanding the electroweak sector of the SM, $SU(2)_L \otimes U(1)_Y$, to $SU(3)_L \otimes U(1)_X$. Furthermore, we assume that, similar to the SM, the color group $SU(3)_c$ is vector-like (i.e., anomaly-free). Left-handed quarks (color triplets) and left-handed leptons (color singlets) transform under the two fundamental representations of $SU(3)_L$ (i.e., 3 and 3^*).

Two categories of models will emerge: universal single-family models, where anomalies cancel within each family similar to the SM, and family models, where anomalies

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are canceled through interactions among multiple families.

In the context of 3-3-1 models, the most complete electric charge operator for this electroweak sector is

$$Q = \alpha\, T_{L3} + \beta T_{L8} + X\mathbb{1}, \tag{1}$$

here, $T_{La} = \lambda_a/2$, where $\lambda_a,; a = 1, 2, \ldots, 8$ represents the Gell-Mann matrices for $SU(3)_L$ normalized as $\mathrm{Tr}(\lambda_a\lambda_b) = 2\delta_{ab}$, and $\mathbb{1} = \mathrm{Diag}(1, 1, 1)$ is the diagonal 3×3 unit matrix. Assuming $\alpha = 1$, the $SU(2)_L$ isospin group of the SM is fully covered in $SU(3)_L$. The parameter $\beta = \frac{2b}{\sqrt{3}}$ is a free parameter that defines the model (β is proportional to b present in the electric charge of the exotic vector boson K_μ). The X values are determined through anomaly cancellation. The 8 gauge fields A^a_μ of $SU(3)_L$ can be expressed as [46, 47]

$$\sum_a \lambda_a A^a_\mu = \sqrt{2}\begin{pmatrix} D^0_{1\mu} & W^+_\mu & K^{(b+1/2)}_\mu \\ W^-_\mu & D^0_{2\mu} & K^{(b-1/2)}_\mu \\ K^{-(b+1/2)}_\mu & K^{-(b-1/2)}_\mu & D^0_{3\mu} \end{pmatrix}, \tag{2}$$

here, $D^0_{1\mu} = A^3_\mu/\sqrt{2} + A^8_\mu/\sqrt{6}$, $D^0_{2\mu} = -A^3_\mu/\sqrt{2} + A^8_\mu/\sqrt{6}$, and $D^0_{3\mu} = -2A^8_\mu/\sqrt{6}$. The superscripts on the gauge bosons in Eq. (2) indicate the electric charge of the particles, some of which are functions of the parameter b.

A. The Minimal Model

In references [14, 50], it was demonstrated that, for $b = 3/2$ (i.e., $\beta = \sqrt{3}$), the following fermion structure is free of all gauge anomalies: $\psi^T_{lL} = (l^-, \nu^0_l, l^+)_L \sim (1, 3^*, 0)$, $Q^T_{iL} = (u_i, d_i, X_i)_L \sim (3, 3, -1/3)$, and $Q^T_{3L}(d_3, u_3, Y) \sim (3, 3^*, 2/3)$, where $l = e, \mu, \tau$ represents the lepton family index, $i = 1, 2$ for the first two quark families, and the quantum numbers after the tilde ($\sim$) denote the 3-3-1 representation. The right-handed fields are $u^c_{aL} \sim (3^*, 1, -2/3)$, $d^c_{aL} \sim (3^*, 1, 1/3)$, $X^c_{iL} \sim (3^*, 1, 4/3)$, and $Y^c_L \sim (3^*, 1, -5/3)$, where $a = 1, 2, 3$ is the quark family index, and there are three exotic quarks with electric charges: $-4/3$ and $5/3$. This version is referred to as *minimal* in the literature because it avoids the use of exotic leptons, including possible right-handed neutrinos.

III. LEPTON AND QUARK GENERATIONS

In what follows, we will propose sets of leptons S_{Li} and quarks S_{Qi} containing triplets (anti-triplets) and singlets of $SU(3)$. These sets must contain at least one SM generation of SM fermions. From Eq. (1), for $\beta = \sqrt{3}$, the electric charges of the 3 and 3^* triplets are: $Q_{\mathrm{QED}}(3) = \mathrm{Diag}(1 + X, X, -1 + X)$ and $Q_{\mathrm{QED}}(3^\star) = \mathrm{Diag}(-1 + X, X, 1 + X)$, respectively. The general expressions for the Z' charges, with the $Z - Z'$ mixing angle equals to zero, are shown in Appendix A. For the SM fields embedded in the sets: S_{L1}, S_{L2},S_{L3}, S_{Q1} and S_{Q2}, the Z' charges are shown in Tables I,II,III, IV and V, respectively.

- Lepton generation $S_{L1} = [(\nu^0_e, e^-, E^{--}_2) \oplus e^+ \oplus E^{++}_2]_L$ with quantum numbers $(1, 3, -1); (1, 1, 1)$ and $(1, 1, 2)$ respectively. The Z' charges for the SM fields are shown in Table I:

$\ell = (\nu_L, e_L)^T \subset 3$, $e_R \subset 1$ (as in S_{L1})		
fields	$g_{Z'}\epsilon^{Z'}_L$	$g_{Z'}\epsilon^{Z'}_R$
ν_e	$\frac{g_L}{\cos\theta_W}\frac{2\cos^2\theta_W - 3}{\sqrt{3(1-4\sin^2\theta_W)}}$	0
e	$\frac{g_L}{\cos\theta_W}\frac{2\cos^2\theta_W - 3}{\sqrt{3(1-4\sin^2\theta_W)}}$	$\frac{g_L}{\cos\theta_W}\frac{\sqrt{3}\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$

TABLE I: Z' chiral charges for the SM leptons and the right-handed neutrino when embedded in S_{L1}. Here, θ_W is the electroweak mixing angle.

- Set $S_{L2} = [(e^-, \nu^0_e, E^+_1) \oplus e^+ \oplus E^-_1]_L$ with quantum numbers $(1, 3^*, 0)$; $(1, 1, 1)$ and $(1, 1, -1)$, respectively. The Z' charges for the SM fields are shown in Table II:

$\ell = (\nu_L, e_L)^T \subset 3^*$, $e_R \subset 1$ (as in S_{L2})		
fields	$g_{Z'}\epsilon^{Z'}_L$	$g_{Z'}\epsilon^{Z'}_R$
ν_e	$\frac{g_L}{\cos\theta_W}\frac{\sqrt{1-4\sin^2\theta_W}}{2\sqrt{3}}$	0
e	$\frac{g_L}{\cos\theta_W}\frac{\sqrt{1-4\sin^2\theta_W}}{2\sqrt{3}}$	$\frac{g_L}{\cos\theta_W}\frac{\sqrt{3}\sin^2\theta_W}{\sqrt{1-4\sin^2\theta_W}}$

TABLE II: Z' chiral charges for the SM leptons and the right-handed neutrino when embedded in S_{L2}. Here, θ_W is the electroweak mixing angle.

- Set $S_{L3} = [(e^-, \nu^0_e, e^+)]_L$ with quantum numbers $(1, 3^*, 0)$. The Z' charges for the SM fields are shown in Table III:

$\ell = (\nu_L, e_L)^T, e_R \subset 3^*$ (as in S_{L3})		
fields	$g_{Z'}\epsilon^{Z'}_L$	$g_{Z'}\epsilon^{Z'}_R$
ν_e	$\frac{g_L}{\cos\theta_W}\frac{\sqrt{1-4\sin^2\theta_W}}{2\sqrt{3}}$	0
e	$\frac{g_L}{\cos\theta_W}\frac{\sqrt{1-4\sin^2\theta_W}}{2\sqrt{3}}$	$\frac{g_L}{\cos\theta_W}\frac{\sqrt{1-4\sin^2\theta_W}}{\sqrt{3}}$

TABLE III: Z' chiral charges for the SM leptons and the right-handed neutrino when embedded in S_{L3}. Here, θ_W is the electroweak mixing angle.

- Set $S_{Q1} = [(d, u, Q^{5/3}_1) \oplus u^c \oplus d^c \oplus Q^c_1]_L$ with quantum numbers $(3, 3^*, 2/3)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ and $(3^*, 1, -5/3)$, respectively. The Z' for the SM fields are shown in Table IV:

$q=(u_L,d_L)^T \subset 3^*$, $u_R, d_R \subset 1$ (as in S_{Q1})		
fields	$g_{Z'}\epsilon_L^{Z'}$	$g_{Z'}\epsilon_R^{Z'}$
u	$\frac{g_L}{\cos\theta_W}\frac{1}{2\sqrt{3\left(1-4\sin^2\theta\right)}}$	$-\frac{g_L}{\cos\theta_W}\frac{2\sin^2\theta_W}{\sqrt{3\left(1-4\sin^2\theta\right)}}$
d	$\frac{g_L}{\cos\theta_W}\frac{1}{2\sqrt{3\left(1-4\sin^2\theta\right)}}$	$\frac{g_L}{\cos\theta_W}\frac{\sin^2\theta_W}{\sqrt{3\left(1-4\sin^2\theta\right)}}$

TABLE IV: Z' chiral charges for the SM quarks when they are embedded in S_{Q1}. Here, θ_W is the electroweak mixing angle.

- Set $S_{Q2}=[(u,d,Q_2^{-4/3})\oplus u^c \oplus d^c \oplus Q_2^c]_L$ with quantum numbers $(3,3,-1/3)$; $(3^*,1,-2/3)$; $(3^*,1,1/3)$ and $(3^*,1,4/3)$, respectively. The Z' charges for the SM fields are shown in Table V:

$q=(u_L,d_L)^T \subset 3$, $u_R, d_R \subset 1$ (as in S_{Q2})		
fields	$g_{Z'}\epsilon_L^{Z'}$	$g_{Z'}\epsilon_R^{Z'}$
u	$\frac{g_L}{\cos\theta_W}\frac{1-2\sin^2\theta_W}{2\sqrt{3\left(1-4\sin^2\theta\right)}}$	$-\frac{g_L}{\cos\theta_W}\frac{2\sin^2\theta_W}{\sqrt{3\left(1-4\sin^2\theta\right)}}$
d	$\frac{g_L}{\cos\theta_W}\frac{1-2\sin^2\theta_W}{2\sqrt{3\left(1-4\sin^2\theta\right)}}$	$\frac{g_L}{\cos\theta_W}\frac{\sin^2\theta_W}{\sqrt{3\left(1-4\sin^2\theta\right)}}$

TABLE V: Z' chiral charges for the SM quarks when they are embedded in S_{Q2}. Here, θ_W is the electroweak mixing angle.

- To cancel anomalies, it is advantageous introducing triplets and anti-triplets of exotic leptons; for example, $S_{E1}=[(N_1^0,E_4^+,E_3^{++})\oplus E_4^- \oplus E_3^{--}]_L$ with quantum numbers $(1,3^*,1)$; $(1,1,-1)$ and $(1,1,-2)$, respectively. We do not report the Z' charges of exotic fermion fields because we assume they have a very high mass.
- Additional exotic lepton sets. $S_{E2}=[(E_5^+,N_2^0,E_6^-)\oplus E_5^- \oplus E_6^+]_L$ with quantum numbers $(1,3,0)$; $(1,1,-1)$ and $(1,1,1)$, respectively. A more economical set is $S_{E3}=[(E_5^+,N_2^0,E_5^-)]$ which has identical contributions to the anomalies as S_{E2} but different Z' charges. However, these details are irrelevant for the low energy phenomenology, so we do not include S_{E3} in Table VI.

IV. IRREDUCIBLE ANOMALY FREE SETS AND MODELS

Table VI shows the contribution of each set to the anomalies. From Table VI, it is possible to obtain the irreducible anomaly-free sets [48], shown in Table VII. The irreducible AFSs Q_i^I, Q_i^{II} and Q_i^{III} in Table VII correspond to fermion sets with one quark family, two quark families, or three quark families, respectively. These sets can be combined to build three family models as shown in Table VIII. There are 33 different models (without considering all the possible embeddings). These models can also be extended by adding vector-like lepton sets, L_i, indicated in the second column of Table VII. To exemplify the possible embeddings we show some cases in Table X. The choice of models in Table X show how the phenomenology depends on the SM fermion embedding in the model. For example, in the case of M10, the embedding determines whether it is strongly coupled. M17 was chosen because it had several embeddings. M3 is the minimal model. M4 is similar to the minimal model but is not universal in the lepton sector.

Anomalías	S_{L1}	S_{L2}	S_{L3}	S_{Q1}	S_{Q2}	S_{E1}	S_{E2}
$[SU(3)_C]^2U(1)_X$	0	0	0	0	0	0	0
$[SU(3)_L]^2U(1)_X$	−1	0	0	2	−1	1	0
$[\text{Grav}]^2U(1)_X$	0	0	0	0	0	0	0
$[U(1)_X]^3$	6	0	0	−12	6	−6	0
$[SU(3)_L]^3$	1	−1	−1	−3	3	−1	1

TABLE VI: Contribution to the anomalies for each family of quarks S_{Q_i}, leptons S_{L_i} and exotics S_{E_i}, for 3-3-1 models with $\beta=\sqrt{3}$.

i	Vector-like lepton set (L_i)	One quark set (Q_i^I)	Two quarks set (Q_i^{II})	Three quarks set (Q_i^{III})
1	$S_{E2}+S_{L2}$	$S_{E2}+2S_{L1}+S_{Q1}$	$S_{L1}+S_{L2}+S_{Q1}+S_{Q2}$	$3S_{L1}+2S_{Q1}+S_{Q2}$
2	$S_{E1}+S_{L1}$	$S_{E1}+2S_{L2}+S_{Q2}$	$S_{L1}+S_{L3}+S_{Q1}+S_{Q2}$	$3S_{L2}+S_{Q1}+2S_{Q2}$
3	$S_{E2}+S_{L3}$	$S_{E1}+S_{L2}+S_{L3}+S_{Q2}$		$3S_{L3}+S_{Q1}+2S_{Q2}$
4		$S_{E1}+2S_{L3}+S_{Q2}$		$2S_{L2}+S_{L3}+S_{Q1}+2S_{Q2}$
5				$S_{L2}+2S_{L3}+S_{Q1}+2S_{Q2}$

TABLE VII: AFSs for $\beta=\sqrt{3}$. We have classified the AFS according to the content of quark families, i.e., Q_i^I, Q_i^{II}, and Q_i^{III}. Combinations of these sets with three SM quark and three SM lepton families can be considered as 3-3-1 models.

	Models	
M1	Q_1^{III}	$3S_{L1}+2S_{Q1}+S_{Q2}$
M2	Q_2^{III}	$3S_{L2}+S_{Q1}+2S_{Q2}$
M3	Q_3^{III}	$3S_{L3}+S_{Q1}+2S_{Q2}$
M4	Q_4^{III}	$2S_{L2}+S_{L3}+S_{Q1}+2S_{Q2}$
M5	Q_5^{III}	$S_{L2}+2S_{L3}+S_{Q1}+2S_{Q2}$
M6	$Q_1^{II}+Q_1^{I}$	$3S_{L1}+S_{L2}+S_{E2}+2S_{Q1}+S_{Q2}$
M7	$Q_1^{II}+Q_2^{I}$	$S_{L1}+3S_{L2}+S_{E1}+S_{Q1}+2S_{Q2}$
M8	$Q_1^{II}+Q_3^{I}$	$S_{L1}+2S_{L2}+S_{L3}+S_{E1}+S_{Q1}+2S_{Q2}$
M9	$Q_1^{II}+Q_4^{I}$	$S_{L1}+S_{L2}+2S_{L3}+S_{E1}+S_{Q1}+2S_{Q2}$
M10	$Q_2^{II}+Q_1^{I}$	$3S_{L1}+S_{L3}+S_{E2}+2S_{Q1}+S_{Q2}$
M11	$Q_2^{II}+Q_2^{I}$	$S_{L1}+2S_{L2}+S_{L3}+S_{E1}+S_{Q1}+2S_{Q2}$
M12	$Q_2^{II}+Q_3^{I}$	$S_{L1}+S_{L2}+2S_{L3}+S_{E1}+S_{Q1}+2S_{Q2}$
M13	$Q_2^{II}+Q_4^{I}$	$S_{L1}+3S_{L3}+S_{E1}+S_{Q1}+2S_{Q2}$
M14	$Q_1^{I}+Q_2^{I}+Q_3^{I}$	$2S_{L1}+3S_{L2}+S_{L3}+2S_{E1}+S_{E2}+S_{Q1}+2S_{Q2}$
M15	$Q_1^{I}+Q_2^{I}+Q_4^{I}$	$2S_{L1}+2S_{L2}+2S_{L3}+2S_{E1}+S_{E2}+S_{Q1}+2S_{Q2}$
M16	$Q_1^{I}+Q_3^{I}+Q_4^{I}$	$2S_{L1}+S_{L2}+3S_{L3}+2S_{E1}+S_{E2}+S_{Q1}+2S_{Q2}$
M17	$Q_2^{I}+Q_3^{I}+Q_4^{I}$	$3S_{L2}+3S_{L3}+3S_{E1}+3S_{Q2}$
M18	$3Q_1^{I}$	$6S_{L1}+3S_{E2}+3S_{Q1}$
M19	$2Q_1^{I}+Q_2^{I}$	$4S_{L1}+2S_{L2}+2S_{E2}+S_{E1}+2S_{Q1}+S_{Q2}$
M20	$2Q_1^{I}+Q_3^{I}$	$4S_{L1}+S_{L2}+S_{L3}+S_{E1}+2S_{E2}+2S_{Q1}+S_{Q2}$
M21	$2Q_1^{I}+Q_4^{I}$	$4S_{L1}+2S_{L3}+S_{E1}+2S_{E2}+2S_{Q1}+S_{Q2}$
M22	$3Q_2^{I}$	$6S_{L2}+3S_{E1}+3S_{Q2}$
M23	$2Q_2^{I}+Q_1^{I}$	$2S_{L1}+4S_{L2}+2S_{E1}+S_{E2}+S_{Q1}+2S_{Q2}$
M24	$2Q_2^{I}+Q_3^{I}$	$5S_{L2}+S_{L3}+3S_{E1}+3S_{Q2}$
M25	$2Q_2^{I}+Q_4^{I}$	$4S_{L2}+2S_{L3}+3S_{E1}+3S_{Q2}$
M26	$3Q_3^{I}$	$3S_{L2}+3S_{L3}+3S_{E1}+3S_{Q2}$
M27	$2Q_3^{I}+Q_1^{I}$	$2S_{L1}+2S_{L2}+2S_{L3}+2S_{E1}+S_{E2}+S_{Q1}+2S_{Q2}$
M28	$2Q_3^{I}+Q_2^{I}$	$4S_{L2}+2S_{L3}+3S_{E1}+3S_{Q2}$
M29	$2Q_3^{I}+Q_4^{I}$	$2S_{L2}+4S_{L3}+3S_{E1}+3S_{Q2}$
M30	$3Q_4^{I}$	$6S_{L3}+3S_{E1}+3S_{Q2}$
M31	$2Q_4^{I}+Q_1^{I}$	$2S_{L1}+4S_{L3}+2S_{E1}+S_{E2}+S_{Q1}+2S_{Q2}$
M32	$2Q_4^{I}+Q_2^{I}$	$2S_{L2}+4S_{L3}+3S_{E1}+3S_{Q2}$
M33	$2Q_4^{I}+Q_3^{I}$	$S_{L2}+5S_{L3}+3S_{E1}+3S_{Q2}$

TABLE VIII: Three-family models built from the irreducible anomaly-free sets (Table VII). It is possible to obtain (trivially) new models by adding vector-like lepton sets; we are not considering these possibilities in our counting unless they are necessary to complete the lepton families.

In general, we obtain three classes of models as we can see below:

- Completely non-universal models: This happens if we embed each of the SM families in different sets; for example, one of the possible embeddings for the M12 model in Table VIII is to put the first lepton family in S_{L3} and the remaining lepton families in S_{L1} and S_{L2}. This class of models usually has very strong restrictions from FCNC and CLFV.

- Universal Models: In several AFSs, there are embeddings with the three families of SM leptons in sets with the same quantum numbers; the same applies for the three families of the SM quarks. For example, in the M26 model in Table VIII, it is possible to embed all the three SM families in the sets $3S_{L3} + 3S_{Q2}$. The remaining fields are considered exotic fermions and are necessary to cancel anomalies.

- The 2 + 1 models: Most AFSs have embeddings where two families are in sets with the same quantum numbers, and the third family is a different set. To avoid the strongest FCNC restrictions, it is necessary that the left-handed doublets of the first two SM quark families have identical quantum numbers. This condition is also desirable for Lepton families, although some models could avoid the FCNC constraints without satisfying this condition. A typical example of these models is the Pisano-Pleitez-Frampton minimal model [13, 14]. $3S_{L3} + S_{Q1} + 2S_{Q2}$ (the M3 model in Table VIII). This model is universal in the lepton sector and non-universal in the quark sector.

V. LHC AND LOW ENERGY CONSTRAINTS

We consider the ATLAS search for high-mass dilepton resonances in the mass range of 250 GeV to 6 TeV in proton-proton collisions at a center-of-mass energy of $\sqrt{s} = 13$ TeV during Run 2 of the LHC with an integrated luminosity of 139 fb^{-1} [51]. This data was collected from searches of Z' bosons decaying dileptons. We obtain the lower limit on the Z' mass from the intersection of the theoretical predictions for the cross-section with the corresponding upper limit reported by ATLAS at a 95% confidence level. We use the expressions given in Ref. [52–54] to calculate the theoretical cross-section. We assume that the $Z - Z'$ mixing angle θ (see appendix D) equals zero for these bounds. In Table IX, the LHC constraints for some models are presented. It is important

Particle content first generation	LHC-Lower limit in TeV
$S_{L3} + S_{Q1}$	7.3
$S_{L3} + S_{Q2}$	6.4

TABLE IX: The lepton families S_{L_1} and S_{L_2} are strongly coupled (For S_{L_1} and S_{L_2} the left-handed lepton doublet ℓ and the right-handed charged lepton singlet e_R have couplings greater than 1, respectively). Therefore only S_{L_3} is phenomenologically viable for the first family. Depending on the quark content, i.e., S_{Q_1} or S_{Q_2}, we have two different constraints.

to stress that the leptons of the first family, i.e., the electron and its neutrino, should be embedded in S_{L3} since it is the only scenario where the right-handed electron has Z' couplings less than 1. In Table IX, this is the best option for models with the first two lepton generations embedded in S_{L3}, as it happens for the minimal model (M3), since having identical quantum numbers for the first and second lepton families avoids possible issues with CLFV and FCNC. To avoid the strongest FCNC constraints in the quark sector, the charges of the left-handed quarks of the first two families should be identical [55]; this feature is assumed to calculate the lower mass limits in Table IX. It is important to stress the non-universal Z' couplings modify processes such as [56]: coherent $\mu - e$ conversion in a muon atom, $K^0 - \bar{K}^0$ and $B - \bar{B}$ mixing, ϵ, and ϵ'/ϵ, lepton, and semileptonic decays (e.g., $\mu \to e\gamma$) which, if observed in the future, the Non-Universal Models will be favored over the Universal ones. For models with a Z' boson coupling in a different way to the third family, there are different predictions for the branching rations $B(t \to Hu)$ and $B(t \to Hc)$. These predictions are strongly constrained by colliders [57]. In Table X, SC stands for strongly coupled, indicating that in the sets S_{L1} and S_{L2}, the coupling of the right-handed electron is greater than one, and therefore, the collider constraints are very strong. Even though Z' with couplings greater than one to the SM fields of the first generation are quite disfavored by colliders [52], strongly coupled models are also attractive in several phenomenological approaches [55, 58]; for this reason, it is important to realize the existence of these models, which naturally appear in 3-3-1 models with large β values. Regarding constraints on exotic particles, the restrictions on the mass of a sequential heavy lepton are above 100 GeV [59]. For exotic quarks t' and b', the allowed mass ranges are above 1370 GeV and 1570 GeV, respectively [59]. The restrictions on fields with exotic electric charges are weaker because the identification algorithms assume the charges are proportional to the charge of the electron [60]. The presence of doubly charged exotic leptons can generate new decay channels in proton-proton collisions at very high energies. In Figure 1, the Feynman diagram for the process $q\bar{q} \to Z' \to E^{++}E^{--} \to \ell^+\ell^-\gamma \to \ell^+\ell^-\mu^+\mu^-(\tau^+\tau^-)$, generating four boosted leptons in the final state (the doubly charged exotic lepton appears in S_{L1}, which strongly couples the Z'; for this reason, to avoid collider constraints, we restrict to leptons of the second or third family). On the other hand, exotic quarks modify the $K^0 - \bar{K}^0$ mixing, as shown in Figure 2. Fermions with exotic electric charges can contribute to several processes; however, an exhaustive study of these processes is beyond the purpose of this work.

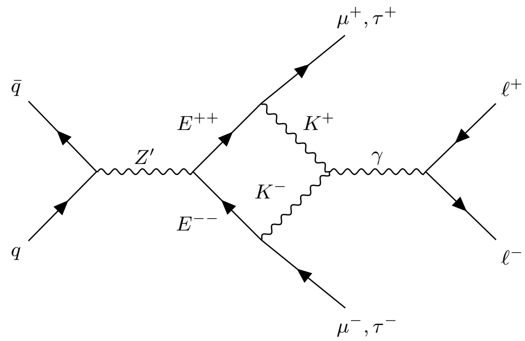

FIG. 1: Doubly charged exotic lepton contribution to the process $q\bar{q} \to Z' \to E^{++}E^{--} \to \ell^+\ell^-\gamma \to \ell^+\ell^-\mu^+\mu^-(\tau^+\tau^-)$.

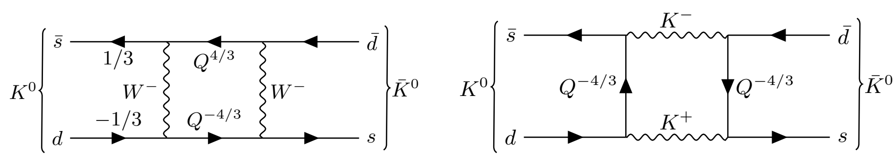

FIG. 2: Exotic quark contribution to the $K^0 - \bar{K}^0$ mixing.

Model	j	SM Lepton Embeddings	Universal	$2+1$	Quark Configuration	LHC-Lower limit
$M3 = Q_3^{III}$(Minimal)	-	$[\,3S_{L3}^{\bar{\ell}+e'^+}]$	✓	×	$2S_{Q2} + S_{Q1}$	6.4 TeV
$M4 = Q_4^{III}$	-	$[2S_{L2}^{\bar{\ell}+e^+} + S_{L3}^{\bar{\ell}+e'^+}]$	×	✓	$2S_{Q2} + S_{Q1}$	6.4 TeV
$M6 = (Q_1^I + Q_1^{II})^j$	1	$[3S_{L1}^{\ell+e^+}]+S_{L2} + S_{E2}$	✓	×	$2S_{Q1} + S_{Q2}$	SC
	2	$[2S_{L1}^{\ell+e^+} + S_{L2}^{\bar{\ell}+e^+}] + S_{L1} + S_{E2}$	×	✓	$2S_{Q1} + S_{Q2}$	SC
$M17 = (Q_2^I + Q_3^I + Q_4^I)^j$	1	$[3S_{L2}^{\bar{\ell}+e^+}]+3S_{L3} + 3S_{E1}$	✓	×	$3S_{Q2}$	SC
	2	$[3S_{L3}^{\bar{\ell}+e'^+}]+3S_{L2} + 3S_{E1}$	✓	×	$3S_{Q2}$	6.4 TeV
	3	$[2S_{L2}^{\bar{\ell}+e^+} + S_{L3}^{\bar{\ell}+e'^+}] + S_{L2} + 2S_{L3} + 3S_{E1}$	×	✓	$3S_{Q2}$	6.4 TeV
	4	$[S_{L2}^{\bar{\ell}+e^+} + 2S_{L3}^{\bar{\ell}+e'^+}]+2S_{L2} + S_{L3} + 3S_{E1}$	×	✓	$3S_{Q2}$	6.4 TeV
$M10 = (Q_1^I + Q_2^{II})^j$	1	$[3S_{L1}^{\ell+e^+}] + S_{L3} + S_{E2}$	✓	×	$2S_{Q1} + S_{Q2}$	SC
	2	$[2S_{L1}^{\ell+e^+} + S_{L3}^{\bar{\ell}+e'^+}]+S_{L1} + S_{E2}$	×	✓	$2S_{Q1} + S_{Q2}$	7.3 TeV

TABLE X: Alternative embeddings of the SM fields for some of the models in Table VIII. The lepton sets in square brackets contain the standard model fields. The superscripts correspond to the particle content of the SM, where ℓ ($\bar{\ell}$) stands for a left-handed lepton doublet embedded in a $SU(3)_L$ triplet (anti-triplet), and e'^+ (e^+) is the right-handed charged lepton embedded in a $SU(3)_L$ triplet (singlet). The check mark ✓ means that at least two (2+1) or three (universal) families have the same charges under the gauge symmetry. The cross × stands for the opposite. LHC constraints are obtained from Table IX for embeddings in which we can choose the same Z' charges for the first two families, otherwise, we leave the space blank. To avoid a strongly coupled model in the Lepton sector, it is necessary to embed the first Lepton family (electron and electron neutrino) in S_{L3}. This feature will be helpful to distinguish between the different embeddings. The embedding also defines the content of exotic particles in each case.

VI. CONCLUSIONS

Since that for 3-3-1 models, the absolute value of the parameter β must be less than $\beta \lessapprox \cot\theta_W = 1.8$ (for $\sin^2\theta_W = 0.231$ in the $\overline{\mathrm{MS}}$ renormalization scheme at the Z-pole energy scale), and the values of β are further limited by the requirement that the vector boson charges be integers, the possible values of this parameter are reduced to a few cases. For a realistic model, the maximum possible value corresponds to $\beta = \sqrt{3} \sim 1.73$. This case is important since it contains the Pleitez-Frampton minimal model. We have constructed three sets of lepton families, S_{Li}, two quark families, S_{Qi}, and two exotic lepton families S_{Ei}, and we calculated their contribution to anomalies. In our analysis, we obtained 14 irreducible AFSs, from which we built 33 non-trivial 3-3-1 models (without considering the different embeddings) with at least three quark and three lepton families for each case. Each of these embeddings constitutes a phenomenologically distinguishable model; however, we limited our analysis of the possible embeddings to a few cases. In the same way, from our analysis of the 3-3-1 models with $\beta = \sqrt{3}$ we report the couplings of the SM fields to the Z' boson for all the possible quark and lepton families and the corresponding lower limits on the Z' mass. We also discuss the conditions under which the reported models avoid FCNC and CLFV. We also observed that strongly coupled models appear naturally and require a high value for the Z' mass. They can be helpful in specific phenomenological approaches based on models with strong dynamics. In the future, a detailed analysis of each model will be necessary; however, this is beyond the scope of the present work.

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Appendix A: Z' charges for a general 3-3-1 model

At low energy, the 3-3-1 models, i.e., the gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ reduces to the low energy effective theory $SU(3)_C \otimes SU(2)_L \otimes U(1)_{8L} \otimes U(1)_X \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. From the covariant derivatives, for the neutral currents, we obtain the interaction Lagrangian

$$-\mathcal{L} \supset g_L J^\mu_{3L} A_{3L\mu} + g_L J^\mu_{8L} A_{8L\mu} + g_X J^\mu_X A_{X\mu}\,, \tag{A1}$$

which can be written as

$$\begin{aligned} -\mathcal{L}_{NC} =& g_i J_{i\mu} A^\mu_i = g_j J_{j\mu} O_{jk} O^T_{kl} A^\mu_l\,, \\ =& \tilde{g}_k \tilde{J}_{k\mu} \tilde{A}^\mu_k\,, \end{aligned} \tag{A2}$$

where $\tilde{A}^\mu_k = O^T_{kl} A^\mu_l$, then $(A^\mu_1, A^\mu_2) = (A^\mu_{8L}, A^\mu_X)$, $(\tilde{A}^\mu_1, \tilde{A}^\mu_2) = (B^\mu, Z'^\mu)$, $(g_1 J^\mu_1, g_2 J^\mu_2) = (g_L A^\mu_{8L}, g_X A^\mu_X)$ and $(\tilde{g}_1 \tilde{A}^\mu_1, \tilde{g}_2 \tilde{A}^\mu_2) = (g_Y J^\mu_Y, g_{Z'} J^\mu_{Z'})$. At high energies, the symmetry is broken following the breaking chain $SU(3)_C \otimes SU(3)_L \otimes U_X(1) \to SU(3)_C \otimes SU(2)_L \otimes U_{8L}(1) \otimes U_X(1) = SU(3)_C \otimes SU(2)_L \otimes U_Y(1) \otimes U'(1)$, i.e.,

$$\begin{pmatrix} A_{3L} \\ B^\mu \\ Z'^\mu \end{pmatrix} = \begin{pmatrix} 1 & 0_{1\times 2} \\ 0_{2\times 1} & O^T_{2\times 2} \end{pmatrix} \begin{pmatrix} A_{3L} \\ A^\mu_{8L} \\ A^\mu_X \end{pmatrix}\,. \tag{A3}$$

Next step $SU(3)_C \otimes SU(2)_L \otimes U_Y(1) \otimes U'(1) \to SU(3)_C \otimes U_{\mathrm{QED}}(1)$, i.e.,

$$\begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix} = \begin{pmatrix} \sin\theta_W & \cos\theta_W & 0 \\ \cos\theta_W & -\sin\theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_{3L} \\ B^\mu \\ Z'^\mu \end{pmatrix}\,. \tag{A4}$$

Where the fields correspond to the SM photon A^μ and the Z^μ boson, and a heavy vector-boson Z'. Proceeding similarly for the currents, and limiting ourselves to the fields on which the orthogonal submatrix $Q_{2\times 2}$ acts, from Eq. (A2) we obtain $\tilde{g}_k \tilde{J}^\mu_k = g_j J^\mu_j O_{jk}$, i.e.,

$$\begin{aligned} \tilde{g}_k \tilde{J}_{k\mu} =& \left(g_Y J^\mu_Y,\ g_{Z'} J^\mu_{Z'}\right) = \left(g_L J^\mu_{L8},\ g_X J^\mu_X\right) \begin{pmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{pmatrix}\,, \\ =& \left(g_L J^\mu_{L8} O_{11} + g_X J^\mu_X O_{21},\ g_L J^\mu_{L8} O_{12} + g_X J^\mu_X O_{22}\right)\,. \end{aligned} \tag{A5}$$

Without further assumption

$$\begin{pmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{pmatrix} = \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix}\,, \tag{A6}$$

so that

$$\begin{aligned} g_Y J^\mu_Y =& \quad g_L J^\mu_{L8} \cos\omega + g_X J^\mu_X \sin\omega\,, \\ g_{Z'} J^\mu_{Z'} =& -g_L J^\mu_{L8} \sin\omega + g_X J^\mu_X \cos\omega\,. \end{aligned} \tag{A7}$$

The charge operator in a 3-dimensional representation is given by

$$Q_{\mathrm{QED}} = T_{L3} + \beta T_{L8} + X\mathbb{1}\,, \tag{A8}$$

hence

$$Y = \beta T_{L8} + X\,. \tag{A9}$$

From this expression, it is possible to obtain a relation between the currents (the currents are proportional to the charges)

$$J^\mu_Y = \beta J^\mu_{L8} + J^\mu_X \,. \tag{A10}$$

Comparing this result with (A7)

$$\beta = \frac{g_L \cos\omega}{g_Y}, \qquad 1 = \frac{g_X \sin\omega}{g_Y}\,. \tag{A11}$$

From $\cos^2\omega + \sin^2\omega = 1$, we obtain

$$\left(\frac{\beta}{g_L}\right)^2 + \left(\frac{1}{g_X}\right)^2 = \frac{1}{g_Y^2}\,. \tag{A12}$$

In the SM, $g_L \approx 0.652$ and $g_Y = g_L \tan\theta_W$,

$$g_X = \frac{g_L \tan\theta_W}{\sqrt{1-\beta^2\tan^2\theta_W}}\,. \tag{A13}$$

This expression shows that the parameter β cannot be arbitrarily large from the matching conditions $\beta \lessapprox \cot\theta_W$; some care must be taken on this approximation since this is a renormalization-scheme dependent inequality. From these expressions, we obtain

$$\cos\omega = \frac{\beta}{g_L} g_Y = \beta\tan\theta_W\,, \quad \sin\omega = \sqrt{1-\beta^2\tan^2\theta_W}\,. \tag{A14}$$

From Eq. (A7), $g_{Z'}\epsilon_{Z'} = -g_L T_{8L}\sin\omega + g_X X_X \cos\omega$, we obtain

$$\begin{aligned} g_{Z'}\epsilon_{Z'} &= -g_L T_{L8}\sqrt{1-\beta^2\tan^2\theta_W} + \beta\frac{g_L\tan^2\theta_W X}{\sqrt{1-\beta^2\tan^2\theta_W}}, \\ &= g_L\left(-T_{L8}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X\right)\,, \end{aligned} \tag{A15}$$

where $\tilde{\alpha} = \sqrt{1-\beta^2\tan^2\theta_W} = \frac{1}{\cos\theta_W}\sqrt{1-4\sin^2\theta_W}$ for $\beta = \sqrt{3}$.

Appendix B: Chiral charges for the 3 representation

In what follows, we propose sets of fermions representing the particle content of a generation of leptons or quarks, for left-handed triplets 3, and for right-handed fermions in an $SU(3)_L$ singlet, in general we have

$$g_{Z'}\epsilon^{Z'}_L(3) = g_L\begin{pmatrix} -\frac{1}{2\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X & 0 & 0 \\ 0 & -\frac{1}{2\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X \end{pmatrix}\,, \quad \epsilon^{Z'}_R = g_L\beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X_R\,. \tag{B1}$$

Here we add the subindex R to the X-charge of the right-handed singlet to emphasize that it differs from the quantum number of the left-handed triplet, i.e., X. If the charge conjugate of the right-handed fermion is identified with the third component of a triplet, then $\epsilon^{Z'}_R = -g_L\left(\frac{1}{\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X\right)$.

Appendix C: The conjugate representation $3^\star$

To cancel the anomalies of $SU(3)_L$, triplets must be put in the conjugate representation. In general, for any set of generators T^a of an $SU(N)$ symmetry with $N \leq 3$ there exists another set of generators $-T^{a*}$, which satisfy the same algebra. This set of generators spawns the so-called conjugate representation of $SU(N)$. With these generators, we can build charge operators and multiplets containing the SM particles. To compare with the conjugate representation, we use the projectors

$$p_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\,, \quad \tilde{p}_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\,. \tag{C1}$$

They should not be confused with permutation operators, as the purpose of these operators is to compare only the first two rows of the charge operators. $\tilde{p}_{12}$ also permutes the first two eigenvalues to make a proper comparison with the conjugate operator. We can obtain the X^C, i.e., the charge of the triplet $3^\star$ in the conjugate representation, from the equation

$$\begin{aligned} &\tilde{p}_{12}\left(T_{L3} + \beta T_{L8} + X\mathbb{1}\right)\tilde{p}^T_{12} \\ =&p_{12}\left(-T_{L3} - \beta T_{L8} + X^C\mathbb{1}\right)p^T_{12}\,, \end{aligned} \tag{C2}$$

only the signs of the $SU(3)$ generators were changed. This matrix equation is equivalent to a couple of linear equations. These equations have the solution $X^C = \left(\frac{\beta}{\sqrt{3}} + X\right) = (1+X)$. An equivalent treatment is to obtain the conjugate representation from $T_{3L} - \beta T_{8L} + X^c$, which generates the exact electric charges but in a different order. We verify that both approaches contribute identically to the anomalies, developing the same particle content and models. For left-handed triplets in the conjugate representation $3^\star$, and right-handed fermions in an $SU(3)_L$ singlet, we have, in general,

$$g_{Z'}\epsilon_L^{Z'}(3^\star) = g_L \begin{pmatrix} +\frac{1}{2\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C & 0 & 0 \\ 0 & +\frac{1}{2\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C & 0 \\ 0 & 0 & -\frac{1}{\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C \end{pmatrix}, \quad \epsilon_R^{Z'} = g_L\beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C . \tag{C3}$$

If the charge conjugate of the right-handed fermion is identified with the third component of a triplet, then $\epsilon_R^{Z'} = -g_L\left(-\frac{1}{\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C\right)$.

Appendix D: Z-Z' Mixing

Mixing angle θ between Z and Z' is tightly constrained [61], i.e., $\theta < 10^{-3}$; however, in several phenomenological analyses, it is still useful delivering expressions for the mass eigenstates.

$$\begin{aligned} Z_1^\mu &= \quad Z^\mu\cos\theta + Z'^\mu\sin\theta, \\ Z_2^\mu &= -Z^\mu\sin\theta + Z'^\mu\cos\theta \ . \end{aligned} \tag{D1}$$

At low energies, Z_1 is identified with the SM Z boson. In order to keep the Lagrangian invariant, this field rotation must be compensated by the corresponding rotation of the currents,

$$\begin{aligned} g_1 J_1^\mu &= \quad g_Z J_Z^\mu\cos\theta + g_{Z'}J_{Z'}^\mu\sin\theta, \\ g_2 J_2^\mu &= -g_Z J_Z^\mu\sin\theta + g_{Z'}J_{Z'}^\mu\cos\theta \ . \end{aligned} \tag{D2}$$

From which we get

$$\begin{aligned} g_1 Q_1 &= \quad g_Z Q_Z\cos\theta + g_{Z'}Q_{Z'}\sin\theta, \\ g_2 Q_2 &= -g_Z Q_Z\sin\theta + g_{Z'}Q_{Z'}\cos\theta \ . \end{aligned} \tag{D3}$$

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