

BÚSQUEDA DE MODELOS 3-3-1 Y
FENOMENOLOGÍA,
Código 2686

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Resumen

Este informe final presenta los objetivos, metodología, resultados y conclusiones del proyecto *Búsqueda de modelos 3-3-1 y fenomenología* (código 2686), ejecutado en la Universidad de Nariño bajo los términos de la VIIS. El trabajo abordó tres frentes complementarios: (i) clasificación y fenomenología de modelos $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) con cargas eléctricas exóticas y acoplamientos no universales, enfatizando el rol del parámetro β y los límites sobre el bosón neutro pesado Z' ; (ii) formulación de una extensión electrodébil efectiva con dos $U(1)$ no universales, con sector escalar tipo 2HDM + singlete y una condición de acople *un fermión derecho* \leftrightarrow *un doblete* que garantiza FCNC= 0 en el sector de Higgs; y (iii) desarrollo de un marco axion-sabor mínimo con cinco ceros de textura que conecta jerarquías de masas y mezclas con la solución de Peccei-Quinn al problema CP fuerte.

Los productos incluyen tres artículos A1, ponencias en COMHEP 2022/2023 y SILAFEA 2024, y acciones de formación (apoyo a estudiante para la 12th AstroTwinCoLO School). Se establecen ventanas de parámetros y firmas experimentales clave (dileptones de alta masa, $t\bar{t}$ resonante, canales $\gamma\gamma$ y $Z\gamma$, mezclas mesónicas), así como lineamientos para ajustes globales y proyecciones HL-LHC. El conjunto de resultados amplía el alcance originalmente comprometido y consolida rutas futuras que integran sabor, gauge y axiones en un marco coherente y reproducible.

Palabras clave: modelos 3-3-1, Z' , FCNC, 2HDM, singlete escalar, axion de Peccei-Quinn, texturas con ceros, teorías efectivas con $U(1)$ no universales.

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1. Introducción

El presente informe final presenta los objetivos, compromisos, metodología y resultados del proyecto de investigación en física de altas energías desarrollado en la Universidad de Nariño, con una duración de 24 meses (con posibilidad de prórroga), de acuerdo con los términos establecidos por la VIIS [1]. El proyecto se enfocó en problemas abiertos más allá del Modelo Estándar (ME), tales como el origen de masas y mezclas fermiónicas, la cancelación controlada de corrientes neutras con cambio de sabor (FCNC), la aparición de bosones neutros pesados y señales tipo Higgs, y la búsqueda de estructuras de sabor compatibles con restricciones fenomenológicas actuales.

La estrategia de trabajo se articuló en tres frentes complementarios: (i) el análisis y la clasificación de modelos $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3–3–1), incluyendo escenarios con cargas eléctricas exóticas y acoplamientos no universales,

con énfasis en el papel del parámetro β en el operador de carga eléctrica; (ii) la construcción de un marco axion–sabor mínimo con texturas con ceros, orientado a reproducir masas y mezclas de quarks y leptones, y a estudiar restricciones de decaimientos raros y acoplos del axión; y (iii) el estudio de teorías efectivas con dos $U(1)$ no universales en el sector electrodébil, con énfasis en el potencial escalar, el patrón de rompimiento y las condiciones de alineamiento del Higgs (implementando la condición *un fermión derecho* \leftrightarrow *un doblete* para FCNC= 0).

Como productos derivados, el proyecto generó publicaciones en revistas indexadas de alto impacto: una clasificación y estudio fenomenológico de modelos 3–3–1 con cargas exóticas [3]; un modelo axion–sabor mínimo con cinco ceros en las matrices de masa, capaz de reproducir el sector de sabor observacional [2]; y un marco efectivo del Modelo Estándar con dos $U(1)$ no universales que organiza de manera coherente las FCNC y la fenomenología gauge/escalar asociada [4].

La socialización de resultados se realizó mediante ponencias nacionales e internacionales, que incluyen un análisis integral de modelos 3–3–1, una clasificación con cargas exóticas y la exploración de señales resonantes tipo Higgs en marcos de dos dobletes (2HDM), presentadas en COMHEP 2022, COMHEP 2023 y SILAFAE 2024 [5, 6, 7]. Todas las publicaciones y ponencias reconocen el apoyo institucional según los compromisos de crédito y divulgación definidos por la VIIS [1]. En las secciones siguientes se detallan la metodología, resultados, discusión, conclusiones, recomendaciones y agradecimientos correspondientes.

2. Metodología

2.1. Enfoque general

El proyecto (código **2686**) se ejecutó con un enfoque teórico–fenomenológico iterativo: (i) construcción y clasificación de modelos gauge más allá del Modelo Estándar (ME); (ii) derivación de contenidos de campos, cargas y acoplamientos; (iii) formulación de lagrangianos y ruptura espontánea de simetrías; (iv) obtención de espectros de masas y corrientes; (v) confrontación con restricciones de colisionadores, precisión electrodébil y sabor; y (vi) difusión de resultados en revistas A1 y eventos científicos, en concordancia con los compromisos institucionales (informes semestrales, socialización y producto académico) [1].

2.2. Ejes de trabajo y procedimiento

Marco teórico y clasificación 3–3–1 con cargas exóticas. Se sistematizó la construcción de modelos $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ con $\beta = \sqrt{3}$, definiendo conjuntos de fermiones (tripletes, antitripletos y singletes), calculando contribuciones a anomalías y construyendo *irreducible anomaly-free sets* (IAFS) como base para modelos de una y tres familias. Se reportó una clasificación general, condiciones para evadir FCNC/CLFV y límites del LHC sobre el bosón neutro pesado [3].

Extensión electrodébil con dos $U(1)$ no universales. Se propuso y estudió un modelo $SU(3)_C \otimes SU(2)_L \otimes U(1)_\alpha \otimes U(1)_\beta$ con cancelación de anomalías por *interplay* entre familias; requiere dos dobletes de Higgs y un singlete escalar. Se demostró la ausencia de FCNC escalar si cada fermión diestro acopla a un único doblete, y se analizaron dos potenciales escalares, contrastando con posibles señales resonantes tipo Higgs [4].

Lagrangianos, ruptura de simetría y matrices de masa. Se construyeron términos de Yukawa y gauge, se implementaron las etapas de ruptura (alta escala mediante el singlete y EWSB con los dobletes), y se derivaron corrientes neutras (incluido Z') y acoplamientos quirales; en el caso no universal se analizó la mezcla Z - Z' bajo el límite de ángulo pequeño consistente con datos de precisión [4].

Restricciones fenomenológicas. Se confrontaron acoplamientos y espectros con: (a) búsquedas de resonancias dileptónicas y $t\bar{t}$ en ATLAS/CMS (límites directos sobre Z'); (b) electrodébil de precisión; y (c) observables de sabor dominados por mezclas de mesones neutros. Este triángulo de pruebas fijó regiones viables y *benchmarks* [4, 3].

Espectro escalar y excesos tipo Higgs. Con dos dobletes de Higgs más un singlete σ (necesario para la masa de Z'), se obtuvieron tres escalares CP-pares, un pseudoescalar y un cargado. Se realizaron escaneos del potencial y se compararon distribuciones de masas con excesos reportados en $\gamma\gamma$, $Z\gamma$ y canales dibosónicos [4].

Sector de sabor y axiones (texturas con ceros). En paralelo, se abordó el problema de sabor con texturas hermiticas y una simetría $U(1)_{PQ}$ mínima, reproduciendo masas y mezclas y aplicando restricciones desde decaimientos leptónicos y $K^\pm \rightarrow \pi^\pm \nu \bar{\nu}$, así como el acople axión-fotón [2].

Difusión y validación por pares. Los resultados de clasificación 3-3-1, de la fenomenología asociada y del análisis escalar se socializaron en COMHEP 2022 y 2023, y SILFAE 2024, cumpliendo la socialización comprometida [5, 6, 7].

2.3. Flujo de trabajo y gestión

El trabajo siguió un ciclo *teoría* \rightarrow *construcción* \rightarrow *restricciones* \rightarrow *ajuste* \rightarrow *difusión*, con seminarios internos periódicos y revisión cruzada de cálculos. Se documentaron supuestos, *benchmarks* y cortes experimentales, manteniendo trazabilidad de entregables según lineamientos institucionales [1].

2.4. Criterios de éxito y reproducibilidad

(i) Consistencia gauge (cancelación de anomalías, cargas y corrientes); (ii) generación de matrices de masas y *mixing* acordes con el ME; (iii) compatibilidad con límites de colisionadores, precisión y sabor; (iv) publicación en A1 y certificaciones de ponencias; y (v) detalles técnicos suficientes para reproducir escaneos (rangos del potencial, jerarquías de VEVs y límites sobre la mezcla $Z-Z'$) [3, 4, 2].

2.5. Gestión de riesgos

Se mitigaron riesgos teóricos (cerramiento de anomalías y ausencia de FCNC) mediante asignaciones de cargas y condiciones de acople; y riesgos fenomenológicos (límites LHC y precisión) mediante escaneos y selección de regiones viables. La ejecución se mantuvo alineada con los compromisos institucionales [1].

3. Resultados

A continuación se consignan los **resultados de investigación** alcanzados por el proyecto *Búsqueda de modelos 3-3-1 y fenomenología* (código 2686), en concordancia con los compromisos establecidos por la VIIS-Universidad de Nariño [1].

3.1. Publicaciones en revistas indexadas (A1)

- **A minimal axion model for mass matrices with five texture-zeros.** Conexión entre simetrías tipo PQ/axiones y ceros de textura en matrices de masa; implicaciones fenomenológicas en el sector fermiónico [2].
- **Alternative 3-3-1 models with exotic electric charges.** Clasificación exhaustiva de modelos 3-3-1 con no universalidad y cargas eléctricas exóticas; análisis de restricciones de colisionadores para nuevas resonancias (Z' , etc.) [3].
- **The Standard Model of Particle Physics as an effective theory from two non-universal $U(1)$.** Formulación efectiva con dos $U(1)$ no universales; contraste con datos de LHC (señales tipo Higgs), electrodébil y sabor [4].

3.2. Socialización de resultados en eventos científicos

- **XV SILAFAE 2024 (Cinvestav, México): *Exploring Higgs-like Resonant Signals within a 2HDM Framework.*** (Certificado y programa en *Ponencias.pdf*, Anexos.)

- **8th ComHEP 2023 (Colombia):** *Classification for Alternative 3–3–1 models with exotic electric charges.* (Certificado en *Ponencias.pdf*, Anexos.)
- **7th ComHEP 2022 (Colombia):** *Alternative 3–3–1 models: a comprehensive analysis.* (Láminas y constancia en *Ponencias.pdf*, Anexos.)

3.3. Aportes científicos del proyecto

- **Catálogo y fenomenología 3–3–1:** construcción y clasificación de familias de modelos libres de anomalía con acoplamientos no universales; estudio de restricciones de colisionadores y posibles señales de nuevas resonancias [3].
- **Dos $U(1)$ no universales como teoría efectiva:** mapeo de parámetros y contraste con datos de LHC, electrodébil y sabor; discusión de señales tipo Higgs [4].
- **Texturas de masa y axiones:** implementación de ceros de textura compatibles con simetrías de sabor y un sector axiónico mínimo con consecuencias en jerarquías de masas [2].

3.4. Formación de talento humano y apoyo estudiantil

- **Apoyo económico a estudiante:** *Maderli Selena Toro* asistió a la *12th AstroTwinCoLO School* (U. de Antioquia, Medellín; tópico: *Weak Gravitational Lensing Techniques*). (Constancia en *Ponencias.pdf*, Anexos.)

3.5. Gestión y ejecución del proyecto

- **Cumplimiento de compromisos VIIS:** productos académicos (artículos A1) y socialización de resultados en evento nacional/internacional, conforme a [1].
- **Duración y prórroga:** ejecución inicial de 24 meses con prórroga autorizada (ver [1] y Actas en Anexos).

Resumen de productos (ver Tabla 1)

4. Discusión

4.1. Qué propusimos y qué logramos

El proyecto aprobado por la VIIS de la Universidad de Nariño (*Búsqueda de modelos 3–3–1 y fenomenología asociada*, código 2686) planteó como objetivo central la clasificación y el análisis fenomenológico de modelos 3–3–1, con productos de divulgación y al menos un artículo indexado con reconocimiento

Tabla 1: Resumen de productos del proyecto.

Tipo	Producto / Referencia
Artículo A1	A minimal axion model for mass matrices with five texture-zeros [2]
Artículo A1	Alternative 3–3–1 models with exotic electric charges [3]
Artículo A1	SM as an effective theory from two non-universal $U(1)$ [4]
Ponencia int.	XV SILAFAE 2024: 2HDM Higgs-like signals (Anexos: <i>Ponencias.pdf</i>)
Ponencia nac.	ComHEP 2023: Classification for Alt 3–3–1 (Anexos: <i>Ponencias.pdf</i>)
Ponencia nac.	ComHEP 2022: Alt 3–3–1 comprehensive analysis (Anexos: <i>Ponencias.pdf</i>)
Formación	Apoyo a estudiante: 12th AstroTwinCoLO School (Anexos)

de la financiación institucional [1]. Durante su ejecución se alcanzaron resultados que no sólo cumplen, sino que además *amplían* el alcance original: (i) una clasificación exhaustiva de modelos alternativos 3–3–1 con cargas eléctricas exóticas y su fenomenología colisionaria y de *flavor* [3]; (ii) un marco efectivo del Modelo Estándar (ME) proveniente de dos simetrías $U(1)$ no universales, con dobletes de Higgs múltiples y condiciones claras para suprimir FCNC [4]; y (iii) un modelo axiónico mínimo que resuelve simultáneamente el problema CP fuerte y reproduce texturas realistas de masas con cinco ceros [2]. Estos productos fueron socializados mediante ponencias nacionales e internacionales y con apoyo directo a formación de talento humano (estudiantil) acorde con los compromisos del proyecto.

4.2. Puntos más álgidos y caminos de solución

(1) Modelos 3–3–1 con cargas exóticas: no universalidad vs. restricciones de *flavor* y colisionarias. *Tensión:* La no universalidad de las cargas bajo el bosón gauge adicional (Z') tiende a inducir transiciones de *flavor* prohibidas a árbol (FCNC) y pone a prueba los acoplamientos leptónicos y hadrónicos mediante búsquedas directas en colisionadores. Además, ciertas incrustaciones de las familias pueden tensionar límites de precisión si no se controlan las asimetrías de carga. *Camino de solución:* Se llevó a cabo una **clasificación sistemática** de los *embeddings* leptónicos y hadrónicos permitidos, identificando configuraciones en las que (a) las dos primeras familias de quarks comparten las mismas cargas para minimizar FCNC y (b) la asignación leptónica evita acoplamientos excesivos al Z' . Con ello se delimitaron *ventanas de parámetros* (incluida la del parámetro β característico de estos modelos) que compatibilizan datos de precisión, pautas de *flavor* y límites colisionarios multi-TeV [3]. *Resultado:* La clasificación obtenida proporciona un *mapa* de escenarios viables y sus firmas experimentales dominantes, que guían nuevas pruebas en LHC y futuros colisionadores.

(2) Dos $U(1)$ no universales y 2HDM: FCNC, mezcla $Z-Z'$, y señales tipo Higgs. *Tensión:* La presencia de dos $U(1)$ no universales, junto con un sector escalar tipo 2HDM (o 2HDM + singlete), puede generar FCNC mediadas

por Higgs y mezcla cinética o masiva entre Z y Z' , sometida a cotas de precisión electrodébil. *Camino de solución:* El análisis mostrado en [4] implementa una **asignación de Yukawas** en la que cada fermión derecho acopla de forma *exclusiva* a un doblete, suprimiendo exactamente las FCNC de Higgs (*alignment* por construcción). Simultáneamente, la mezcla Z - Z' se restringe a valores pequeños compatibles con datos de precisión. El espectro escalar resultante (tres CP-par, uno CP-impar y cargados) deja *abierta* la interpretación de señales tipo Higgs reportadas por ATLAS/CMS dentro de un marco con no universalidad bien controlada. *Resultado:* El ME como teoría efectiva emergente de $U(1) \times U(1)'$ no universal queda *coherente* con observables de precisión y sugiere canales de búsqueda específicos en resonancias escalares y en Z' ligeros a intermedios, sin requerir ajuste fino severo.

(3) Axión + texturas con cinco ceros: sabor y CP fuerte con naturalidad. *Tensión:* Los *ansätze* con seis ceros (tipo Fritzsche) no acomodan simultáneamente todos los ángulos y jerarquías del sector quark; a la vez, resolver el problema CP fuerte con un axiÓN de Peccei–Quinn sin arruinar la fenomenología de sabor es desafiante. *Camino de solución:* En [2] se construye un **marco axiónico mínimo** con matrices de masa hermiticas (vía descomposición polar) y cinco ceros, normalizando las cargas PQ al anómalo QCD. Las texturas resultantes ajustan masas y mezclas de quarks y leptones con *Yukawas más naturales* que en el ME, mientras que el acople axiÓN–materia está controlado por f_a^{-1} y respeta límites indirectos de baja energía. *Resultado:* Se obtiene un escenario predictivo que conecta sabor y CP fuerte, compatible con los límites axiónicos contemporáneos y con espacio para pruebas experimentales en búsquedas de axiones y observables raros de *flavor*.

4.3. Convergencias y siguientes pasos

1. **Observables discriminantes.** Las tres líneas convergen en firmas claras: conversión coherente $\mu \rightarrow e$ en núcleos, mezclas K^0 - \bar{K}^0 y B^0 - \bar{B}^0 , canales raros de top ($t \rightarrow Hc, Hu$) y búsquedas directas de Z' en el rango multi-TeV. Priorizar límites de *flavor* con correlaciones gauge/escalares puede cerrar regiones de parámetros.
2. **Ajustes globales.** Integrar de forma sistemática datos de precisión electrodébil, límites de mezcla Z - Z' , y espectros escalares (2HDM + singlete), realizando *scans* conjuntos que identifiquen islas viables y proyecciones de sensibilidad.
3. **Puentes sabor–gauge.** Explorar cómo texturas con cinco ceros y simetrías PQ pueden emerger de simetrías horizontales (gaugeadas o discretas) ya presentes en extensiones 3–3–1, unificando las tres líneas del proyecto en una única narrativa de sabor+gauge.
4. **Fenomenología extendida.** En 3–3–1, estudiar leptones doblemente cargados y quarks exóticos en canales multileptón de alta energía (y topo-

logías *boosted*); en $U(1) \times U(1)'$, detallar correlaciones entre resonancias escalares y firmas de Z' con no universalidad controlada; en el marco axiónico, cuantificar proyecciones de futuros experimentos sobre f_a .

4.4. Reflexión final y coherencia del documento

El programa de trabajo abordó el núcleo comprometido en [1] (clasificación 3–3–1 y su fenomenología) y, además, avanzó más allá con (a) un mecanismo constructivo para $FCNC=0$ en 2HDM no universal [4] y (b) un marco axiónico mínimo y predictivo [2]. Para mantener la coherencia global del informe, se recomienda que la **Introducción** mencione explícitamente: (i) el papel del parámetro β en la clasificación 3–3–1 [3]; (ii) la condición de acople *un fermión derecho* \leftrightarrow *un doblete* como vía de $FCNC=0$ en el sector escalar [4]; y (iii) el doble objetivo *sabor + CP fuerte* en el modelo con cinco ceros y simetría PQ [2]. Con ello, la Discusión aquí presentada mantiene continuidad con la Metodología, los Resultados y las Conclusiones.

5. Conclusiones

1. **Aportes teóricos principales.** El proyecto consolidó y amplió la clasificación sistemática de modelos 3–3–1, incluyendo escenarios con *cargas eléctricas exóticas*, construyendo conjuntos irreducibles libres de anomalía y modelos fenomenológicamente distinguibles; se reportaron los acoplamientos del bosón neutro pesado Z' a los campos del SM y límites inferiores para su masa, así como las condiciones para evadir FCNC y CLFV [3]. En paralelo, se propuso y estudió un marco con dos simetrías $U(1)$ no universales que reproduce al Modelo Estándar como teoría efectiva y presenta un *mecanismo de cancelación exacta* de FCNC en el sector de Higgs cuando cada fermión derecho del SM acopla sólo a un doblete [4]. Finalmente, se desarrolló un modelo mínimo axión–sabor con cinco ceros de textura que, con cuatro dobletes de Higgs, ajusta masas y mezclas de quarks y leptones dentro de los valores experimentales, abordando de manera simultánea el problema CP fuerte a la Peccei–Quinn [2].
2. **Implicaciones fenomenológicas y límites.** La fenomenología de los modelos 3–3–1 con cargas exóticas indica límites para $M_{Z'}$ en la franja multi-TeV, consistentes con restricciones de colisionadores y con la posible aparición natural de escenarios fuertemente acoplados; en los casos analizados, los encajes y límites resumidos implican $M_{Z'}$ en el rango de varios TeV, coherente con la clasificación y las condiciones de ausencia de FCNC/CLFV reportadas [3]. En el marco con dos $U(1)$ no universales, se mostró que es posible ajustar *todas* las masas y ángulos de mezcla con dos dobletes de Higgs y un singlete, preservando la cancelación de anomalías por interacción entre familias y suprimiendo FCNC en el sector escalar [4]. En el modelo axiotal, los decaimientos semileptónicos de mesones cargados (como $K^\pm \rightarrow \pi^\pm \nu \bar{\nu}$) dominan la exclusión del espacio de parámetros

(ε, f_a) , mientras que los acoples axi3n–fot3n refinan la regi3n permitida; el enfoque mejora el *fine-tuning* de Yukawas en el ajuste de texturas [2].

3. **Socializaci3n, validaci3n por pares y formaci3n de talento humano.** Los resultados fueron socializados en eventos nacionales e internacionales (SILAFEA, COMHEP, entre otros), incluyendo contribuciones sobre clasificaci3n 3–3–1 y se3ales tipo Higgs en 2HDM extensos, con constancias de participaci3n/ponente. Adem3s, se apoy3 a estudiantado en su formaci3n, con asistencia certificada a escuelas especializadas. Estos hitos fortalecen la apropiaci3n social de los resultados y el relevo generacional.
4. **Cumplimiento de compromisos del proyecto.** El proyecto *B3squeda de modelos 3–3–1 y fenomenolog3a* (c3digo 2686) fue aprobado con duraci3n de 24 meses, y cont3 con pr3rroga hasta la culminaci3n establecida, cumpliendo la entrega de productos cient3ficos y la socializaci3n programada, en concordancia con los acuerdos y el acta de cumplimiento de la VIIS. Los art3culos A1 publicados reconocen el apoyo de la VIIS y de Minciencias/ICETEX, evidenciando la trazabilidad institucional de los resultados.

En conjunto, el proyecto (*i*) ampl3a el mapa de modelos 3–3–1—incluidos casos no universales y con cargas ex3ticas—con caracterizaci3n fenomenol3gica y l3mites robustos; (*ii*) propone y valida un mecanismo novedoso de supresi3n de FCNC en marcos $U(1) \times U(1)$ no universales, compatible con masas y mezclas ferm3nicas; y (*iii*) presenta una v3a axi3nal m3nima coherente con datos de sabor y astro/fenomenol3gicos. Todo ello fue acompa3ado de publicaci3n en revistas A1, disseminaci3n en congresos y fortalecimiento de capacidades humanas, superando el alcance m3nimo originalmente trazado en el plan de trabajo.

6. Recomendaciones

A partir de los resultados del proyecto y de los productos obtenidos, proponemos las siguientes l3neas de acci3n y mejora:

1. **Profundizar la fenomenolog3a de los modelos 3–3–1 con cargas el3ctricas ex3ticas.** Ampliar el an3lisis detallado de los 33 modelos construidos a partir de 14 conjuntos irreducibles libres de anomal3a (AFSs), incluyendo *scans* sistem3ticos de par3metros y un estudio comparativo de acoplamientos del Z' y l3mites de colisionadores para cada incrustaci3n ferm3nica reportada [3]. Priorizar:
 - Ajustes globales con restricciones de FCNC/CLFV y mezclas mes3nicas ($K^0 - \bar{K}^0$), ya motivadas por la presencia de quarks ex3ticos [3].
 - B3squedas dirigidas de $Z' \rightarrow \ell^+ \ell^-$ y $Z' \rightarrow t\bar{t}$, y firmas con leptones doblemente cargados en el LHC y HL–LHC [3].

- Actualizar y homogeneizar los límites inferiores de masas en un mismo marco estadístico.
- 2. **Consolidar la extensión con dos $U(1)$ no universales como EFT del SM.** A partir del marco con dos $U(1)$ no universales y su bosón Z' efectivo, recomendamos:
 - Realizar un *fit* conjunto de los parámetros (x, y, z, w) a datos de bajas energías y colisionadores, usando límites de ATLAS en di-leptones y $t\bar{t}$ para acotar los acoplamientos quirales [4].
 - Estudiar sistemáticamente modelos no universales (dos familias con cargas idénticas y una distinta) frente a casos universales como $B-L$ [4].
 - Integrar proyecciones HL-LHC para definir regiones de interés experimental (dileptones de alta masa y $t\bar{t}$ resonante).
- 3. **Enlazar el sector escalar 2HDM + singlete con anomalías del LHC.**
 - Cerrar la versión de artículo con el espectro $(H_{1,2,3}, A, C^\pm)$ y el mapa de parámetros que reproduce indicios en 95 y 151 GeV, y su conexión con Z' [4, 1].
 - Anclar el análisis numérico a restricciones de ATLAS/CMS y proyecciones HL-LHC para resonancias escalares y Z' (canales $\gamma\gamma, Z\gamma, b\bar{b}, WW/ZZ, hh$).
- 4. **Programa de axiones y texturas de masas (vía PQ/flavor).**
 - Explorar la región de parámetros (f_a, m_a) y los acoplos $g_{a\gamma}$ y g_{ae} compatibles con límites astrofísicos y helioscopios, usando las expresiones de referencia para $g_{a\gamma}$ y m_a [2].
 - Estudiar la *UV completion* donde la simetría PQ/flavor se incruste en escenarios 3-3-1 o en el marco con dos $U(1)$, buscando predicciones correlacionadas en flavor y cosmología [2].
- 5. **Herramientas, reproducibilidad y sinergias.**
 - Implementar todos los escenarios en SARAH/SPHENO/MADGRAPH con un repositorio público (parámetros, *cards* y scripts de *fits*), facilitando replicabilidad de códigos y figuras.
 - Integrar tablas comparativas de límites de masas (leptones/quarks exóticos, Z') y acoplamientos, armonizando supuestos experimentales a lo largo de los tres ejes de trabajo [3].
- 6. **Formación, colaboración y difusión.** Potenciar la participación estudiantil (tesis, *internships*) y la presentación de resultados en eventos de alto impacto, consolidando redes con los coautores y grupos afines; continuar la socialización nacional e internacional iniciada en las ponencias del proyecto.

7. **Gestión y plazos.** Alinear los productos por escribir (al menos un artículo por línea: 3–3–1 detallado; EFT con dos $U(1)$; axiones+texturas) con la prórroga autorizada hasta el 14 de septiembre de 2025, reportando avances semestrales conforme a la VIIS [1].

Nota de coherencia interna. Si se incorporan estos ejes (por ejemplo, la conexión 2HDM + singlete con dos $U(1)$ y la extensión PQ/flavor en 3–3–1), recomendamos ajustar la *Introducción* y los *Objetivos específicos* para reflejar claramente el alcance ampliado del proyecto a la luz de los resultados obtenidos.

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Referencias

- [1] VIIS–Universidad de Nariño, *Acuerdo N° 53 y Acta de Cumplimiento N° 18 (14 de marzo de 2023). Búsqueda de modelos 3–3–1 y fenomenología asociada (código 2686)*. Documento institucional, Universidad de Nariño, San Juan de Pasto, 2023. [PDF].
- [2] Y. Giraldo, R. Martínez, E. Rojas, J. C. Salazar, *A minimal axion model for mass matrices with five texture-zeros*. *Eur. Phys. J. C* **83**, 638 (2023). doi:10.1140/epjc/s10052-023-11808-0.
- [3] E. Suarez, R. H. Benavides, Y. Giraldo, W. A. Ponce, E. Rojas, *Alternative 3–3–1 models with exotic electric charges*. *J. Phys. G: Nucl. Part. Phys.* **51** (2024) 035004. doi:10.1088/1361-6471/ad1e21.
- [4] R. H. Benavides, Y. Giraldo, W. A. Ponce, O. Rodríguez, E. Rojas, *The Standard Model of Particle Physics as an effective theory from two*

non-universal $U(1)$'s. *J. Phys. G: Nucl. Part. Phys.* **52** (2025) 055002.
doi:10.1088/1361-6471/add281.

- [5] Y. Giraldo, *Alternative 3-3-1 models: a comprehensive analysis*. Presentación en COMHEP 2022, 28 de noviembre de 2022.
- [6] Y. Giraldo, *Classification for Alternative 3-3-1 models with exotic electric charges*. Presentación en COMHEP 2023 (8th Colombian Meeting on High Energy Physics), 6 de diciembre de 2023.
- [7] Y. Giraldo, *Exploring Higgs-like Resonant Signals within a 2HDM Framework*. Presentación en SILAFEA 2024, 5 de noviembre de 2024, Cinvestav, Ciudad de México, México.



Búsqueda de Modelos 3-3-1 y Fenomenología

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Resumen

En esta investigación vamos a estudiar de manera sistemática todos los modelos posibles $SU(3)_c \otimes SU(3)_L \otimes U(1)_x$, libres de anomalías y sin cargas eléctricas exóticas, y su correspondiente fenomenología. Estamos considerando una extensión del Modelo Estándar, en el que el sector electrodébil $SU(2)_L \otimes U(1)_Y$ es extendido a $SU(3)_L \otimes U(1)_X$, lo cual enriquece la teoría con un nuevo contenido de partículas exóticas e interacciones que pueden explicar algunas de las incógnitas desconocidas actualmente en la física. Los modelos $SU(3)_c \otimes SU(3)_L \otimes U(1)_x$ han sido estudiados, pero aún hay aspectos por resolver: no se ha profundizado en la fenomenología de muchos de estos modelos, falta descubrir nuevos modelos interesantes y, además, se carece de una estructura analítica para identificarlos, tomando en cuenta que pueden existir estructuras "Vector-Like".

Palabras Claves: Modelos 3-3-1, Extensiones del Modelo Estándar.

1. Planteamiento

Revisaremos de forma sistemática cómo construir modelos $SU(3)_c \otimes SU(3)_L \otimes U(1)_x$ (modelos 3-3-1) libres de anomalías y sin cargas eléctricas exóticas. Haremos fenomenología con algunos de estos modelos nuevos. Ya existen amplios estudios de los modelos 3-3-1, desde los trabajos originales de Pisano y Pleitez [1] y Frampton [2], hasta los trabajos del profesor Ponce [3, 4]. La idea es sistematizar el proceso para encontrar todos los modelos posibles libres de anomalías, universales y no universales, con tres o más familias en el sector de quarks y leptónico.

2. Justificación

La razón de esta propuesta de investigación es darle un mejor sentido y tener una mejor comprensión de la cantidad de modelos 3-3-1 que han sido propuestos en la literatura, por ejemplo, el modelo mínimo [1, 2] que contiene cargas eléctricas exóticas, y modelos sin cargas eléctricas exóticas como el modelo 3-3-1 con neutrinos derechos [5, 6, 7] y el modelo 3-3-1 con leptones cargados exóticos [8, 9, 10], modelos que han sido extensamente estudiados desde el punto de vista fenomenológico. Pero hay varios modelos 3-3-1 sin cargas eléctricas exóticas que no han recibido la suficiente atención y que sólo se han identificado como modelos C, D, E, F,

G, H, I y J [3, 4, 11]. Esperamos entonces que un análisis sistemático nos permita encontrar más modelos, y confiamos que sean todos los posibles modelos libres de anomalías, y así poder identificar modelos interesantes que ameriten profundizar en su análisis fenomenológico.

3. Objetivo general

Encontrar un método de construcción de modelos 3-3-1, en especial sin cargas eléctricas exóticas, e identificar modelos 3-3-1 nuevos e interesantes para su análisis fenomenológico.

3.1. Objetivo específicos

- Identificar las estructuras fermiónicas cerradas —Resultado—>encontrar las estructuras que contengan las antipartículas de todas las partículas cargadas eléctricamente, y que consistan de tripletes $SU(3)_L$ izquierdos y singletes derechos.
- Encontrar las estructuras básicas libres de anomalías —>con estas estructuras se podrían construir todos los modelos 3-3-1.
- Identificar modelos con una fenomenología interesante —>tendríamos a nuestra disposición

modelos nuevos que pasarían a enriquecer la literatura científica.

4. Marco teórico

El impresionante éxito del Modelo Estándar (ME) basado en el grupo gauge local $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ con el sector de color $SU(3)_c$ confinado y el sector de sabor $SU(2)_L \otimes U(1)_Y$ oculto y roto espontáneamente por el mecanismo mínimo de Higgs [12], no ha sido capaz de proporcionar una explicación de varios problemas fundamentales, entre ellos: la jerarquía de masas y los ángulos de mezcla para ambos casos, el sector de quarks y leptónico, la cuantización de carga, la fuerte violación de la simetría carga-paridad (CP), masas de neutrinos y sus oscilaciones, y por último pero no menos importante, la abundancia de materia oscura y energía oscura en el universo. Debido a esto, muchos físicos creen que el ME no representa la teoría final, representando solo un modelo efectivo originado a partir de uno más fundamental. Las extensiones mínimas del ME surgen o bien añadiendo nuevos campos, o bien ampliando el grupo gauge local (añadiendo un campo de neutrinos de quiralidad derecha constituye una extensión simple, algo que mejora, pero no resuelve los problemas mencionados anteriormente). Las extensiones simples del grupo gauge local consideran un sector electrodébil con una simetría abeliana extra $SU(2)_L \otimes U(1)_x \otimes U(1)_z$ [13, 14], o bien el llamado modelo simétrico izquierda-derecha $SU(2)_L \otimes SU(2)_R \otimes U(1)_{(B-L)}$ [15, 16, 17], o también una simetría de la forma $SU(3)_L \otimes U(1)_X$, siendo este último caso el que vamos a considerar en este estudio [1, 2, 3, 4].

En lo que sigue suponemos que el grupo gauge electrodébil es $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (para abreviar 3-3-1) en el cual el sector electrodébil del ME $SU(2)_L \otimes U(1)_Y$ es extendido a $SU(3)_L \otimes U(1)_X$. También suponemos que, como en el ME, el grupo de color $SU(3)_c$ es vectorial (libre de anomalías) y que los quarks izquierdos (tripletes de color) y los leptones izquierdos (singletes de color) transforman bajo las dos representaciones fundamentales de $SU(3)_L$ (la representación 3 y 3^*).

Aparecen dos clases de modelos: los modelos universales de una familia en los que las anomalías se cancelan en cada familia como en el ME, y los modelos de varias familias en los que las anomalías se cancelan por una interacción entre las diversas familias.

Para los modelos 3-3-1, el operador de carga eléctrica más general en el sector electrodébil extendido es

$$Q = a\lambda_3 + \frac{1}{\sqrt{3}}b\lambda_8 + XI_3 \quad (1)$$

donde λ_α , $\alpha = 1, 2, \dots, 8$ son las matrices de Gell-Mann para $SU(3)_L$ normalizado como $\text{Tr}(\lambda_\alpha\lambda_\beta) = 2\delta_{\alpha\beta}$ e $I_3 = \text{Dg}(1, 1, 1)$ es la matriz unitaria 3×3 . $a = 1/2$ si se asume que el isospín $SU(2)_L$ del ME está enteramente embebido en $SU(3)_L$; b es un parámetro libre que fija el modelo y los valores de X se obtienen por

cancelación de anomalías. Para los 8 campos gauge A_μ^α de $SU(3)_L$, $X = 0$ se pueden escribir como [3, 4]

$$\sum_\alpha \lambda_\alpha A_\mu^\alpha = \sqrt{2} \begin{pmatrix} D_{1\mu}^0 & W_\mu^+ & K_\mu^{(b+1/2)} \\ W_\mu^- & D_{2\mu}^0 & K_\mu^{(b-1/2)} \\ K_\mu^{-(b+1/2)} & K_\mu^{-(b-1/2)} & D_{3\mu}^0 \end{pmatrix} \quad (2)$$

donde $D_{1\mu}^0 = A_\mu^3/\sqrt{2} + A_\mu^8/\sqrt{6}$, $D_{2\mu}^0 = -A_\mu^3/\sqrt{2} + A_\mu^8/\sqrt{6}$, y $D_{3\mu}^0 = -2A_\mu^8/\sqrt{6}$. Los índices superiores de los bosones gauge representan la carga eléctrica de las partículas, siendo algunos de ellos funciones del parámetro b .

4.1. El modelo mínimo

En la referencia [1, 2] se ha demostrado que, para $b = 3/2$, las siguientes estructuras fermiónicas están libres de todas las anomalías gauge: $\psi_{lL}^T = (\nu_l^0, l^-, l^+)_{L} \sim (1, 3, 0)$, $Q_{iL}^T = (d_i, u_i, X_i)_{L} \sim (3, 3^*, -1/3)$, $Q_{3L}^T = (u_3, d_3, Y) \sim (3, 3, 2/3)$, donde $l = e, \mu, \tau$ es un índice para las familias leptónicas, $i = 1, 2$ para las dos primeras familias de quarks, y los números después del signo de similaridad significa representaciones 3-3-1. Los campos derechos son $u_{aL}^c \sim (3^*, 1, -2/3)$, $d_{aL}^c \sim (3^*, 1, 1/3)$, $X_{iL}^c \sim (3^*, 1, 4/3)$ y $Y_L^c \sim (3^*, 1, -5/3)$, donde $a = 1, 2, 3$ es el índice de familias de quarks, y hay dos quarks exóticos con cargas eléctricas $-4/3$ (X_i) y otro con carga eléctrica $5/3$ (Y). Esta versión es llamada modelo mínimo en la literatura, porque no hace uso de leptones exóticos, incluyendo posibles neutrinos derechos.

4.2. Modelos 3-3-1 sin cargas eléctricas exóticas

Si se desea evitar cargas eléctricas exóticas en los sectores de fermiones y bosones como las presentes en el modelo mínimo, se debe elegir $b = 1/2$ en la ecuación (1) como se muestra en la referencia [3, 4].

Para comenzar nuestro análisis sistemático, empezamos con estructuras cerradas de fermiones que consisten sólo de un triplete $SU(3)_L$ izquierdo y singletes derechos, donde por cerradas entendemos estructuras que contienen las antipartículas de todas las partículas cargadas eléctricamente. Siguiendo la notación de las referencias [3, 4], sólo existen las siguientes seis estructuras de este tipo que contienen al menos todos los campos fermiónicos de una familia del ME:

- $S_1 = [(\nu_e^0, e^-, E_1^-) \oplus e^+ \oplus E_1^+]_L$ con números cuánticos $(1, 3, -2/3)$; $(1, 1, 1)$ y $(1, 1, 1)$ respectivamente.
- $S_2 = [(e^-, \nu_e^0, N_1^0) \oplus e^+]_L$ con números cuánticos $(1, 3^*, -1/3)$ y $(1, 1, 1)$ respectivamente.
- $S_3 = [(d, u, U) \oplus u^c \oplus d^c \oplus U^c]_L$ con números cuánticos $(3, 3^*, 1/3)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ y $(3^*, 1, -2/3)$ respectivamente.
- $S_4 = [(u, d, D) \oplus u^c \oplus d^c \oplus D^c]_L$ con números cuánticos $(3, 3, 0)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ y $(3^*, 1, 1/3)$ respectivamente.

- $S_5 = [(N_2^0, E_2^+, e^+) \oplus E_2^- \oplus e^-]_L$ con números cuánticos $(1, 3^*, 2/3)$; $(1, 1, -1)$ y $(1, 1, -1)$ respectivamente.
- $S_6 = [(E_3^+, N_3^0, N_4^0) \oplus E_3^-]_L$ con números cuánticos $(1, 3, 1/3)$ y $(1, 1, -1)$ respectivamente.

donde por razones fenomenológicas permitimos la presencia de varios leptones exóticos (cargados y neutros), pero sólo un quark exótico de cada tipo. En los primeros conjuntos, N_1^0 y N_4^0 pueden desempeñar el papel del campo de neutrino derecho ν_e^{0c} en una base $SO(10)$.

Anomalías	S_1	S_2	S_3	S_4	S_5	S_6
$[SU(3)_C]^2 U(1)_X$	0	0	0	0	0	0
$[SU(3)_L]^2 U(1)_X$	-2/3	-1/3	1	0	2/3	1/3
$[\text{Grav}]^2 U(1)_X$	0	0	0	0	0	0
$[U(1)_X]^3$	10/9	8/9	-4/3	-2/3	-10/9	-8/9
$[SU(3)_L]^3$	1	-1	-3	3	-1	1

Tabla 1: Anomalías para algunas estructuras de campos fermiónicas 3-3-1.

Ahora bien, si queremos considerar una sola familia de quarks, basta con los conjuntos S_3 o S_4 , pero para 3 familias de quarks hay que utilizar una de las siguientes combinaciones $3S_3$, $3S_4$, $(2S_3+S_4)$ y (S_3+2S_4) , donde las dos primeras están asociadas a modelos universales.

A partir de la Tabla I es sencillo leer los siguientes conjuntos libres de anomalías:

- **Modelo A:** $3S_2 + S_3 + 2S_4$
- **Modelo B:** $3S_1 + 2S_3 + S_4$
- **Modelo I:** $2S_2 + S_4 + S_5$

$$[(e^-, \nu_e^0, N_1^0) \oplus e^+ \oplus (E_1^-, N_2^0, N_3^0) \oplus E_1^+ \oplus (N_4^0, E_2^+, E_3^+) \oplus E_2^- \oplus E_3^- \oplus (u, d, D) \oplus u^c \oplus d^c \oplus D^c]_L. \quad (3)$$

Por por muy feo que sea, debido a la presencia de leptones exóticos, algunos con los mismos números cuánticos de los ordinarios, podemos decir que este mo-

$$[(\nu_e^0, e^-, E_1^-) \oplus e^+ \oplus E_1^+ \oplus (N_1, E_2^-, E_3^-) \oplus E_2^+ \oplus E_3^+ \oplus (E_4^+, N_2^0, N_3^0) \oplus E_4^- \oplus (d, u, U) \oplus u^c \oplus d^c \oplus U^c]_L. \quad (4)$$

Las otras dos estructuras **A** y **B** corresponden a dos modelos no universales bien conocidos y ya presentes en la literatura; **A** se denomina “modelo 3-3-1 con neutrinos derechos” [5, 6, 7] y **B** se denomina “modelo 3-3-1 con leptones exóticos cargados” [8, 9, 10].

5. Antecedentes

Como se pudo observar en el marco teórico, sección 4, la riqueza física y matemática es bastante amplia de lo que pretendemos investigar, desde la construcción de modelos, estudiados ampliamente en la li-

Nótese que en este punto, nuestro enfoque es diferente al presentado en la referencia [3, 4], siendo la diferencia que sólo se utiliza un triplete $SU(3)_L$ en cada conjunto, en lugar de los compuestos presentes en la referencia original (tales estructuras leptónicas compuestas aparecerán más adelante en nuestro análisis sistemático).

Las diversas anomalías gauge calculadas para estos seis conjuntos se indican en la Tabla I; donde nótese que los valores de anomalía para S_1 , S_2 , S_3 y S_4 coinciden con los presentados en la referencia [3, 4], siendo los valores para S_5 y S_6 nuevos resultados.

- **Modelo J:** $2S_1 + S_3 + S_6$

donde las estructuras **I** y **J** [11] contienen sólo una familia de quarks, y **A** y **B** son modelos de tres familias de quarks. Pero, ¿podemos ver a **I** y **J** como modelos libres de anomalías de una familia (“Universal”)? la respuesta es sí, si permitimos modelos con electrones exóticos y nuevas partículas eléctricas neutras. De hecho, podemos escribir el contenido de partículas para el modelo **I** como:

delo aún no está excluido de la fenomenología actual.

De forma similar, el contenido de partículas de la estructura **J** puede escribirse como [11]:

6. Metodología

El método que pretendemos seguir en nuestra investigación consiste de varios pasos:

1. Construir más estructuras básicas (S 's), con sus respectivas anomalías, como las indicadas en la Tabla 1.

2. Construir más modelos libres de anomalías, jugando con estos conjuntos, S . Estos modelos pueden ser universales o no universales.
3. Finalmente, identificar modelos nuevos interesantes a los cuales poderles hacer la fenomenología, identificación de cargas y corrientes, y hacer física con el nuevo bosón exótico Z' .

7. Novedad en el aporte a la ciencia, la tecnología, la innovación o la creación artística y cultural

Esta investigación es en física básica, su aporte va dirigido hacia la ciencia pura. Nuestra investigación va dirigido al estudio de los constituyentes últimos de la materia, de qué está hecha y cómo interactúa ésta para formar los cuerpos macroscópicos. La novedad está en que aún hay muchas preguntas sin responder; por ejemplo, no hemos sido capaces de proporcionar una explicación de varios problemas fundamentales, entre ellos: la jerarquía de masas y los ángulos de mezcla para ambos casos, el sector de quarks y leptónico, la cuantización de la carga eléctrica, la fuerte violación de CP, masas de neutrinos y sus oscilaciones, y por último pero no menos importante, la abundancia de materia oscura y energía oscura en el universo.

8. Impactos esperados

- **Impactos científico y tecnológicos del proyecto en las entidades participantes** — descripción—> Esperamos publicar los resultados

de nuestra investigación en revistas de alto impacto científico tipo A1. Lo que generará en la entidades participantes, la Vicerrectoría de investigación e Interacción Social (viis) y la Universidad de Nariño una mayor visibilidad y mejor posicionamiento en el escalafón investigativo y académico.

- **Formación de recursos humanos en investigación, nuevas tecnologías y en gestión tecnológica** —descripción—>Respecto a este impacto, tendremos dos estudiantes vinculados al proyecto, que recibirán una preparación invaluable en los aspectos relacionados de cómo se investiga. Tendrán la oportunidad de entrar en contacto con los distintos canales de comunicación y tecnológicos disponibles actualmente, y cómo trabajar en grupo. Se les incentivará el uso de software especializado como Mathematica, SARAH (cálculos de decaimientos), ROOT, etc. que usaremos a lo largo de la investigación. Cómo preparar artículos, uso del inglés, dónde y cómo publicar, en fin una serie de enseñanzas fructíferas relacionadas con el hecho de investigar.

- **Redes de información y colaboración científico-tecnológico**—descripción—>En este aspecto nuestro grupo de investigación está en permanente contacto con otros grupos de investigación a fin de consolidar colaboraciones. Actualmente tenemos colaboraciones con investigadores de la Universidad Nacional de Bogotá, Universidad de Antioquia, Universidad Santiago de Cali, Universidad del Tolima, Universidad de Sao Paulo en Brasil (UNICID), etc. Los fondos de este proyecto nos permitiría entrar en contacto con más grupos a fin de lograr más redes de colaboración.

Referencias

- [1] F. Pisano and V. Pleitez, Phys. Rev. D **46** (1992), 410-417 doi:10.1103/PhysRevD.46.410 [arXiv:hep-ph/9206242 [hep-ph]].
- [2] P. H. Frampton, Phys. Rev. Lett. **69** (1992), 2889-2891 doi:10.1103/PhysRevLett.69.2889
- [3] W. A. Ponce, J. B. Florez and L. A. Sanchez, Int. J. Mod. Phys. A **17** (2002), 643-660 doi:10.1142/S0217751X02005815 [arXiv:hep-ph/0103100 [hep-ph]].
- [4] W. A. Ponce, Y. Giraldo and L. A. Sanchez, Phys. Rev. D **67** (2003), 075001 doi:10.1103/PhysRevD.67.075001 [arXiv:hep-ph/0210026 [hep-ph]].
- [5] J. C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D **47** (1993), 2918-2929 doi:10.1103/PhysRevD.47.2918 [arXiv:hep-ph/9212271 [hep-ph]].
- [6] R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D **50** (1994) no.1, R34-R38 doi:10.1103/PhysRevD.50.R34 [arXiv:hep-ph/9402243 [hep-ph]].
- [7] R. H. Benavides, Y. Giraldo and W. A. Ponce, Phys. Rev. D **80** (2009), 113009 doi:10.1103/PhysRevD.80.113009 [arXiv:0911.3568 [hep-ph]].
- [8] M. Ozer, Phys. Rev. D **54** (1996), 1143-1149 doi:10.1103/PhysRevD.54.1143

- [9] W. A. Ponce and O. Zapata, Phys. Rev. D **74** (2006), 093007 doi:10.1103/PhysRevD.74.093007 [arXiv:hep-ph/0611082 [hep-ph]].
- [10] J. C. Salazar, W. A. Ponce and D. A. Gutierrez, Phys. Rev. D **75** (2007), 075016 doi:10.1103/PhysRevD.75.075016 [arXiv:hep-ph/0703300 [hep-ph]].
- [11] Ponce, William A., Rev. Acad. Colomb. Cienc. Ex. Fis. Nat. Vol. 42 Número 165 (2018), páginas 319-322
- [12] J. F. Donoghue, E. Golowich and B. R. Holstein, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **2** (1992), 1-540 doi:10.1017/CBO9780511524370
- [13] W. A. Ponce, Phys. Rev. D **36** (1987), 962-965 doi:10.1103/PhysRevD.36.962
- [14] R. H. Benavides, L. Muñoz, W. A. Ponce, O. Rodríguez and E. Rojas, J. Phys. G **47** (2020) no.7, 075003 doi:10.1088/1361-6471/ab8d8d [arXiv:1812.05077 [hep-ph]].
- [15] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11** (1975), 2558 doi:10.1103/PhysRevD.11.2558
- [16] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980), 912 doi:10.1103/PhysRevLett.44.912
- [17] R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44** (1980), 1316-1319 [erratum: Phys. Rev. Lett. **44** (1980), 1643] doi:10.1103/PhysRevLett.44.1316



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VICERRECTORÍA DE INVESTIGACIONES E INTERACCIÓN SOCIAL

ACUERDO N° 53
marzo 14 de 2023

Por el cual se aprueba un Proyecto de Investigación para su ejecución y financiación

EL COMITÉ DE INVESTIGACIONES DE LA UNIVERSIDAD DE NARIÑO
En uso de sus atribuciones estatutarias y reglamentarias y

CONSIDERANDO:

Que mediante Acuerdo No. 162 de 17/06/2022, emanado de este organismo, se expidió la Reglamentación y el Calendario de la Convocatoria de Investigación Docente 2022.

Que los profesores YITHSBY GERALDO USUGA, EDUARDO ROJAS , GERMAN ENRIQUE RAMOS ZAMBRANO y JUAN CARLOS SALAZAR MONTENEGRO adscritos al(los) Departamento(s) de Física presentaron el proyecto "BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA ", registrado con código 2686, el cual, es financierable, una vez surtido el proceso de la convocatoria.

Que es necesario expedir el acto administrativo de aprobación del proyecto y definir las condiciones básicas para su ejecución.

Que el proyecto cumple con los requisitos exigidos en la Convocatoria de Investigación Docente 2022.

ACUERDA:

- ARTÍCULO 1°.-** Aprobar el proyecto de investigación "BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA "a los profesores YITHSBY GERALDO USUGA, EDUARDO ROJAS , GERMAN ENRIQUE RAMOS ZAMBRANO y JUAN CARLOS SALAZAR MONTENEGRO adscritos al(los) Departamento(o) de Física el cual tendrá una duración de 24 meses para su ejecución.
- ARTÍCULO 2°.-** El proyecto de investigación será coordinado por el profesor YITHSBY GERALDO USUGA, identificado con cédula de ciudadanía No 71.741.558.
- ARTÍCULO 3°.-** Adjudicar recursos por valor de VENTICINCO MILLONES DE PESOS MDA. CTE. (\$ 25.000.000), según certificado de disponibilidad presupuestal No. 2162-1 de 28/02/2023, para garantizar el desarrollo del proyecto.
- ARTÍCULO 4°.-** El desarrollo del proyecto se regirá de acuerdo con lo establecido en el Acuerdo No. 162 de 17/06/2022, emanado de este organismo y la ejecución de los recursos se sujetará al presupuesto presentado en el proyecto, cumpliendo con las normas fiscales nacionales e institucionales establecidas para el manejo de dineros públicos. Los avances y el plazo para la legalización de los mismos serán autorizados mediante resolución emitida por el Vicerrector de Investigaciones e Interacción Social.
- Parágrafo:** El primer desembolso debe solicitarse hasta el 15 de septiembre de 2023, de lo contrario se asume que el proyecto no se desarrollará y será cancelado.
- ARTÍCULO 5°.-** Se iniciará con la ejecución de recursos una vez esté firmada el acta de cumplimiento, por el(la) Coordinador(a) del proyecto y los Coinvestigadores.
- ARTÍCULO 6°.-** Los investigadores deben cumplir con la presentación de un informe parcial cada 6 meses y uno final, además de los demás compromisos establecidos en el acta de cumplimiento suscrita por el docente responsable.



Universidad de
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VICERRECTORÍA DE INVESTIGACIONES E INTERACCIÓN SOCIAL

ACUERDO N° 53
marzo 14 de 2023

- ARTÍCULO 7°-** Los docentes con vinculación por hora cátedra y tiempo completo ocasional que lideren proyectos, deberán suscribir una póliza de cumplimiento del 40% del valor total del proyecto, la cual debe tener una vigencia que comprenda el tiempo establecido para el desarrollo del proyecto más seis meses. Dicha póliza deberá adquirirse y cargarse al sistema de información hasta el 22 de marzo de 2023, de lo contrario se asume que el proyecto no se desarrollará y será cancelado.
- ARTÍCULO 8°-** La Vicerrectoría de Investigaciones e Interacción Social, y el(los) Departamento(s) de Física , anotará(n) lo de su cargo.

COMUNÍQUESE Y CÚMPLASE

Dado en San Juan de Pasto, a los 14 días del mes de marzo de 2023.

Firmado digitalmente
WILLIAM ALBARRACÍN HERNÁNDEZ
Presidente

NATHALY LORENA SANTACRUZ RECALDE
Secretaria



Universidad de Nariño
**VICERRECTORÍA DE INVESTIGACIONES E
INTERACCIÓN SOCIAL**

**ACTA DE CUMPLIMIENTO No. 18
marzo 14 de 2023**

Entre los suscritos WILLIAM ALBARRACÍN HERNÁNDEZ identificado con cédula de ciudadanía número 79.788.448, en calidad de Vicerrector de Investigaciones e Interacción Social de la Universidad de Nariño y quien para la presente acta se denominará LA VICERRECTORÍA DE INVESTIGACIÓN E INTERACCIÓN SOCIAL, de una parte y de la otra los profesores YITHSBEY GIRALDO USUGA, EDUARDO ROJAS , GERMAN ENRIQUE RAMOS ZAMBRANO Y JUAN CARLOS SALAZAR MONTENEGRO adscritos al(los) Departamento(s) de Física quienes para los fines de la misma se denominarán LOS INVESTIGADORES, se celebra la presente acta de cumplimiento la cual se registrá por las siguientes cláusulas: PRIMERA.- LA VICERRECTORÍA DE INVESTIGACIÓN E INTERACCIÓN SOCIAL a través del Fondo de Investigaciones se compromete a financiar el proyecto de investigación "BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA ", registrado con código 2686 aprobado mediante Acuerdo No. 53, de 14/03/2023 por un valor de \$25.000.000 , los cuales serán ejecutados según el presupuesto aprobado en un término de 24 meses, contados a partir de la fecha de la firma de la presente acta.- SEGUNDA.- LOS INVESTIGADORES se comprometen a desarrollar todos los objetivos de esta investigación, contenidos en el proyecto aprobado.- TERCERA.- LOS INVESTIGADORES implementarán las medidas necesarias para asegurar la participación de TODOS los integrantes, de acuerdo con los compromisos de cada uno.- CUARTA. - LOS INVESTIGADORES se comprometen a entregar como resultados de la investigación: a) Informes semestrales los cuales serán requisito para solicitar recursos financieros e informe final a través Sistema de Información del Sistema de Investigaciones. b) Socialización de los resultados de la investigación en un evento científico de carácter nacional o internacional. c) Generación de un producto académico de los establecidos en los literales a, b, c, d, e, g, i, j, ó k exceptuando el subliteral B "Otras modalidades de publicaciones en revistas especializadas" del artículo 10 del decreto 1279 de 2002. Parágrafo 1. En el caso del literal A del Artículo 10 del Decreto 1279 de 2002, los investigadores podrán presentar el artículo científico preferiblemente a una revista actualmente indexada u homologada por MINCIENCIAS (PUBLINDEX), o en su defecto en una revista indexada en uno o más repertorios, repositorios, índices o bases de datos de revistas científicas reconocido internacionalmente, dependiendo de su área de profundización como: Scopus, EBSCO, PubMed, Scielo, ISI, CUIDENplus, Psycodoc, Clase, DOAJ, IRESIE, IDEAS, LILACS, Mathematical Reviews. PERIÓDICA, SSCI, DIALNET, REDALYC, Web of Science, LATINDEX, REDIB, Clarivate Analytics, Elsevier , Crossref, Philosopher's Index, Publons, COSMOS, ESJI, ICMJE, ISIFI, DRJI, ISI, Scilit. En el caso de que la revista esté indexada en un repertorio, repositorio, índice o base de datos diferente a los estipulados en este parágrafo, los investigadores deben consultarlo con el Comité de Investigaciones para obtener el aval, antes de enviar el artículo a la revista. Parágrafo 2. El número de productos a los que se refiere el literal c) será igual o superior al número de grupos que realicen el proyecto, con coautorías de los grupos participantes en todos ellos y se deberá hacer referencia a la financiación otorgada por la Universidad de Nariño a través de la Vicerrectoría de Investigaciones e Interacción Social. Parágrafo 3. Para el caso de los libros de investigación, ensayo o texto, se cumplirá el requisito una vez se encuentre certificada su aceptación para publicación por el Consejo Editorial de la Universidad de Nariño o por una editorial de reconocido prestigio, las anteriores deben estar avaladas por el Consejo Editorial de la Universidad de Nariño. Parágrafo 4. Para los productos académicos de los literales b, i y k del Artículo 10 del Decreto 1279 de 2002, se cumplirá el requisito una vez el Comité de Asignación de Puntaje los haya aceptado como producción académica de los docentes. Para el caso de docentes hora cátedra dichos productos se evaluarán por pares designados por el Comité de Investigaciones, teniendo en cuenta los criterios establecidos por el Comité de Asignación de Puntaje (CAP). Una vez se entregue dicha información, se podrá expedir el acto administrativo de terminación del proyecto. QUINTA.- LA VICERRECTORÍA DE INVESTIGACIÓN E INTERACCIÓN SOCIAL se reserva la facultad de supervisar y auditar el desarrollo integral de la investigación en sus diferentes fases, lo mismo que el cumplimiento de las observaciones y recomendaciones establecidas por los evaluadores y el Comité de Ética en Investigaciones, tanto en el proyecto como en el desarrollo del trabajo de campo y en el informe final.- SEXTA.- Los bienes no fungibles que se adquieran en el desarrollo de este trabajo son de propiedad de la Universidad, los cuales quedarán a cargo de uno de los docentes de tiempo completo que hacen parte del proyecto de investigación.- SEPTIMA.- LOS INVESTIGADORES, cuando sin causa justificada no cumplan la totalidad de los objetivos aprobados en el proyecto de investigación, o se abstengan de legalizar uno o varios desembolsos o los ejecuten en contra de lo establecido por la ley o por los reglamentos de la Universidad o no remitan oportunamente los informes parciales o final dentro de los plazos estipulados, deberán reintegrar en forma solidaria en los términos de la Ley Civil, los dineros destinados por la Universidad para el desarrollo de la investigación, independientemente de los procesos disciplinarios a que hubiere lugar.- OCTAVA.- Si un INVESTIGADOR hizo uso de los beneficios de la asignación académica por investigación y no culminó a satisfacción con la entrega de los productos correspondientes en los tiempos estipulados en el proyecto, sin justificación oportuna o de fuerza mayor, deberá reintegrar los recursos ejecutados, según lo establecido en el Artículo 28 del Estatuto del Investigador. De igual manera deberá reintegrar los dineros equivalentes al valor de las horas asignadas, indexados a la fecha en la cual se inicia el proceso de reintegro, además de someterse a las sanciones a que hubiese lugar. NOVENA.- El desarrollo del proyecto se registrá según lo establecido en la Convocatoria de Investigación Docente aprobada mediante Acuerdo No. 162 de 17/06/2022, y Acuerdo No. 070 de 2017, emanados de este organismo, así como las demás normas institucionales vigentes.

COMUNÍQUESE Y CÚMPLASE

Dada en San Juan de Pasto, a los 14 días del mes de marzo de 2023.

WILLIAM ALBARRACÍN HERNÁNDEZ
Vicerrector

Yithsbey Giraldo Usuga
Investigador

Eduardo Rojas
Investigador

German Enrique Ramos Zambrano
Investigador

Juan Carlos Salazar Montenegro
Investigador

ACUERDO No. 0137 (Mayo 31 de 2023)

Por el cual se autoriza el retiro de co-investigadores en proyecto de investigación.

EL VICERRECTOR DE INVESTIGACIONES E INTERACCIÓN SOCIAL DE LA UNIVERSIDAD DE NARIÑO

En uso de sus atribuciones estatutarias y reglamentarias y

CONSIDERANDO:

Que mediante *Acuerdo No. 53 del 14 de marzo de 2023*, fue aprobado el proyecto de investigación "BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA", registrado con código 2686 en la Convocatoria Docente 2022. Presentado por las docentes YITHSBEY GIRALDO USUGA, EDUARDO ROJAS, GERMAN ENRIQUE RAMOS ZAMBRANO y JUAN CARLOS SALAZAR MONTENEGRO adscritos al(los) Departamento(s) de Física de la Universidad de Nariño.

Que mediante oficio del 4 de mayo de 2023, el Docente Juan Carlos Salazar Montenegro identificado con c.c. 98.398.864. Solicita ante el Comité de Investigaciones la aprobación de su retiro como coinvestigador en el marco del proyecto de investigación "BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA". La solicitud obedece a que el mencionado docente se encuentra como investigador principal en un proyecto de la convocatoria 2022 y en otro proyecto de la convocatoria Docente 2023, por el carácter de su participación en dichas investigaciones requiere mayor tiempo para el cumplimiento de los objetivos propuestos, aunado a ello menciona las múltiples ocupaciones académicas y administrativas del docente coinvestigador.

Que el artículo 12 del Acuerdo No. 0162 del 17 de junio de 2022 **Por el cual se expide la Reglamentación de la Convocatoria de Investigación Docente 2022**, establece:

Artículo 12. El reemplazo, adición o retiro de investigadores a partir de la suscripción del acta de cumplimiento se podrán realizar, con solicitud al Comité o Consejo de Investigaciones, únicamente, hasta transcurrido el 50% del tiempo de ejecución del proyecto. Las solicitudes que se reciban superado este porcentaje serán tratadas exclusivamente por fuerza mayor o caso fortuito.

Parágrafo. El retiro de un investigador se aprobará siempre y cuando el proyecto siga cumpliendo con las condiciones de la convocatoria, previa aprobación del Comité de Investigaciones. El investigador que se retira suscribirá oficio manifestando su renuncia al proyecto y a la propiedad intelectual generada después de su retiro.

Que mediante consulta 008 del 5 de mayo de 2023, el Comité de Investigaciones aprobó el retiro del coinvestigador Juan Carlos Salazar Montenegro del proyecto denominado: "BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA"

Que mediante **Acuerdo 0154 de 2 de noviembre de 2021**, el Comité de Investigaciones delega al presidente de dicho organismo para resolver algunas solicitudes, como: tramitar prorrogas de proyectos de investigación, tramitar el registro de proyectos en la Vicerrectoría de Investigaciones que son financiados y/o no corresponden a las convocatorias internas, resolver las solicitudes de aceptación de renunciaciones o inclusión de investigadores a los proyectos de investigación docente y la recepción y análisis de informes finales y productos de investigación. La realización de estas funciones se hace con el fin de descongestionar las solicitudes ante el Comité de Investigaciones y dar celeridad a los procesos del Sistema de Investigación.

RESUELVE:

ARTICULO 1º Retirar del proyecto de investigación **2686** denominado: **“BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA”** al docente **JUAN CARLOS SALAZAR MONTENEGRO** identificado con c.c. 98.398.864, adscrito al Departamento de Física de la Universidad de Nariño.

ARTICULO 2º La Vicerrectoría de Investigación e Interacción Social, el Sistema de Investigaciones y el Departamento de Física de la Universidad de Nariño, anotarán lo de su competencia.

COMUNÍQUESE Y CÚMPLASE.

Dado en San Juan de Pasto, a los treintaún (31) días del mes de mayo de dos mil veintitrés (2023).



WILLIAM ALBARRACÍN HERNÁNDEZ
Presidente



NATHALY LORENA SANTACRUZ
Secretaría



Vicerrectoría de Investigación e Interacción Social

ACUERDO N°. 072

(12 de febrero de 2025)

Por el cual se autoriza Prórroga a un proyecto de Investigación

EL COMITÉ DE INVESTIGACIONES DE LA UNIVERSIDAD DE NARIÑO

En uso de sus atribuciones estatutarias y reglamentarias y

CONSIDERANDO:

Que según **Acuerdo No. 080 del 23 de diciembre de 2019** del Consejo Superior se expide el Estatuto General de la Universidad de Nariño.

Que el **Acuerdo No. 080 del 23 de diciembre de 2019**, establece en el "ARTÍCULO 143. Derogatorias y Vigencia. El presente Estatuto General rige a partir de su publicación y deroga todas las disposiciones que le sean contrarias, en especial el Acuerdo 194 del 20 de diciembre de 1993, expedido por el Consejo Superior Universitario".

Que el **Acuerdo No. 080 del 23 de diciembre de 2019**, establece en el "ARTÍCULO 141. Periodo de Transición. Mientras se promulgan los Estatutos relacionados en los artículos anteriores y se reglamentan por parte del Consejo Superior, el Consejo Académico, la rectoría y las vicerrectorías, según sea el caso, las funciones de los cargos contemplados en este Estatuto, seguirán vigentes los estatutos y demás normas internas actuales".

Que mediante **Acuerdo 053 de 14 de marzo de 2023** se aprueba el proyecto de investigación titulado "BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA" (**Cod 2686**) a los profesores Yithsbey Giraldo Usuga CC 71741558, Eduardo Rojas CC 13957295 y German Enrique Ramos Zambrano CC 98388559 adscritos al(los) Departamento(s) de Física respectivamente, el cual tendrá una duración de 24 meses para su ejecución, por valor de VENTICINCO MILLONES DE PESOS MDA. CTE. (\$ 25.000.000)

Que el **Artículo 2 del Acuerdo 0149 de junio 29 de 2023** reza:

Prórroga Ordinaria Se asigna un tiempo adicional correspondiente el 50% del tiempo inicialmente otorgado para la terminación del proyecto, previa verificación de la justificación. Cuenta con el respectivo acto administrativo.

El investigador deberá estar al día en legalizaciones de avances e informes, a fin de solicitar la prórroga.

Para otorgar la prórroga, una vez culminado el cronograma del proyecto, el investigador tendrá hasta 15 días hábiles para solicitar la misma. De lo contrario, el proyecto pasará a estado vencido de manera definitiva y tendrá máximo 15 días hábiles para cumplir con los compromisos establecidos.

Que mediante **Resolución Rectoral No. 004 de 01 de enero de 2025**, la Rectora de la Universidad de Nariño resuelve designar a partir del día tres (3) de enero de 2025, al Doctor ÁLVARO JAVIER BURGOS ARCOS, identificado con cedula de ciudadanía No. 12.978.849, como Vicerrector de Investigación e Interacción Social de la Universidad de Nariño.

Que mediante **Acuerdo 0154 de 2 de noviembre de 2021**, el Comité de Investigaciones delega al presidente de dicho organismo para resolver algunas solicitudes, como: tramitar prórrogas de proyectos de investigación, tramitar el registro de proyectos en la Vicerrectoría de Investigaciones que son financiados y/o no corresponden a las convocatorias internas y la recepción y análisis de informes finales y productos de investigación. La realización de estas funciones, se hace con el fin de descongestionar las solicitudes ante el Comité de Investigaciones y dar celeridad a los procesos del Sistema de Investigación.

Que el investigador principal se solicita una prórroga, con el fin de culminar el proyecto, y que la misma es viable de acuerdo a la norma y la información presentada.

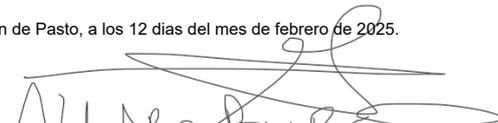
Que, en virtud de lo anterior,

ACUERDA:

ARTICULO 1 °. Autorizar una prórroga ordinaria de 6 meses, solicitados por el investigador, esto es hasta 14 de septiembre de 2025. al proyecto de investigación titulado: "BÚSQUEDA DE MODELOS 3-3-1 Y FENOMENOLOGÍA" (**Cod 2686**), teniendo en cuenta la parte motiva del presente acto administrativo.

COMUNÍQUESE Y CÚMPLASE

Dado en San Juan de Pasto, a los 12 días del mes de febrero de 2025.


ÁLVARO JAVIER BURGOS ARCOS
Presidente

Universidad de Nariño VIIS - Bloque 5 - Oficina 102 - Carrera 33 No. 5-121 - Barrio Las Acacias
Teléfono 6027244309 - 6027311449 Ext. 2390 - Celular 3216365289 - Línea Gratuita 018000957071
Correo electrónico: secretariaviis@udenar.edu.co - www.udenar.edu.co - San Juan de Pasto - Nariño - Colombia

Institución de Educación Superior | Vigilada por MINEDUCACIÓN - Fundada mediante Decreto No. 049 del 4 de noviembre de 1904.
Acreditada en Alta Calidad mediante Resolución No. 000022 MINEDUCACIÓN

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NATHALY LORENA SANTACRUZ R.
Secretaria



SC-CER110449



A minimal axion model for mass matrices with five texture-zeros

Yithsbey Giraldo^{1,a}, R. Martinez^{2,b}, Eduardo Rojas^{1,c}, Juan C. Salazar^{1,d} 

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Abstract A model with fermion and scalar fields charged under a Peccei–Queen (PQ) symmetry is proposed. The PQ charges are chosen in such a way that they can reproduce mass matrices with five texture zeros, which can generate the fermion masses, the CKM matrix, and the PMNS matrix of the Standard Model (SM). To obtain this result, at least 4 Higgs doublets are needed. As we will see in the manuscript this is a highly non-trivial result since the texture zeros of the mass matrices impose a large number of restrictions. This model shows a route to understand the different scales of the SM by extending it with a multi-Higgs sector and an additional PQ symmetry. Since the PQ charges are not universal, the model predicts flavor-changing neutral currents (FCNC) at the tree level, a feature that constitutes the main source of restrictions on the parameter space. We report the allowed regions by lepton decays and compare them with those coming from the semileptonic decays $K^\pm \rightarrow \pi \bar{\nu} \nu$. We also show the excluded regions and the projected bounds of future experiments for the axion–photon coupling as a function of the axion mass and compare it with the parameter space of our model.

1 Introduction

The discovery of the Higgs with a mass of 125 GeV, by the ATLAS [1] and CMS [2] collaborations, is very important because it provides experimental support for spontaneous symmetry breaking, which is the mechanism that explains the origin of the masses of fermions and gauge bosons. Additionally, it opens up the possibility of new physics in the scalar sector, such as the two Higgs doublet model [3–

7], models with additional singlet scalar fields [8–10], or scalar fields that could be candidates for Dark Matter [11–14]. On the other hand, in the Standard Model (SM) [15–17], symmetry breaking generates a coupling of the Higgs to fermions, proportional to their masses, which is consistent with experimental data. However, there are several orders of magnitude between the fermion mass hierarchies that cannot be explained within the context of the SM. Six masses must be defined for the up and down quarks, three Cabibbo–Kobayashi–Maskawa (CKM) mixing angles, and a complex phase that involves CP violation. On the other hand, in the lepton sector, there are three masses for charged leptons, two squared mass differences for neutrinos, three mixing angles, and a complex phase that involves CP violation in the lepton sector. In this case, it is necessary to determine the mass of the lightest neutrino and the character of neutrinos, whether they are Dirac or Majorana fermions.

In the Davis experiment [18], which was designed to detect solar neutrinos, a deficiency in the solar neutrino flux was first observed. According to the results of Bahcall, only one-third of solar neutrinos would reach the Earth [19]. Neutrino oscillation was first proposed by Pontecorvo [20], and the precise mechanism of solar neutrino oscillations was proposed by Mikheyev, Smirnov, and Wolfenstein, involving a resonant enhancement of neutrino oscillations due to matter effects [21,22]. These observations have been confirmed by many experiments from four different sources: solar neutrinos as in Homestake [18], SAGE [18], GALLEX & GNO [23,24], SNO [25], Borexino [26,27] and Super-Kamiokande [28,29] experiments, atmospheric neutrinos as in IceCube [30], neutrinos from reactors as KamLAND [31], CHOOZ [32], Palo Verde [33], Daya Bay [34], RENO [35] and SBL [36], and from accelerators as in MINOS [37], T2K [38] and NO ν A [39]. Neutrino oscillations depend on squared mass differences. On the other hand, the lightest neutrino mass has not been determined yet, but from cosmological considerations, none of the neutrino masses can exceed 0.3 eV, which implies

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that the neutrino masses are much smaller than the charged fermion masses. However, unlike quarks and charged leptons, in the SM the neutrinos are massless, which is explained by assuming that neutrinos are left-handed. Therefore, the discovery of neutrino masses implies new physics beyond the SM. By adding right-handed neutrinos, the Higgs mechanism of the SM can give neutrinos the same type of mass acquired by charged leptons and quarks. It is possible to add right-handed neutrinos ν_R to the SM, as long as they do not participate in weak interactions. With the presence of right-handed neutrinos, it would be possible to generate Dirac masses m_D , similar to those of charged leptons and quarks. In principle, it is also possible to give Majorana masses to left-handed neutrinos, and similarly, right-handed neutrinos can have Majorana masses M_R . For a very large M_R , it would give effective Majorana masses for left-handed neutrinos as $m_{\text{eff}} \approx m_D^2/M_R$. The presence of large Majorana masses allows to explain the tiny neutrino masses compared to the charged fermion masses [38]. To explain the smallness of neutrino masses, there are three types of seesaw mechanisms in the literature: type I with three electroweak neutrinos and three heavy right-handed neutrinos, type II [40,41], type III [42], and inverse seesaw [43,44]. One way to explain the fermion mass hierarchies and the CKM and PMNS mixing angles is through zeros in the Yukawa couplings of fermions (this is known as texture-zeros or simply textures of the mass matrices, and these zeros are usually chosen by hand). It is common in the literature to consider Fritzsch-type textures [45,46], or similar [47–52], for the neutrino and charged lepton mass matrices.

There is no theory that provides values for the entries of the Yukawa Lagrangian, and consequently, there is no a first-principle explanation for the masses and their large differences in the SM. The mass hierarchy between fermions is unnatural because it requires Yukawa constants that differ by many orders of magnitude; this feature is known as the flavor problem or flavor puzzle [53–57]. In this direction, a way that has been explored in the literature is to propose a sector with multiple scalar doublets along with discrete symmetries [58,59], to reduce the number of Yukawa couplings, or equivalently, by introducing texture-zeros in the mass matrices [60–65]. It is also possible to consider global symmetry groups that prohibit certain Yukawas, which somehow generate the texture-zeros mentioned [53–57]. Another way of obtaining these textures is through horizontal gauge symmetries, with the assignment of quantum numbers to the fermion sector, which can break the universality of the SM [62,66–81]. This gauge symmetry generates textures that produce flavor changes in the neutral currents and that, in principle, could be seen in future colliders. There are models with electroweak extensions of the SM such as $SO(14)$, $SU(9)$, $3-3-1$, $U(1)_X$, etc. [82–97] that attempt to explain the flavor and the mass hierarchy problem of the SM. Another mechanism to

generate textures in the Yukawa Lagrangian is through additional discrete or global symmetries. Some groups that have been used in the literature are: S_3 , A_4 , Δ_{27} , Z_2 , etc. [98–110]. The simplest symmetries are of abelian type, which can be used to impose texture-zeros in the mass matrices to make them predictive. On the other hand, given fermion mass matrices with texture-zeros, it is possible to find an extended scalar sector so that the texture-zeros can be generated from abelian symmetries [58,59].

Due to the fact that there are three up-type quarks and three down-type quarks, the mass operators are 3×3 complex matrices with 36 degrees of freedom. If we consider these operators to be Hermitian [111–113], the number of free parameters reduces to 18, which cannot be fully determined from the 10 available physical quantities, namely masses and mixing angles [114]. This provides freedom to reduce the number of free parameters in the matrices and search for matrix structures with zeros that provide eigenvalues and mixing angles consistent with the masses and mixing matrices of the fermions. One way to find zeros in the mass matrices that is automatically consistent with experimental data is based on weak basis transformations (WBT) for quarks and leptons [112,113,115,116]. Fritzsch proposed an ansatz with six zeros [117,118,118–122], but the value of $|V_{ub}/V_{cb}| \approx 0.06$ is too small compared to the experimental value $|V_{ub}/V_{cb}|_{\text{exp}} \approx 0.09$ [123]. For this reason, the use of 4 and 5 zero-textures was proposed [111,112,122,124–127]. References [111,113] showed that matrices with five zero-textures could reproduce the mass hierarchy and mixing angles of the CKM matrix.

The strong CP problem arises from the fact that the QCD Lagrangian has a non-perturbative term (“ θ -term”) that explicitly violates CP in strong interactions. On the other hand, the possible connection between the strong CP problem and flavor problems was first mentioned in [128], and in later works [129–133]. Some recent studies and further references in the same direction are found in [59,129,134–146]. Peccei and Quinn proposed a solution to the strong CP problem [147,148], where it is assumed that the SM has an additional global chiral symmetry $U(1)$, which is spontaneously broken at a large energy scale f_a . One consequence of this breaking is the existence of a particle called the axion, which is the Goldstone boson of the broken $U(1)_{PQ}$ symmetry [149,150]. Due to the fact that the PQ symmetry is not exact at the quantum level, as a result of a chiral anomaly, the axion is massive and its mass (see Appendix D) is given by:

$$m_a = \frac{f_\pi m_\pi}{f_a} \frac{\sqrt{z}}{1+z} \approx 6\mu\text{eV} \left(\frac{10^{12}\text{GeV}}{f_a} \right), \quad (1)$$

where $z = 0.56$ is assumed for the up and down quark mass ratio, while $f_\pi \approx 92$ MeV and $m_\pi = 135$ MeV are the pion decay constant and mass, respectively.

The effective couplings of axions to ordinary particles are inversely proportional to f_a , and also depend on the model. It was originally thought that the PQ symmetry breaking occurred at the electroweak scale, but experiments have ruled this out. The mass of the axion and its coupling to matter and radiation scale as $1/f_a$, making its direct detection extremely difficult. The combined limits from unsuccessful searches in nuclear and particle physics experiments and from stellar evolution imply that $f_a \geq 3 \times 10^9$ GeV [151]. Furthermore, there is an upper limit of $f_a \leq 10^{12}$ GeV that comes from cosmology, since light axions are produced in abundance during the QCD phase transition [152–156]. Hence, these models are generically referred to as “invisible” axion models and remain phenomenologically viable. There are two classes of invisible axion models in the literature: KSVZ (Kim, Shifman, Vainshtein, and Zakharov) [151, 157] and DFSZ (Dine, Fischler, Srednicki, and Zhitnitsky) [158, 159]. The main difference between KSVZ-type and DFSZ-type axions is that the former do not couple to ordinary quarks and leptons at tree level, but instead require an exotic quark that ensures a nonzero QCD anomaly to generate CP violation. Depending on the assumed value of f_a , the existence of axions could have interesting consequences in astrophysics and cosmology. The emission of axions produced in stellar plasma through their coupling to photons, electrons, and nucleons would provide a new mechanism for energy loss in stars. This could accelerate the evolutionary process of stars and, therefore, shorten their lifespan. Axions can also exist as primordial cosmic relics produced copiously in early times and could be candidates for dark matter. From numerous laboratory experiments and astrophysical observations, together with the cosmological requirement that the contribution to the mass density of the Universe from relic axions does not saturate the Universe. In post-inflationary scenarios, these constraints restrict the allowed values of the axion mass to a range of of [160] 10^{-5} eV $< m_a < 10^{-4}$ eV. One source of axions would be the Sun, which, coupled to two photons, could be produced through the Primakoff conversion of thermal photons in the electric and magnetic fields of the solar plasma. The limits are primarily useful for complementing the arguments of stellar energy loss [161] and the searches for solar axions by CAST at CERN [162] and the Tokyo axion helioscope [163].

The axion–photon coupling (see Appendix D) can be calculated in chiral perturbation theory as [147, 148].

$$g_{a\gamma} = -\frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - \frac{2z+4}{3z+1} \right). \quad (2)$$

This coupling and the axion mass are related to each other through the relation E/N , which depends on the model and can be tested in experiments.

The strongest limits on the axion–electron coupling are derived from observations of stars with a dense core, where bremsstrahlung is very effective. These conditions are real-

ized in White Dwarfs and Red Giant Stars, where the evolution of a White Dwarf is a cooling process by photon radiation and neutrino emission, with the possible addition of new energy loss channels such as axions. Current numerical analysis suggest a limit of $g_{ae} \leq 2.8 \times 10^{-13}$ [160]. In particular, using data from the Sloan Digital Sky Survey (SDSS) and SuperCOSMOS Sky Survey (SCSS) [164], they showed that the axion–electron coupling is approximately 1.4×10^{-13} . In a more recent analysis of the data in Ref. [164] by interpreting anomalous cooling observations in White Dwarfs and Red Giant Stars as a consequence of additional cooling channels induced by axions, the axion–electron coupling is determined to lie within the 2σ confidence interval $g_{ae} = 1.5_{-0.9}^{+0.6} \times 10^{-13}$ (95% CL) [165, 166]. The two groups studying the axion–electron coupling are M5 [161] and M3 [167]. Their combination yields the limit $g_{ae} = 1.6_{-0.34}^{+0.29} \times 10^{-13}$. For a recent and comprehensive review of axion physics, see [160].

This document is organized as follows: In Sect. 2, we review the textures for the quark and lepton mass matrices that will be used in this work. We also write the real parameters of these matrices in terms of the masses of the SM fermions and two free parameters. In Sect. 3, we present the particle content of our model and the necessary PQ charges to generate the mass matrix textures presented in Sect. 2. In Sect. 4, we adjust the Yukawa couplings to obtain the masses of the charged leptons and neutrinos. It is important to note that we cannot use the VEVs to adjust the lepton masses, as these were already adjusted to reproduce the quark masses. It is also important to note that by using a seesaw mechanism, we can avoid adjusting the Yukawas, however, that is not our purpose in the present work. In Sect. 5, we show the Lagrangian of our model. In Sect. 6, we present some constraints in the parameter space, as well as projected constraints for upcoming experimental results, both for experiments under construction and in the data-taking phase.

2 The five texture-zero mass matrices

The reason for dealing with texture zeros in the Standard Model (SM) and its extensions is to simplify as much as possible the number of free parameters that allow us to see relationships between masses and mixings present in these models. The Yukawa Lagrangian is responsible for giving mass to SM fermions after spontaneous symmetry breaking. A first simplification, without losing generality, is to consider that the fermion mass matrices are Hermitian, so the number of free parameters for each sector of quarks and leptons is reduced to 18, but there is still an excess of parameters to reproduce the experimental data provided in the literature. Due to the lack of a model to make predictions, discrete symmetries can be used to prohibit some components in the

Yukawa matrix, generating the so-called texture zeros for the mass matrices. In many works, instead of proposing discrete symmetries, texture zeros are proposed as practical and direct alternatives. The advantage of this approach is that it is possible to choose each mass matrix in an optimal way for the analytical treatment of the problem, and at the same time adjust the mixing angles and the masses of the fermions.

2.1 Quark sector

We should keep in mind that six-zero textures in the SM have already been discarded because their predictions are outside the experimental ranges allowed; but, five-zero textures for quark mass matrices is a viable possibility [55, 115, 168–171]. Specifically, we chose the following five-zero textures because they fit well with experimental quark masses and mixing parameters [111, 113, 172]:

$$\begin{aligned}
 M^U &= \begin{pmatrix} 0 & 0 & C_u \\ 0 & A_u & B_u \\ C_u^* & B_u^* & D_u \end{pmatrix}, \\
 M^D &= \begin{pmatrix} 0 & C_d & 0 \\ C_d^* & 0 & B_d \\ 0 & B_d^* & A_d \end{pmatrix}.
 \end{aligned}
 \tag{3}$$

In addition, the phases in M^D can be removed by a weak basis transformation (WBT) [111, 112, 116], so that they are absorbed by the off-diagonal terms in M^U . In this way, the mass matrices (3) can be rewritten as:

$$\begin{aligned}
 M^U &= \begin{pmatrix} 0 & 0 & |C_u|e^{i\phi_{C_u}} \\ 0 & A_u & |B_u|e^{i\phi_{B_u}} \\ |C_u|e^{-i\phi_{C_u}} & |B_u|e^{-i\phi_{B_u}} & D_u \end{pmatrix}, \\
 M^D &= \begin{pmatrix} 0 & |C_d| & 0 \\ |C_d| & 0 & |B_d| \\ 0 & |B_d| & A_d \end{pmatrix},
 \end{aligned}
 \tag{4}$$

By applying the trace and the determinant to the mass matrices (4), before and after the diagonalization process, the free real parameters of M^U and M^D can be written in terms of their masses:

$$D_u = m_u - m_c + m_t - A_u, \tag{5a}$$

$$|B_u| = \sqrt{\frac{(A_u - m_u)(A_u + m_c)(m_t - A_u)}{A_u}}, \tag{5b}$$

$$|C_u| = \sqrt{\frac{m_u m_c m_t}{A_u}}, \tag{5c}$$

$$A_d = m_d - m_s + m_b, \tag{5d}$$

$$|B_d| = \sqrt{\frac{(m_b - m_s)(m_d + m_b)(m_s - m_d)}{m_d - m_s + m_b}}, \tag{5e}$$

$$|C_d| = \sqrt{\frac{m_d m_s m_b}{m_d - m_s + m_b}}. \tag{5f}$$

A possibility that works very well is to consider the second generation of quark masses to be negative, i.e., with eigenvalues $-m_c$ and $-m_s$. And A_u is a free parameter, whose value, determined by the quark mass hierarchy, must be in the following range:

$$m_u \leq A_u \leq m_t. \tag{6}$$

The exact analytical procedure for diagonalizing the mass matrices (4) is indicated in Appendix C.

2.2 Lepton sector

In this work, we will consider Dirac neutrinos. This is achieved, in part, by extending the SM with right-handed neutrinos. In this way, we can carry out a treatment similar to that of the quark sector, that is, the mass matrices of the lepton sector can be considered Hermitian and the weak basis transformation (WBT) can be applied [111, 112]. In the literature, work has been done considering various texture-zeros for the Dirac mass matrices of the lepton sector [53, 173–184]. In our treatment, we are going to consider the following five-zero texture model studied in the paper [126], which can accurately reproduce the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix V_{PMNS} (mixing angles and the CP violating phase), the charged lepton masses, and the squared mass differences in the normal mass ordering.

$$\begin{aligned}
 M^N &= \begin{pmatrix} 0 & |C_\nu|e^{ic_\nu} & 0 \\ |C_\nu|e^{-ic_\nu} & E_\nu & |B_\nu|e^{ib_\nu} \\ 0 & |B_\nu|e^{-ib_\nu} & A_\nu \end{pmatrix}, \\
 M^E &= \begin{pmatrix} 0 & |C_\ell| & 0 \\ |C_\ell| & 0 & |B_\ell| \\ 0 & |B_\ell| & A_\ell \end{pmatrix}.
 \end{aligned}
 \tag{7}$$

Without loss of generality, by using a WBT, the phases of the charged lepton mass matrix, M^E , can be absorbed into the entries C_ν and B_ν of the neutrino mass matrix, M^N . Similarly, as was done in the case of the quark sector, the parameters present in the mass matrices of the lepton sector (7) can be expressed in terms of the masses of the charged leptons m_e, m_μ and m_τ and the masses of the neutrinos m_1, m_2 and m_3 , in the normal ordering ($m_1 < m_2 < m_3$):

$$A_\ell = m_e - m_\mu + m_\tau, \tag{8a}$$

$$|B_\ell| = \sqrt{\frac{(m_\tau - m_\mu)(m_e + m_\tau)(m_\mu - m_e)}{m_e - m_\mu + m_\tau}}, \tag{8b}$$

$$|C_\ell| = \sqrt{\frac{m_e m_\mu m_\tau}{m_e - m_\mu + m_\tau}}, \tag{8c}$$

$$E_\nu = m_1 - m_2 + m_3 - A_\nu, \tag{8d}$$

$$|B_\nu| = \sqrt{\frac{(A_\nu - m_1)(A_\nu + m_2)(m_3 - A_\nu)}{A_\nu}}, \tag{8e}$$

$$|C_\nu| = \sqrt{\frac{m_1 m_2 m_3}{A_\nu}}, \tag{8f}$$

where the values of the masses and the parameter A_ν are given in Table 5. Furthermore, for the adjustment of the mass matrices (7) it is very convenient to assume that the eigenvalues associated with the masses of the second family, $-m_2$ and $-m_\mu$, are negative quantities. The exact diagonalizing matrices of the mass matrices (7) are shown in Appendix C, Eqs. (49), (50) and (51).

3 PQ symmetry and the minimal particle content

3.1 Yukawa Lagrangian and the PQ symmetry

The texture-zeros of the mass matrices defined in the Eqs. (4) and (7) can be generated by imposing a Peccei–Queen symmetry $U(1)_{PQ}$ on the Lagrangian model, Eq. (9) [59, 185, 186]. As will be explained below, the minimal Lagrangian that allows us to implement this symmetry is given by [58, 187]

$$\begin{aligned} \mathcal{L}_{LO} \supset & (D_\mu \Phi^\alpha)^\dagger D^\mu \Phi^\alpha + \sum_\psi i \bar{\psi} \gamma^\mu D_\mu \psi + \sum_{i=1}^2 (D_\mu S_i)^\dagger D^\mu S_i \\ & - \left(\bar{q}_{Li} \gamma_{ij}^{D\alpha} \Phi^\alpha d_{Rj} + \bar{q}_{Li} \gamma_{ij}^{U\alpha} \tilde{\Phi}^\alpha u_{Rj} \right. \\ & \left. + \bar{\ell}_{Li} \gamma_{ij}^{E\alpha} \Phi^\alpha e_{Rj} + \bar{\ell}_{Li} \gamma_{ij}^{N\alpha} \tilde{\Phi}^\alpha \nu_{Rj} + \text{h.c.} \right) \\ & + (\lambda_Q \bar{Q}_R Q_L S_2 + \text{h.c.}) - V(\Phi, S_1, S_2). \end{aligned} \tag{9}$$

As it was shown in Ref. [58], at least four Higgs doublets are required to generate the quark mass textures, therefore $\alpha = 1, 2, 3, 4$. In (9) i, j are family indices (there is an implicit sum over repeated indices). The superscripts U, D, E, N refer to up-type quarks, down-type quarks, electron-like and neutrino-like fermions, respectively; and $D_\mu = \partial_\mu + i\Gamma_\mu$ is the covariant derivative in the SM. The scalar potential $V(\Phi, S_1, S_2)$ is shown in appendix A (for further details, see Ref. [58]). In Eq. (9) ψ stands for the SM fermion fields plus the heavy quark Q (see Tables 1 and 2). As it is shown in Table 2 the PQ charges of the heavy quark can be chosen in such a way that only the interaction with the scalar singlet S_2 is allowed. We assign Q_{PQ} charges for the left-handed quark doublets (q_L): x_{qi} , right-handed up-type quark singlets (u_R): x_{ui} , right-handed down-type quark singlets (d_R): x_{di} , left-handed lepton doublets (ℓ_L): $x_{\ell i}$, right-handed charged leptons (e_R): x_{ei} and right-handed Dirac neutrinos (ν_R): $x_{\nu i}$ for each family ($i = 1, 2, 3$). We follow a similar notation for the scalar doublets, x_{ϕ_α} ($\alpha = 1, 2, 3, 4$), and the scalar singlets $x_{S_{1,2}}$.

In this work, the PQ charges assigned to the quark sector and the scalar sector, as well as the VEVs assigned to the scalar doublets, will be the same as those assigned in [58] (Tables 1 and 2), and we will adjust the PQ charges of the lepton sector to reproduce the texture-zeros given in Eq. (7). To forbid a given entry in the lepton mass matrices, the corresponding sum of PQ charges must be different from zero, so that we can obtain texture-zeros by imposing the following conditions:

$$M^N = \begin{pmatrix} 0 & x & 0 \\ x & x & x \\ 0 & x & x \end{pmatrix} \longrightarrow \begin{pmatrix} S_{11}^{N\alpha} \neq 0 & S_{12}^{N\alpha} = 0 & S_{13}^{N\alpha} \neq 0 \\ S_{21}^{N\alpha} = 0 & S_{22}^{N\alpha} = 0 & S_{23}^{N\alpha} = 0 \\ S_{31}^{N\alpha} \neq 0 & S_{32}^{N\alpha} = 0 & S_{33}^{N\alpha} = 0 \end{pmatrix}, \tag{10}$$

$$M^E = \begin{pmatrix} 0 & x & 0 \\ x & 0 & x \\ 0 & x & x \end{pmatrix} \longrightarrow \begin{pmatrix} S_{11}^{E\alpha} \neq 0 & S_{12}^{E\alpha} = 0 & S_{13}^{E\alpha} \neq 0 \\ S_{21}^{E\alpha} = 0 & S_{22}^{E\alpha} \neq 0 & S_{23}^{E\alpha} = 0 \\ S_{31}^{E\alpha} \neq 0 & S_{32}^{E\alpha} = 0 & S_{33}^{E\alpha} = 0 \end{pmatrix}, \tag{11}$$

where $S_{ij}^{N\alpha} = (-x_{\ell i} + x_{\nu j} - x_{\phi_\alpha})$ and $S_{ij}^{E\alpha} = (-x_{\ell i} + x_{e_j} + x_{\phi_\alpha})$.

Since the PQ charges of the Higgs doublets ($\alpha = 1, 2, 3, 4$) are already given, the possible solutions of (10) and (11) are strongly constrained. Table 1 provides a solution for the PQ charges of the lepton sector.

In our model we include two scalar singlets S_1 and S_2 that break the global symmetry $U(1)_{PQ}$. The QCD anomaly of the PQ charges is

$$N = 2 \sum_i^3 x_{qi} - \sum_i^3 x_{ui} - \sum_i^3 x_{di} + A_Q, \tag{12}$$

where $A_Q = x_{QL} - x_{QR}$ is the contribution to the anomaly of the heavy quark Q , which is a singlet under the electroweak gauge group, with left (right) Peccei-Quinn charges $x_{QL,R}$, respectively. We can write the charges as a function of N (since N must be different from zero), such that

$$s_1 = \frac{N}{9} \hat{s}_1, \quad s_2 = \frac{N}{9} (\epsilon + \hat{s}_1), \quad \text{with } \epsilon = 1 - \frac{A_Q}{N}, \tag{13}$$

where \hat{s}_1 and ϵ are arbitrary real numbers. To solve the strong CP problem with $N \neq 0$ and simultaneously generate the texture-zeros in the mass matrices, it is necessary to maintain $\epsilon = \frac{9(s_2 - s_1)}{N} \neq 0$. With these definitions for Flavor-Changing Neutral Currents (FCNC) observables, the relevant parameters are \hat{s}_1 and ϵ . This parameterization is quite convenient (for those cases where the parameters α_q and α_ℓ are not relevant) because by fixing N and f_a , we can vary \hat{s}_1 and ϵ for a fixed $\Lambda_{PQ} = f_a N$ in such a way that the parameter space naturally reduces to two dimensions.

Table 1 Particle content. The subindex $i = 1, 2, 3$ stand for the family number in the interaction basis. Columns 6–8 are the Peccei-Quinn charges, Q_{PQ} , for each family of quarks and leptons in the SM. s_1, s_2 and α are real parameters, with $s_1 \neq s_2$

Particles	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U_{PQ}(i = 1)$	$U_{PQ}(i = 2)$	$U_{PQ}(i = 3)$	Q_{PQ}
q_{Li}	1/2	3	2	1/6	$-2s_1 + 2s_2 + \alpha_q$	$-s_1 + s_2 + \alpha_q$	α_q	x_{q_i}
u_{Ri}	1/2	3	1	2/3	$s_1 + \alpha_q$	$s_2 + \alpha_q$	$-s_1 + 2s_2 + \alpha_q$	x_{u_i}
d_{Ri}	1/2	3	1	-1/3	$2s_1 - 3s_2 + \alpha_q$	$s_1 - 2s_2 + \alpha_q$	$-s_2 + \alpha_q$	x_{d_i}
ℓ_{Li}	1/2	1	2	-1/2	$-2s_1 + 2s_2 + \alpha_\ell$	$-s_1 + s_2 + \alpha_\ell$	α_ℓ	x_{ℓ_i}
e_{Ri}	1/2	1	1	-1	$2s_1 - 3s_2 + \alpha_\ell$	$s_1 - 2s_2 + \alpha_\ell$	$-s_2 + \alpha_\ell$	x_{e_i}
ν_{Ri}	1/2	1	1	0	$-4s_1 + 5s_2 + \alpha_\ell$	$-s_1 + 2s_2 + \alpha_\ell$	$s_2 + \alpha_\ell$	x_{ν_i}

Table 2 Beyond the SM fields and their respective PQ charges. The parameters s_1, s_2 are reals, with $s_1 \neq s_2$ and $x_{Q_R} \neq x_{Q_L}$

Particles	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	U_{PQ}	Q_{PQ}
Φ_1	0	1	2	1/2	s_1	x_{ϕ_1}
Φ_2	0	1	2	1/2	s_2	x_{ϕ_2}
Φ_3	0	1	2	1/2	$-s_1 + 2s_2$	x_{ϕ_3}
Φ_4	0	1	2	1/2	$-3s_1 + 4s_2$	x_{ϕ_4}
Q_L	1/2	3	1	0	x_{Q_L}	x_{Q_L}
Q_R	1/2	3	1	0	x_{Q_R}	x_{Q_R}
S_1	0	1	1	0	$s_1 - s_2$	x_{s_1}
S_2	0	1	1	0	$x_{Q_R} - x_{Q_L}$	x_{s_2}

4 Naturalness of Yukawa couplings

4.1 The mass matrices in the quark sector

In Ref. [58], it was shown that to generate five texture zeros in the quark mass matrices (3), as a consequence of a PQ symmetry, it is necessary to include at least four scalar doublets in the model. After spontaneous symmetry breaking, the quark sector mass matrices take on the following form:

$$\begin{aligned}
 M^U &= \hat{v}_\alpha y_{ij}^{U\alpha} = \begin{pmatrix} 0 & 0 & y_{13}^{U1} \hat{v}_1 \\ 0 & y_{22}^{U1} \hat{v}_1 & y_{23}^{U2} \hat{v}_2 \\ y_{13}^{U1*} \hat{v}_1 & y_{23}^{U2*} \hat{v}_2 & y_{33}^{U3} \hat{v}_3 \end{pmatrix}, \\
 M^D &= \hat{v}_\alpha y_{ij}^{D\alpha} = \begin{pmatrix} 0 & |y_{12}^{D4}| \hat{v}_4 & 0 \\ |y_{12}^{D4}| \hat{v}_4 & 0 & |y_{23}^{D3}| \hat{v}_3 \\ 0 & |y_{23}^{D3}| \hat{v}_3 & y_{33}^{D2} \hat{v}_2 \end{pmatrix}, \tag{14}
 \end{aligned}$$

where the \hat{v}_i are defined in terms of the vacuum expectation values, $\hat{v}_i = v_i/\sqrt{2}$. In [58] it was shown that the five-texture zeros (4) are flexible enough to set the quark Yukawa couplings close to 1 for most of them (except for y_{23}^{U2}, y_{23}^{D3} and y_{13}^{U1}), in this way we obtain:

$$\begin{aligned}
 \hat{v}_1 &= 1.71 \text{ GeV}, & \hat{v}_2 &= 2.91 \text{ GeV}, \\
 \hat{v}_3 &= 174.085 \text{ GeV}, & \hat{v}_4 &= 13.3 \text{ MeV}. \tag{15}
 \end{aligned}$$

As we can see, the hermiticity of the mass matrices is not fully achieved, but it is good to impose it for several reasons:

(i) In the SM and its extensions, in which the right chirality fields are singlets under $SU(2)$, the mass matrices can be assumed Hermitian without losing generality, (ii) the previous fact allows us to consider Hermitian mass matrices, even after imposing an additional PQ symmetry in the model, (iii) we can implement the WBT method [111], and (iv) there is an extensive literature on physically viable Hermitian mass matrices. It is important to noticing that the mass matrices in Eq. (14) are Hermitian.

4.2 The mass matrices in the lepton sector

We can obtain the lepton mass matrices by starting from the Yukawa Lagrangian (9), which is invariant under the Peccei-Quinn $U(1)_{PQ}$ symmetry, and taking into account the Yukawa parameters and expectation values (15). After the spontaneous symmetry breaking, the mass matrices for neutral and charged leptons are given respectively by [126, 188]:

$$M^N = \hat{v}_\alpha y_{ij}^{N\alpha} = \begin{pmatrix} 0 & y_{12}^{N1} \hat{v}_1 & 0 \\ y_{21}^{N4} \hat{v}_4 & y_{22}^{N2} \hat{v}_2 & y_{23}^{N1} \hat{v}_1 \\ 0 & y_{32}^{N3} \hat{v}_3 & y_{33}^{N2} \hat{v}_2 \end{pmatrix}, \tag{16}$$

$$M^E = \hat{v}_\alpha y_{ij}^{E\alpha} = \begin{pmatrix} 0 & |y_{12}^{E4}| \hat{v}_4 & 0 \\ |y_{12}^{E4}| \hat{v}_4 & 0 & |y_{23}^{E3}| \hat{v}_3 \\ 0 & |y_{23}^{E3}| \hat{v}_3 & y_{33}^{E2} \hat{v}_2 \end{pmatrix}. \tag{17}$$

As we previously mentioned, at least four Higgs doublets are needed to obtain the five texture-zeros for the chosen quark

mass matrices. Our goal in this work is to keep the same number of Higgs doublets and their respective PQ charges to generate the mass matrices and texture zeros for the lepton sector, Eq. (7). To get an Hermitian mass matrix M^N , it is necessary to impose $y_{21}^{N4}/y_{12}^{N1*} = \hat{v}_1/\hat{v}_4$ and $y_{32}^{N3}/y_{23}^{N1*} = \hat{v}_1/\hat{v}_3$, requiring that the diagonal elements be real, i.e., $y_{22}^{N2} = y_{22}^{N2*}$ and $y_{33}^{N2} = y_{33}^{N2*}$. On the other hand, to obtain a symmetric mass matrix, M^E , for the charged leptons, it is sufficient to assume that the Yukawa couplings are Hermitian. Through these choices it is possible to avoid additional Higgs doublets.

Based on the results of Table 5, Appendix C, and the relationships established in (8), we find the following values for the Yukawa couplings of the lepton sector:

$$\begin{aligned}
 |y_{12}^{E4}| &= 0.569582, & |y_{23}^{E3}| &= 0.00248291, \\
 y_{33}^{E2} &= 0.574472, & |y_{12}^{N1}| &= 4.74362 \times 10^{-6}, \\
 |y_{21}^{N4}| &= 0.000609894, & y_{22}^{N2} &= 6.68808 \times 10^{-6}, \\
 |y_{23}^{N1}| &= 0.0000159881, & |y_{32}^{N3}| &= 1.57047 \times 10^{-7}, \\
 y_{33}^{N2} &= 8.65364 \times 10^{-6}.
 \end{aligned}$$

To reproduce the neutrino masses quoted in [126], in the SM is required a Yukawa coupling around 10^{-14} . In our case, the smallest Yukawa coupling is 10^{-7} , which significantly reduces the fine-tuning in comparison to that given by the SM.

5 The effective Lagrangian

The strongest constraints on non-universal PQ charges come from the FCNC. To determine these constraints, we start by writing the most general next-to-leading order (NLO) effective Lagrangian as [189, 190]:

$$\mathcal{L}_{\text{NLO}} = c_{a\Phi^\alpha} O_{a\Phi^\alpha} + c_1 \frac{\alpha_1}{8\pi} O_B + c_2 \frac{\alpha_2}{8\pi} O_W + c_3 \frac{\alpha_3}{8\pi} O_G, \tag{18}$$

$c_{a\Phi^\alpha}$ and $c_{1,2,3}$ are Wilson coefficients; $\alpha_{1,2,3} = \frac{g_{1,2,3}^2}{4\pi}$, where $g_{1,2,3}$ are the coupling strengths of the electroweak and strong interactions in the interaction basis; and the Wilson operators are:

$$\begin{aligned}
 O_{a\Phi} &= i \frac{\partial^\mu a}{\Lambda_{\text{PQ}}} \left((D_\mu \Phi^\alpha)^\dagger \Phi^\alpha - \Phi^{\alpha\dagger} (D_\mu \Phi^\alpha) \right), \\
 O_B &= -\frac{a}{\Lambda_{\text{PQ}}} B_{\mu\nu} \tilde{B}^{\mu\nu},
 \end{aligned}$$

$$\begin{aligned}
 O_W &= -\frac{a}{\Lambda_{\text{PQ}}} W_{\mu\nu}^a \tilde{W}^{a\mu\nu}, \\
 O_G &= -\frac{a}{\Lambda_{\text{PQ}}} G_{\mu\nu}^a \tilde{G}^{a\mu\nu},
 \end{aligned} \tag{19}$$

where B , W^a and G^a correspond to the gauge fields associated with the SM gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. a is the axion field which corresponds to the CP odd component of S_1 . It is possible to redefine the fields by multiplying by a phase

$$\begin{aligned}
 \Phi^\alpha &\longrightarrow e^{i \frac{x_{\Phi^\alpha} a}{\Lambda_{\text{PQ}}}} \Phi^\alpha, \\
 \psi_L &\longrightarrow e^{i \frac{x_{\psi_L} a}{\Lambda_{\text{PQ}}}} \psi_L, \\
 \psi_R &\longrightarrow e^{i \frac{x_{\psi_R} a}{\Lambda_{\text{PQ}}}} \psi_R, \\
 S_i &\longrightarrow e^{i \frac{x_{S_i} a}{\Lambda_{\text{PQ}}}} S_i.
 \end{aligned} \tag{20}$$

In this expression, x_ψ corresponds to the PQ charges of the SM fermions, i.e., $\{x_{\psi_{L,R}}\} = \{x_{q_i}, x_{u_i}, x_{d_i}, x_{l_i}, x_{e_i}, x_{\nu_i}\}$ and $\{x_{\Phi^\alpha}\}$ are the PQ charges of the Higgs doublets $\{\Phi^\alpha\}$. Replacing these definitions in the kinetic terms of Eq. (9), we obtain new contributions to the effective Lagrangian Eq. (18) (the NLO contributions in the non-derivative terms cancel out). The leading order (LO) terms in Λ_{PQ}^{-1} can be written as [187, 189]:

$$\mathcal{L}_{\text{NLO}} \longrightarrow \mathcal{L}_{\text{NLO}} + \Delta \mathcal{L}_{\text{NLO}}, \tag{21}$$

where

$$\Delta \mathcal{L}_{\text{NLO}} = \Delta \mathcal{L}_{K\Phi} + \Delta \mathcal{L}_{K\psi} + \Delta \mathcal{L}_{K^S} + \Delta \mathcal{L}(F_{\mu\nu}), \tag{22}$$

with

$$\begin{aligned}
 \Delta \mathcal{L}_{K\Phi} &= i x_{\Phi^\alpha} \frac{\partial^\mu a}{\Lambda_{\text{PQ}}} \left[(D_\mu \Phi^\alpha)^\dagger \Phi^\alpha - \Phi^{\alpha\dagger} (D_\mu \Phi^\alpha) \right], \\
 \Delta \mathcal{L}_{K\psi} &= \frac{\partial^\mu a}{2\Lambda_{\text{PQ}}} \sum_\psi (x_{\psi_L} - x_{\psi_R}) \bar{\psi} \gamma^\mu \gamma^5 \psi \\
 &\quad - (x_{\psi_L} + x_{\psi_R}) \bar{\psi} \gamma^\mu \psi, \\
 \Delta \mathcal{L}_{K^S} &= i x_{S_i} \frac{\partial^\mu a}{\Lambda_{\text{PQ}}} \left[(D_\mu S_i)^\dagger S_i - S_i^\dagger (D_\mu S_i) \right] + \text{h.c.}
 \end{aligned} \tag{23}$$

The field redefinitions (20) induce a modification in the measure of the functional path integral whose effects can be obtained from the divergence of the axial-vector current: $J_\mu^{PQ5} = \sum_\psi (x_{\psi_L} - x_{\psi_R}) \bar{\psi} \gamma_\mu \gamma^5 \psi$ [191],

$$\begin{aligned}
 \partial^\mu J_\mu^{PQ5} &= \sum_\psi 2im_\psi (x_{\psi_L} - x_{\psi_R}) \bar{\psi} \gamma^5 \psi \\
 &\quad - \sum_\psi (x_{\psi_L} - x_{\psi_R}) \frac{\alpha_1 Y^2(\psi)}{2\pi} B_{\mu\nu} \tilde{B}^{\mu\nu}
 \end{aligned}$$

$$\begin{aligned}
 & - \sum_{SU(2)_L \text{ doublets}} x_{\psi_L} \frac{\alpha_2}{4\pi} W_{\mu\nu}^a \tilde{W}^{a\mu\nu} \\
 & - \sum_{SU(3) \text{ triplets}} (x_{\psi_L} - x_{\psi_R}) \frac{\alpha_3}{4\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \tag{24}
 \end{aligned}$$

where the hypercharge is normalized by $Q = T_{3L} + Y$. The relation (24) is an on-shell relation, which is consistent with the momentum of an on-shell axion.

Substituting this result into $\mathcal{L}_{K\psi} = \frac{\partial^\mu a}{2\Lambda_{PQ}} J_\mu^{PQ5} = -\frac{a}{2\Lambda_{PQ}} \partial^\mu J_\mu^{PQ5}$ we obtain new contributions to the leading-order Wilson coefficients [192]

$$\begin{aligned}
 c_1 & \longrightarrow c_1 - \frac{1}{3}\Sigma q + \frac{8}{3}\Sigma u + \frac{2}{3}\Sigma d - \Sigma\ell + 2\Sigma e, \\
 c_2 & \longrightarrow c_2 - 3\Sigma q - \Sigma\ell, \\
 c_3 & \longrightarrow c_3 - 2\Sigma q + \Sigma u + \Sigma d - A_Q, \tag{25}
 \end{aligned}$$

where $\Sigma q \equiv x_{q_1} + x_{q_2} + x_{q_3}$. The corresponding NLO Lagrangian is

$$\begin{aligned}
 \Delta\mathcal{L}(F_{\mu\nu}) &= \frac{a}{\Lambda_{PQ}} \frac{\alpha_1}{8\pi} B_{\mu\nu} \tilde{B}^{\mu\nu} \left(\frac{1}{3}\Sigma q - \frac{8}{3}\Sigma u - \frac{2}{3}\Sigma d + \Sigma\ell - 2\Sigma e \right) \\
 &+ \frac{a}{\Lambda_{PQ}} \frac{\alpha_2}{8\pi} W_{\mu\nu}^a \tilde{W}^{a\mu\nu} (3\Sigma q + \Sigma\ell) \\
 &+ \frac{a}{\Lambda_{PQ}} \frac{\alpha_3}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} (2\Sigma q - \Sigma u - \Sigma d + A_Q). \tag{26}
 \end{aligned}$$

It is convenient to define $c_3^{\text{eff}} = c_3 - 2\Sigma q + \Sigma u + \Sigma d - A_Q = -N$. In our case, $c_i = 0$ and the only contributions to c_i^{eff} come from the anomaly. It is customary to define $\Lambda_{PQ} = f_a |c_3^{\text{eff}}|$ to include the factor c_3^{eff} in the normalization of the PQ charges. From now on, we will assume that all the PQ charges are normalized in this way, so that x_ψ corresponds to x_ψ/c_3^{eff} . For normalized charges, $c_3^{\text{eff}} = 1$, therefore, we still maintain the general form despite writing all the expressions in terms of the effective scale f_a .

The scalar fields and their PQ charges are the same as in the Ref. [58], so the scalar potential $V(\Phi, S)$ is identical to that of the mentioned reference. With the VEVs and couplings given in [58], the model reproduces the mass of the SM Higgs, while the masses of the exotic scalars are above the TeV scale. This potential has the appropriate number of Goldstone bosons to give masses to the SM gauge bosons Z^0, W^\pm and has an extra field that can be identified with the axion a .

6 Low energy constraints

6.1 Flavor changing neutral currents

Due to the non-universal PQ charges in our model, a tree-level analysis of flavor-changing neutral currents is necessary. As mentioned in Ref. [160], the strongest limits on the axion-quark FCNC couplings come from meson decays in light mesons and missing energy.

The decays $K^\pm \rightarrow \pi^\pm a$ provide the tightest limits (NA62 Collaboration [193]) for the axion mass [160]. Currently the most restrictive limits come from the semileptonic decays of kaons $K^\pm \rightarrow \pi^\pm \bar{\nu} \nu$ and leptons $\ell_1 \rightarrow \ell_2 + \text{missing energy}$. From the term $\Delta\mathcal{L}_{K\psi}$, we obtain the vector and axial couplings for a multi-Higgs sector model, as shown in Refs. [58, 160]

$$\Delta\mathcal{L}_{K\psi} = -\partial_\mu a \bar{f}_i \gamma^\mu \left(g_{af_i f_j}^V + \gamma^5 g_{af_i f_j}^A \right) f_j, \tag{27}$$

where

$$g_{af_i f_j}^{V,A} = \frac{1}{2f_a c_3^{\text{eff}}} \Delta_{V,A}^{Fij}, \tag{28}$$

where:

$$\Delta_{V,A}^{Fij} = \Delta_{RR}^{Fij}(d) \pm \Delta_{LL}^{Fij}(q), \tag{29}$$

with $\Delta_{LL}^{Fij}(q) = \left(U_L^F x_q U_L^{F\dagger} \right)^{ij}$ and $\Delta_{RR}^{Fij}(d) = \left(U_R^F x_d U_R^{F\dagger} \right)^{ij}$. In these expressions, F stands for U, D, N or E and the $U_{L,R}^F$ are de diagonalizing matrices (see Appendix C). In Eq. (28), we normalize the charges using c_3^{eff} , as explained in the last paragraph of Sect. 5 (in other references $|c_3^{\text{eff}}| = |N|$ is considered, corresponding to the $SU(3) \times U(1)_{PQ}$ anomaly). The branching ratio for lepton decays $\ell_i \rightarrow \ell_j a$ is given by [134]

$$\text{Br}(\ell_1 \rightarrow \ell_2 a) = \frac{m_{\ell_1}^3}{16\pi \Gamma(\ell_1)} \left(1 - \frac{m_{\ell_2}^2}{m_{\ell_1}^2} \right)^3 |g_{a\ell_1 \ell_2}|^2,$$

in this expression, the vector and axial couplings contribute in the same way

$$|g_{a\ell_1 \ell_2}|^2 = |g_{a\ell_1 \ell_2}^V|^2 + |g_{a\ell_1 \ell_2}^A|^2.$$

where m is the mass of the leptons and $\Gamma(\ell_i)$ is the total decay width of the particle ℓ_j .

For the lepton decay $\ell_i \rightarrow \ell_j a \gamma$, we can relate this branching ratio to the branching ratio of the process without the photon in the final state, according to the expression:

$$\text{Br}(\ell_1 \rightarrow \ell_2 a \gamma) = \left(\frac{\alpha}{2\pi} \int dx dy f(x, y) \right) \text{Br}(\ell_1 \rightarrow \ell_2 a),$$

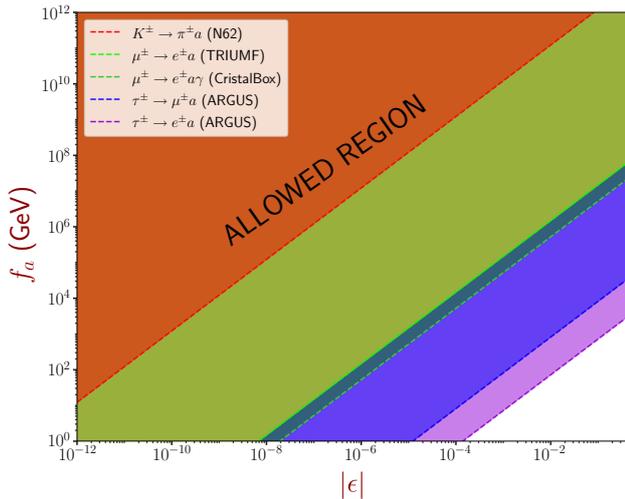


Fig. 1 Allowed regions by lepton decays. For the down-type quarks and charged leptons the non-universal part of the PQ charges just depend on the difference $s_2 - s_1 = N\epsilon/9$, hence the flavor-changing neutral-current couplings (the off diagonal elements) just depend on ϵ

where α is the fine structure constant, and the function:

$$f(x, y) = \frac{(1-x)(2-y-xy)}{y^2(x+y-1)} \tag{30}$$

depends on the mass and the energies $x = 2E_{\ell_2}/m_{\ell_1}$ and $y = 2E_\gamma/m_{\ell_1}$. For the lepton decay $\mu \rightarrow e a \gamma$, the constraints come from the Crystal Box experiment [194], with cut energies $E_\gamma, E_e > 30 \text{ MeV}$, $\theta_{e\gamma} > 140^\circ$, where:

$$\cos \theta_{e\gamma} = 1 + \frac{2(1-x-y)}{xy}, \tag{31}$$

so that $\int dx dy f(x, y) \approx 0.011$ (Table 3).

In our model, there is a natural alignment between the Φ_3 (which is quite similar to H_1 in the Georgi basis [198]) and the standard model Higgs boson as a consequence of the large suppression of the VEVs of the scalar doublets v_i , with $i = 1, 2, 4$, respect to v_3 , the VEV of Φ_3 . To some extent, this alignment avoids FCNC involving the SM Higgs boson [198]; however, after alignment, there are other sources of FCNC associated with the additional scalar doublets, which cannot be avoided by any means; however, as argued in Ref. [58] they are suppressed by a factor $1/M^4$ (where $M > 1 \text{ TeV}$ is the mass of the exotic scalar doublets), and therefore, our model avoids these potential sources of FCNC in agreement with the general argument presented in [198].

From astrophysical considerations we have: bounds from black holes superradiance and the SN 1987A upper limit on the neutron electric dipole moment, which, when combined, impose a constraint on the axion decay constant in the range [160] (see Fig. 1): $0.8 \times 10^6 \text{ GeV} \leq f_a \leq 2.8 \times 10^{17} \text{ GeV}$.

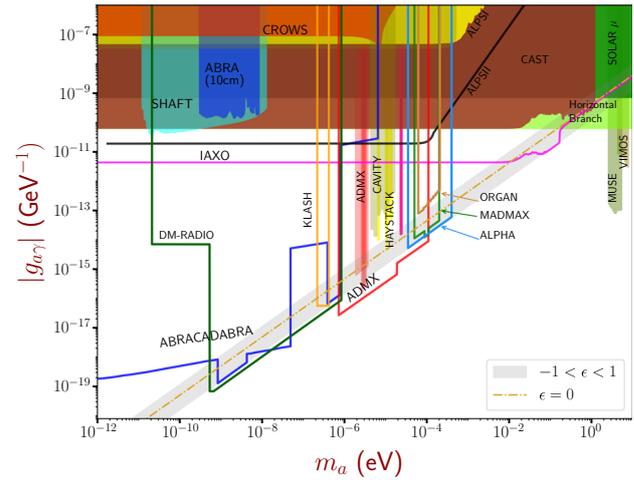


Fig. 2 The excluded parameter space by various experiments corresponds to the colored regions, the dashed-lines correspond to the projected bounds of coming experiments looking for axion signals [199]. The gray region corresponds to the parameter space scanned by our model

6.2 Constraints on the axion–photon coupling

There are several experiments designed to look for exotic particles. The sources studied in the search for axions are: the solar axion flux (helioscopes experiments), dark matter halo (haloscopes experiments), and axions produced in the laboratory.

Among the experiments with the potential to search for evidence of axions in regions that cover areas within the limits established by the parameters of our model are: DM-Radio [200], KLASH [201,202], ADMX [203], ALPHA [204], MADMAX [205], IAXO [206,207] and ABRACADABRA [208]. Similarly, some experiments have already ruled out regions established by the parameters of our model, among which are: ADMX [209–211], CAST [212,213], CAPP [214–216], HAYSTACK [217,218], Solar ν [219], Horizontal Branch [220], MUSE [221] and VIMOS [222]

7 Discussion and conclusions

We have presented a model in which the fermion and scalar fields are charged under a $U(1)_{PQ}$ Peccei-Quinn symmetry. A recent work [58] showed that at least four Higgs doublets are required to generate Hermitian mass matrices in the quark sector with five texture-zeros, reproducing the quark masses, the mixing angles, and the CP-violating phase of the CKM mixing matrix. In this work, we show that using the same number of Higgs doublets, without changing the PQ charges in the quark and Higgs sectors, it is possible to generate Hermitian mass matrices in the lepton sector that reproduce the neutrino mass-squared differences in the normal mass

Table 3 These inequalities come from the window for new physics in the branching ratio uncertainty of the meson decay in a pair $\bar{\nu}\nu$

Collaboration	Upper bound
N62 collaboration [193]	$\mathcal{B}(K^+ \rightarrow \pi^+ a) < (10.6_{-3.4}^{+4.0})_{stat} \pm 0.9_{syst} \times 10^{-11}$
TRIUMF [195]	$\mathcal{B}(\mu^+ \rightarrow e^+ a) < 2.6 \times 10^{-6}$
Crystal Box [196]	$\mathcal{B}(\mu^+ \rightarrow e^+ \gamma a) < 1.1 \times 10^{-9}$
ARGUS [197]	$\mathcal{B}(\tau^+ \rightarrow e^+ a) < 1.5 \times 10^{-2}$
ARGUS [197]	$\mathcal{B}(\tau^+ \rightarrow \mu^+ a) < 2.6 \times 10^{-2}$

ordering, the mixing angles, and the CP-violating phase of the PMNS mixing matrix. This result is quite non-trivial as we maintain the same four Higgs doublets required in the quark sector to generate a different texture pattern in the lepton sector. When compared to the SM, our model has almost all Yukawa couplings close to 1 in the quark sector. In the neutrino sector, the smallest Yukawa coupling is of the order of 1.6×10^{-7} , which is seven orders of magnitude larger than the corresponding Yukawa coupling in the SM, so it requires less fine-tuning than the SM.

The polar decomposition theorem [223, 224] allows any matrix to be written as the product of a Hermitian matrix and a unitary matrix. In the SM and in theories where the right-handed fermion fields are singlets under the gauge group, it is possible to absorb the unitary matrix into the right-handed fields by redefining them; from this procedure, we can write any mass matrix as a Hermitian matrix. In our work, we assume that the mass matrices are Hermitian in the interaction space, this hypothesis has been used in previous studies on textures [55, 111, 115, 225–227], and it is quite useful for studying the flavor problem. In our work, we have normalized the PQ charges with the QCD anomaly $-N$ in such a way that by keeping the parameter $\epsilon \neq 0$, we obtain the textures of the mass matrices, addressing the flavor and strong CP problems simultaneously.

If nature is not fine-tuned in a more fundamental high-energy theory, we expect that, eventually, it will be possible to find a texture that allows us to obtain all the scales of the SM from the VEVs of a Higgs sector with a minimal scalar content without the need to adjust the Yukawa couplings.

In our analysis, we report the constraints from lepton decays and compare them with the constraints from the search for neutrino pairs in charged Kaon decays $K^\pm \rightarrow \pi^\pm \bar{\nu}\nu$. The results are shown in Fig. 1, where the allowed region in the parameter space generated by ϵ and the axion decay constant f_a is displayed. This figure shows that the strongest constraints come from the semileptonic meson decay $K^\pm \rightarrow \pi \nu \bar{\nu}$. It is important to note that the lepton decays do not further constrain the parameter space of our model (compared to the region excluded by the meson decay). We also show the excluded regions for the axion–photon coupling as a function of the axion mass; these results

are summarized in Fig. 2; the gray region corresponds to the parameter space of our model in the interval $-1 < \epsilon < 1$.

In this article, we have demonstrated that with four Higgs doublets, it is possible to fit the textures of the mass matrices, both in the lepton and quark sectors. These matrices generate the masses and the mixing matrices for quarks and leptons within the experimentally reported values in the literature. The introduction these doublets improves the fine-tuning problem of the Yukawa couplings and shows that this approach is a viable way to tackle the flavor problem. We hope to improve our results in future work by using the See-saw mechanism in the lepton sector.

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Appendix A: The mass operator matrices

The most general Yukawa Lagrangian for the interaction of four Higgs doublets Φ_α with the SM fermions is given by

$$\mathcal{L} = -\bar{q}_L^i \Phi_\alpha y_{ij}^{D\alpha} d_R^j - \bar{q}_L^i \tilde{\Phi}_\alpha y_{ij}^{U\alpha} u_R^j - \bar{\ell}_L^i \Phi_\alpha y_{ij}^{E\alpha} e_R^j$$

$$- \bar{\ell}_L^i \tilde{\Phi}_\alpha y_{ij}^{N\alpha} \nu_R^j + \text{h.c.} \tag{32}$$

where a sum is assumed on repeated indices. Here i, j run over 1, 2, 3 and α over 1, 2, 3, 4. The Higgs boson doublet fields are parameterized as follows:

$$\Phi_\alpha = \begin{pmatrix} \phi_\alpha^+ \\ \frac{v_\alpha + h_\alpha + i\eta_\alpha}{\sqrt{2}} \end{pmatrix}, \quad \tilde{\Phi}_\alpha = i\sigma_2 \Phi_\alpha^*. \tag{33}$$

Similar to the two Higgs doublet model [228] we rotate the Higgs fields to the (generalized) Georgi basis, that is,

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{pmatrix} = R_1(\beta_1) R_2(\beta_2) R_3(\beta_3) \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix} \equiv R_{\beta\alpha} \Phi_\alpha, \tag{34}$$

where the orthogonal matrices

$$R_1(\beta_1) = \begin{pmatrix} \cos \beta_1 & \sin \beta_1 & 0 & 0 \\ -\sin \beta_1 & \cos \beta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{35a}$$

$$R_2(\beta_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \beta_2 & \sin \beta_2 & 0 \\ 0 & -\sin \beta_2 & \cos \beta_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{35b}$$

$$R_3(\beta_3) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \beta_3 & \sin \beta_3 \\ 0 & 0 & -\sin \beta_3 & \cos \beta_3 \end{pmatrix}, \tag{35c}$$

where $\tan \beta_1 = \frac{\sqrt{v_2^2 + v_3^2 + v_4^2}}{v_1}$, $\tan \beta_2 = \frac{\sqrt{v_3^2 + v_4^2}}{v_2}$ and $\tan \beta_3 = \frac{v_4}{v_3}$, and $H_\beta = (H_\beta^+, (H_\beta^0 + iH_\beta^{\text{odd}})/\sqrt{2})^T$. This basis is chosen in such a way that only the neutral component of H_1 acquires a vacuum expectation value

$$\langle H_1^0 \rangle = \sqrt{v_1^2 + v_2^2 + v_3^2 + v_4^2} \equiv v, \tag{36}$$

$$\langle H_2^0 \rangle = 0, \quad \langle H_3^0 \rangle = 0, \quad \langle H_4^0 \rangle = 0.$$

In this way $\Phi_\alpha y_{ij}^{F\alpha} = y_{ij}^{F\alpha} R_{\alpha\beta}^T R_{\beta\gamma} \Phi_\gamma = \mathcal{Y}_{ij}^{F\beta} H_\beta$, and $F = U, D, N, E$; where we have defined

$$\mathcal{Y}_{ij}^{F\beta} = R_{\beta\alpha} y_{ij}^{F\alpha}. \tag{37}$$

With these definitions, Eq. (32) becomes

$$\mathcal{L} = -\bar{q}_L^i H_\beta \mathcal{Y}_{ij}^{D\beta} d_R^j - \bar{q}_L^i \tilde{H}_\beta \mathcal{Y}_{ij}^{U\beta} u_R^j - \bar{\ell}_L^i H_\beta \mathcal{Y}_{ij}^{E\beta} e_R^j - \bar{\ell}_L^i \tilde{H}_\beta \mathcal{Y}_{ij}^{N\beta} \nu_R^j + \text{h.c.} \tag{38}$$

It is necessary to rotate to the fermion mass eigenstates, i.e.,

$$f_{L,R} = U_{L,R}^F f'_{L,R}, \tag{39}$$

where the diagonalization matrices $U_{L,R}$ are defined below, in Appendix C. From the Lagrangian for the charged currents

$$\begin{aligned} \mathcal{L}_{CC} &= -\frac{g}{\sqrt{2}} \bar{u}'_{Li} \gamma^\mu d'_{Li} W^+ - \frac{g}{\sqrt{2}} \bar{e}'_{Li} \gamma^\mu \nu'_{Li} W^- + \text{h.c.} \\ &= -\frac{g}{\sqrt{2}} \bar{u}_{Li} \gamma^\mu (V_{\text{CKM}})_{ij} d_{Lj} W^+ \\ &\quad - \frac{g}{\sqrt{2}} \bar{e}_{Li} \gamma^\mu (V_{\text{PMNS}})_{ij} \nu_{Lj} W^- + \text{h.c.} \end{aligned} \tag{40}$$

it is possible to obtain the CKM ($V_{\text{CKM}} = U_L^U U_L^{D\dagger}$) and PMNS ($V_{\text{PMNS}} = U_L^E U_L^{\nu\dagger}$) mixing matrices by rotating to the fermion mass eigenstates. In particular, we are interested in the coupling of the axial neutral current to the axion in the mass eigenstates.

$$\begin{aligned} \mathcal{L}_{H^0} &= -\frac{1}{\sqrt{2}} \bar{d}_L^i H_\beta^0 \mathcal{Y}_{ij}^{D\beta} d_R^j - \frac{1}{\sqrt{2}} \bar{u}_L^i H_\beta^{0*} \mathcal{Y}_{ij}^{U\beta} u_R^j \\ &\quad - \frac{1}{\sqrt{2}} \bar{e}_L^i H_\beta^0 \mathcal{Y}_{ij}^{E\beta} e_R^j - \frac{1}{\sqrt{2}} \bar{\nu}_L^i H_\beta^{0*} \mathcal{Y}_{ij}^{N\beta} \nu_R^j + \text{h.c.}, \\ &= -\frac{1}{\sqrt{2}} \bar{d}_L^i H_\beta^0 Y_{ij}^{D\beta} d_R^j - \frac{1}{\sqrt{2}} \bar{u}_L^i H_\beta^{0*} Y_{ij}^{U\beta} u_R^j \\ &\quad - \frac{1}{\sqrt{2}} \bar{e}_L^i H_\beta^0 Y_{ij}^{E\beta} e_R^j - \frac{1}{\sqrt{2}} \bar{\nu}_L^i H_\beta^{0*} Y_{ij}^{N\beta} \nu_R^j + \text{h.c.}, \end{aligned}$$

where $Y_{ij}^{F\beta} = (U_L^F \mathcal{Y}^{F\beta} U_R^{F\dagger})_{ij}$. In these expressions the mass functions in the interaction basis are:

$$\begin{aligned} M_{ij}^D &= \frac{v}{\sqrt{2}} \mathcal{Y}_{ij}^{D1}, & M_{ij}^U &= \frac{v}{\sqrt{2}} \mathcal{Y}_{ij}^{U1}, \\ M_{ij}^E &= \frac{v}{\sqrt{2}} \mathcal{Y}_{ij}^{E1}, & M_{ij}^N &= \frac{v}{\sqrt{2}} \mathcal{Y}_{ij}^{N1}, \end{aligned} \tag{41}$$

where $v = \langle H_1^0 \rangle$ is the Higgs vacuum expectation value.

Appendix B: Scalar potential

As studied in [58], the scalar sector requires four scalar doublets ϕ^α to reproduce the mass textures of the fermion sector correctly, and two scalar singlets S_1 and S_2 that break the PQ symmetry while generating a phenomenologically viable scalar mass spectrum. The S_2 singlet also gives mass to the heavy quark. The most general potential allowed by the PQ symmetry according to the charges established in Table 2 is:

$$V(\Phi, S_i) = \sum_{i=1}^4 \mu_i^2 \Phi_i^\dagger \Phi_i + \sum_{k=1}^2 \mu_{S_k}^2 S_k^* S_k + \sum_{i=1}^4 \lambda_i (\Phi_i^\dagger \Phi_i)^2$$

$$\begin{aligned}
 & + \sum_{k=1}^2 \lambda_{s_k} (S_k^* S_k)^2 + \sum_{i=1}^4 \sum_{k=1}^2 \lambda_{i s_k} (\Phi_i^\dagger \Phi_i) (S_k^* S_k) \\
 & + \sum_{\substack{i,j=1 \\ i < j}}^4 \left(\lambda_{ij} (\Phi_i^\dagger \Phi_i) (\Phi_j^\dagger \Phi_j) + J_{ij} (\Phi_i^\dagger \Phi_j) (\Phi_j^\dagger \Phi_i) \right) \\
 & + \lambda_{s_1 s_2} (S_1^* S_1) (S_2^* S_2) \\
 & + K_1 \left((\Phi_1^\dagger \Phi_2) (\Phi_3^\dagger \Phi_2) + h.c. \right) \\
 & + K_2 \left((\Phi_3^\dagger \Phi_4) (\Phi_3^\dagger \Phi_1) + h.c. \right) \\
 & + F_1 \left((\Phi_2^\dagger \Phi_3) S_1 + h.c. \right) \\
 & + F_2 \left((\Phi_1^\dagger \Phi_2) S_1 + h.c. \right) \\
 & + \frac{1}{2} (m_{\xi S_2})_{SB}^2 \xi_{S_2}^2 + \frac{1}{2} (m_{\epsilon S_2})_{SB}^2 \xi_{S_2}^2. \tag{42}
 \end{aligned}$$

where the terms proportional to F_i are allowed by the particular choice of PQ charges and these couplings F_i have units of mass. After spontaneous symmetry breaking (SSB), the four Higgs doublets acquire VEVs that give mass to all the SM particles. The scalar doublets and singlets are written as follows:

$$\begin{aligned}
 \Phi_\alpha & = \left(\frac{\phi_\alpha^+}{\frac{v_\alpha + h_\alpha + i\eta_\alpha}{\sqrt{2}}} \right), & \tilde{\Phi}_\alpha & = i\sigma_2 \Phi_\alpha^*, \alpha = 1, 2, 3, 4, \\
 S_i & = \frac{v_{S_i} + \xi_{S_i} + i\zeta_{S_i}}{\sqrt{2}}; & & i = 1, 2, \tag{43}
 \end{aligned}$$

where the VEVs satisfy the following hierarchy: $v_4 \ll v_1, v_2 \ll v_3 \ll v_{S_1} \sim v_{S_2}$. The scalar singlets S_1 and S_2 break the PQ symmetry at the high energy scale given by $v_{S_1} \approx v_{S_2}$. The last two terms in Eq. (42) correspond to the soft-breaking masses of the imaginary and the real parts of S_2 , which are generated at one loop in the Coleman-Weinberg potential from the interaction term $\lambda_Q S_2 \tilde{Q}_R Q_L + h.c.$ Additionally, we choose numerical values for the parameters of the potential (42) in order to obtain a scalar sector mass spectrum consistent with the existing phenomenology. The values of these parameters are:

$$\begin{aligned}
 \lambda_1 & = \lambda_2 = \lambda_4 = \lambda_{s_1} = \lambda_{s_2} = \lambda_{s_1 s_2} = 1, \\
 \lambda_3 & = 0.463 \\
 \lambda_{ij} & = 1 \text{ for any } i, j, \\
 \lambda_{j s_1} & = \lambda_{j s_2} = 1 \text{ for any } j, \\
 J_{12} & = J_{13} = J_{23} = J_{24} = -1, \text{ otherwise } J_{ij} = 1, \\
 K_1 & = K_2 = -1, \\
 F_1 & = F_2 = -1 \text{ GeV}. \tag{44}
 \end{aligned}$$

In particular, the value of λ_3 adjusts the SM Higgs mass. The v_i are determined from the SM fermion masses and the quark mass matrix textures, Eq. (15). The VEV v_{s_1} remains a free parameter; however, this parameter is important for the axion

physics due to the relationship [165],

$$f_a = \frac{v_{s_1}}{2N}. \tag{45}$$

In our calculations we took $v_{s_1} \approx v_{s_2} \approx 10^6 \text{ GeV}$. It is important to emphasize that in our model, f_a can take arbitrary values; nevertheless, a small f_a restricts ϵ (Eq. 13) to values close to zero. Taking into account all these considerations, including Eq. (44), the scalar mass spectrum (in GeV) is:

$$\begin{aligned}
 \text{CP even} & = \{1.73 \times 10^6, 1. \times 10^6, 6.54 \times 10^3, 1.97 \times 10^3, \\
 & \quad 1.09 \times 10^3, 125\}, \\
 \text{CP odd} & = \{6.54 \times 10^3, 1.97 \times 10^3, 1.09 \times 10^3, 0, 0, m_{\xi S_2}\}, \\
 \text{Charged fields} & = \{6.54 \times 10^3, 1.97 \times 10^3, 1.11 \times 10^3, 0\}. \tag{46}
 \end{aligned}$$

The mass spectrum of the scalar fields is above the TeVs scale, except for the SM Higgs, which is at 125 GeV. The pseudoscalar sector (CP odd fields) have two massless eigenstates, the axion field and the Goldstone boson which is absorbed by the longitudinal component of the SM Z boson. A similar result is obtained in the charged sector, where it is possible to identify the two Goldstone bosons required to give mass to the SM W^\pm fields.

Appendix C: diagonalization matrices

To compare with physical quantities, it is necessary to rotate fields to the mass eigenstates, i.e., $f_{L,R} = U_{L,R}^F f'_{L,R}$, where the prime symbol stands for the interaction basis. In our formalism the quark mass matrices are Hermitian, so the right- and left-handed diagonalizing matrices are identical; additionally, we establish that the eigenvalues of the second family of quarks are negative in order to generate texture-zeros in some diagonal terms of the mass matrices, as indicated in [112]. This sign is taken into account by introducing the identity matrix written as $I_2 I_2 = 1$ with $I_2 = \text{diag}(1, -1, 1)$, i.e.,

$$\begin{aligned}
 M_{ij}^F & = \left(U^{F\dagger} \lambda^F U^F \right)_{ij} = \left(U_L^{F\dagger} m^F U_R^F \right)_{ij} \\
 & = \frac{v}{\sqrt{2}} \mathcal{Y}_{ij}^{F1} = \frac{v}{\sqrt{2}} R_{1\alpha} \mathcal{Y}_{ij}^{F\alpha}, \tag{47}
 \end{aligned}$$

where $\mathcal{Y}_{ij}^{F\beta}$ and $R_{\alpha\beta}$ were defined in Appendix A, $\lambda^{U,D} = \text{diag}(m_{u,d}, -m_{c,s}, m_{t,b})$ and $m^{U,D} = \text{diag}(m_{u,d}, m_{c,s}, m_{t,b})$, with similar definitions in the lepton sector, i.e., $\lambda^{N,E} = \text{diag}(m_{1,e}, -m_{2,\mu}, m_{3,\tau})$, $m^{N,E} = \text{diag}(m_{1,e}, m_{2,\mu}, m_{3,\tau})$, and

$$U_L^F = U^F, \quad U_R^F = I_2 U^F, \tag{48}$$

where the U^F diagonalization matrices are defined below. It is important to stress that the texture-zeros pattern in the

matrix \mathcal{Y}_{ij}^{F1} are identical to those in the original Yukawa couplings $y_{ij}^{F\alpha}$, since the sum over α does not mix the i, j indices. In fact, according to Eqs. (14) and (17), $M^F = \frac{v_\alpha}{\sqrt{2}} y_{ij}^{F\alpha} = \frac{v}{\sqrt{2}} R_{1\alpha} y_{ij}^{F\alpha}$, therefore $R_{1\alpha} = \frac{v_\alpha}{v}$. The diagonalization matrices are:

$$\begin{aligned}
 U^{U\dagger} &= \begin{pmatrix} e^{i(\phi_{C_u} + \theta_{1u})} \sqrt{\frac{m_c m_t (A_u - m_u)}{A_u (m_c + m_u) (m_t - m_u)}} & -e^{i(\phi_{C_u} + \theta_{2u})} \sqrt{\frac{(A_u + m_c) m_t m_u}{A_u (m_c + m_t) (m_c + m_u)}} & e^{i(\phi_{C_u} + \theta_{3u})} \sqrt{\frac{m_c (m_t - A_u) m_u}{A_u (m_c + m_t) (m_t - m_u)}} \\ -e^{i(\phi_{B_u} + \theta_{1u})} \sqrt{\frac{(A_u + m_c) (m_t - A_u) m_u}{A_u (m_c + m_u) (m_t - m_u)}} & -e^{i(\phi_{B_u} + \theta_{2u})} \sqrt{\frac{m_c (m_t - A_u) (A_u - m_u)}{A_u (m_c + m_t) (m_c + m_u)}} & e^{i(\phi_{B_u} + \theta_{3u})} \sqrt{\frac{(A_u + m_c) m_t (A_u - m_u)}{A_u (m_c + m_t) (m_t - m_u)}} \\ e^{i\theta_{1u}} \sqrt{\frac{m_u (A_u - m_u)}{(m_c + m_u) (m_t - m_u)}} & e^{i\theta_{2u}} \sqrt{\frac{m_c (A_u + m_c)}{(m_c + m_t) (m_c + m_u)}} & e^{i\theta_{3u}} \sqrt{\frac{m_t (m_t - A_u)}{(m_c + m_t) (m_t - m_u)}} \end{pmatrix}, \quad (49) \\
 U^{D\dagger} &= \begin{pmatrix} e^{i\theta_{1d}} \sqrt{\frac{m_b (m_b - m_s) m_s}{(m_b - m_d) (m_d + m_s) (m_b + m_d - m_s)}} & -e^{i\theta_{2d}} \sqrt{\frac{m_b (m_b + m_d) m_d}{(m_d + m_s) (m_b + m_d - m_s) (m_b + m_s)}} & \sqrt{\frac{m_d (m_s - m_d) m_s}{(m_b - m_d) (m_b + m_d - m_s) (m_b + m_s)}} \\ e^{i\theta_{1d}} \sqrt{\frac{m_d (m_b - m_s)}{(m_b - m_d) (m_d + m_s)}} & e^{i\theta_{2d}} \sqrt{\frac{(m_b + m_d) m_s}{(m_d + m_s) (m_b + m_s)}} & \sqrt{\frac{m_b (m_s - m_d)}{(m_b - m_d) (m_b + m_s)}} \\ -e^{i\theta_{1d}} \sqrt{\frac{m_d (m_b + m_d) (m_s - m_d)}{(m_b - m_d) (m_d + m_s) (m_b + m_d - m_s)}} & -e^{i\theta_{2d}} \sqrt{\frac{(m_b - m_s) m_s (m_s - m_d)}{(m_d + m_s) (m_b + m_d - m_s) (m_b + m_s)}} & \sqrt{\frac{m_b (m_b + m_d) (m_b - m_s)}{(m_b - m_d) (m_b + m_d - m_s) (m_b + m_s)}} \end{pmatrix}, \quad (50)
 \end{aligned}$$

where $\theta_{1u}, \theta_{2u}, \theta_{3u}, \theta_{1d}$ and θ_{2d} are arbitrary phases (a third phase for the diagonalization matrix (50) can be absorbed by the remaining phases) that are useful for conforming to the $V_{CKM} = U_L^U U_L^{D\dagger}$ matrix convention. Taking as input the SM parameters at the Z pole, the best fit values are given in Table 4.

Similarly, in the lepton sector, the diagonalization matrices of the mass matrices (7) are:

$$\begin{aligned}
 U^{N\dagger} &= \begin{pmatrix} e^{i(\theta_{1\nu} + c_\nu)} \sqrt{\frac{m_2 m_3 (A_\nu - m_1)}{A_\nu (m_2 + m_1) (m_3 - m_1)}} & -e^{i(\theta_{2\nu} + c_\nu)} \sqrt{\frac{m_1 m_3 (m_2 + A_\nu)}{A_\nu (m_2 + m_1) (m_3 + m_2)}} & e^{i(\theta_{3\nu} + c_\nu)} \sqrt{\frac{m_1 m_2 (m_3 - A_\nu)}{A_\nu (m_3 - m_1) (m_3 + m_2)}} \\ e^{i\theta_{1\nu}} \sqrt{\frac{m_1 (A_\nu - m_1)}{(m_1 + m_2) (m_3 - m_1)}} & e^{i\theta_{2\nu}} \sqrt{\frac{m_2 (A_\nu + m_2)}{(m_2 + m_1) (m_3 + m_2)}} & e^{i\theta_{3\nu}} \sqrt{\frac{m_3 (m_3 - A_\nu)}{(m_3 - m_1) (m_3 + m_2)}} \\ -e^{i(\theta_{1\nu} - b_\nu)} \sqrt{\frac{m_1 (A_\nu + m_2) (m_3 - A_\nu)}{A_\nu (m_1 + m_2) (m_3 - m_1)}} & -e^{i(\theta_{2\nu} - b_\nu)} \sqrt{\frac{m_2 (A_\nu - m_1) (m_3 - A_\nu)}{A_\nu (m_2 + m_1) (m_3 + m_2)}} & e^{i(\theta_{3\nu} - b_\nu)} \sqrt{\frac{m_3 (A_\nu - m_1) (A_\nu + m_2)}{A_\nu (m_3 - m_1) (m_3 + m_2)}} \end{pmatrix}, \\
 U^{E\dagger} &= \begin{pmatrix} e^{i\theta_{1\ell}} \sqrt{\frac{m_\mu m_\tau (m_\tau - m_\mu)}{(m_e - m_\mu + m_\tau) (m_\mu + m_e) (m_\tau - m_e)}} & -e^{i\theta_{2\ell}} \sqrt{\frac{m_e m_\tau (m_e + m_\tau)}{(m_e - m_\mu + m_\tau) (m_\mu + m_e) (m_\tau + m_\mu)}} & \sqrt{\frac{m_e m_\mu (m_\mu - m_e)}{(m_e - m_\mu + m_\tau) (m_\tau - m_e) (m_\tau + m_\mu)}} \\ e^{i\theta_{1\ell}} \sqrt{\frac{m_e (m_\tau - m_\mu)}{(m_\mu + m_e) (m_\tau - m_e)}} & e^{i\theta_{2\ell}} \sqrt{\frac{m_\mu (m_e + m_\tau)}{(m_\tau - m_e) (m_\tau + m_\mu)}} & \sqrt{\frac{m_\tau (m_\mu - m_e)}{(m_\tau - m_e) (m_\tau + m_\mu)}} \\ -e^{i\theta_{1\ell}} \sqrt{\frac{m_e (m_e + m_\tau) (m_\mu - m_e)}{(m_e - m_\mu + m_\tau) (m_\mu + m_e) (m_\tau - m_e)}} & -e^{i\theta_{2\ell}} \sqrt{\frac{m_\mu (m_\tau - m_\mu) (m_\mu - m_e)}{(m_e - m_\mu + m_\tau) (m_\mu + m_e) (m_\tau + m_\mu)}} & \sqrt{\frac{m_\tau (m_\tau - m_\mu) (m_e + m_\tau)}{(m_e - m_\mu + m_\tau) (m_\tau - m_e) (m_\tau + m_\mu)}} \end{pmatrix}, \quad (51)
 \end{aligned}$$

where $\theta_{1\ell}, \theta_{2\ell}, \theta_{1\nu}, \theta_{2\nu}, \theta_{3\nu}$ are necessary phases in order to adjust to the established convention for the PMNS mixing matrix [229]¹; and c_ν and b_ν are the phases of C_ν and B_ν in the neutral mass matrix M^N in Eq. (7). The best fit values for these quantities are shown in Table 5.

Appendix D: Axion decay into photons

In the SM, $B^\mu = \cos \theta_W A^\mu - \sin \theta_W Z^\mu$ and $W^{3\mu} = \sin \theta_W A^\mu + \cos \theta_W Z^\mu$, where A^μ and Z^μ are the SM fields for the photon and Z gauge bosons, replacing these expressions in Eq. (26) we obtain

$$\begin{aligned}
 \mathcal{L} \supset & -c_1^{\text{eff}} \frac{\alpha_1}{8\pi} \frac{a}{\Lambda_{\text{PQ}}} B_{\mu\nu} \tilde{B}^{\mu\nu} - c_2^{\text{eff}} \frac{\alpha_2}{8\pi} \frac{a}{\Lambda_{\text{PQ}}} W_{\mu\nu}^3 \tilde{W}^{3\mu\nu} \\
 & = -\frac{\alpha}{8\pi} (c_1^{\text{eff}} + c_2^{\text{eff}}) \frac{a}{\Lambda_{\text{PQ}}} F_{\mu\nu} \tilde{F}^{\mu\nu}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{\alpha}{8\pi c_W^2 s_W^2} (s_W^4 c_1^{\text{eff}} + c_W^4 c_2^{\text{eff}}) \frac{a}{f_a} Z_{\mu\nu} \tilde{Z}^{\mu\nu} \\
 & -\frac{2\alpha}{8\pi c_W s_W} (c_W^2 c_2^{\text{eff}} - c_W^2 c_1^{\text{eff}}) \frac{a}{\Lambda_{\text{PQ}}} F_{\mu\nu} \tilde{Z}^{\mu\nu} \\
 & = e^2 C_{\gamma\gamma} \frac{a}{\Lambda_{\text{PQ}}} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{e^2 C_{ZZ}}{c_W^2 s_W^2} \frac{a}{\Lambda_{\text{PQ}}} Z_{\mu\nu} \tilde{Z}^{\mu\nu}
 \end{aligned}$$

$$+ \frac{2e^2 C_{\gamma Z}}{c_W s_W} \frac{a}{\Lambda_{\text{PQ}}} F_{\mu\nu} \tilde{Z}^{\mu\nu} \quad (52)$$

$$c_1^{\text{eff}} = c_1 - \frac{1}{3} \Sigma q + \frac{8}{3} \Sigma u + \frac{2}{3} \Sigma d - \Sigma l + 2 \Sigma e \quad (53)$$

$$c_2^{\text{eff}} = c_2 - 3 \Sigma q - \Sigma l \quad (54)$$

where $\Sigma f \equiv f_1 + f_2 + f_3$ is the sum of the PQ charges of the three families. There are similar definitions for the interaction of the axion with the gluons

$$-c_3^{\text{eff}} \frac{\alpha_3}{8\pi} \frac{a}{\Lambda_{\text{PQ}}} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} = g_s^2 C_{GG} \frac{a}{\Lambda_{\text{PQ}}} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad (55)$$

¹ NuFIT collaboration (<http://www.nu-fit.org/?q=node/211>) (with SK atmospheric data).

Table 4 Best-fit point of the mass matrix parameters with respect to experimental data for the masses and mixing angles of the quark sector at the Z pole

θ_{1u}	θ_{2u}	θ_{3u}	θ_{1d}	θ_{2d}	ϕ_{C_u}	ϕ_{B_u}
-2.84403	1.85606	-0.00461668	1.93013	-0.976639	-1.49697	0.301461
A_u	m_u	m_c	m_t	m_d	m_s	m_b
1690.29 MeV	1.2684 MeV	633.197 MeV	171268 MeV	3.14751 MeV	56.1169 MeV	2910.01 MeV

Table 5 Best fit values

$\theta_{1\ell}$	$\theta_{2\ell}$	$\theta_{1\nu}$	$\theta_{2\nu}$	$\theta_{3\nu}$	c_ν	b_ν
0.154895	2.01797	-0.835504	2.21169	1.81786	1.01608	2.03726
A_ν (eV)	m_e (MeV)	m_μ (MeV)	m_τ (MeV)	m_1 (eV)	m_2 (eV)	m_3 (eV)
0.0251821	0.5109989461	105.6583745	1776.86	0.00353647	0.00929552	0.0504034

where $c_3^{\text{eff}} = c_3 - 2\Sigma q + \Sigma u + \Sigma d - A_Q$, in our particular case $c_i = 0$. In axion phenomenology, it is usual to define

$$C_{\gamma\gamma} = -\frac{1}{32\pi^2}(c_1^{\text{eff}} + c_2^{\text{eff}}), \quad C_{ZZ} = -\frac{1}{32\pi^2}(s_W^4 c_1^{\text{eff}} + c_W^4 c_2^{\text{eff}}),$$

$$C_{\gamma Z} = -\frac{1}{32\pi^2}(c_W^2 c_2^{\text{eff}} - c_W^2 c_1^{\text{eff}}), \quad C_{GG} = -\frac{1}{32\pi^2}c_3^{\text{eff}}. \tag{56}$$

The decay widths of an axion decaying in two photons and a Z decaying in an axion and a photon are [191]

$$\Gamma(a \rightarrow \gamma\gamma) = \frac{4\pi\alpha^2 m_a^3}{\Lambda_{\text{PQ}}^2} |C_{\gamma\gamma}^{\text{eff}}|^2,$$

$$\Gamma(Z \rightarrow \gamma a) = \frac{8\pi\alpha(m_Z)m_Z^3}{3s_W^2 c_W^2 \Lambda_{\text{PQ}}^2} |C_{\gamma Z}^{\text{eff}}|^2 \left(1 - \frac{m_a^2}{m_Z^2}\right)^3. \tag{57}$$

Another possible decay channel of the axion in two photons is due to the mixing between the axion and the pion since the latter can decay in two photons, this decay mode generates an additional correction that only depends on the couplings of the axion to the gluons [230]

$$C_{\gamma\gamma}^{\text{eff}} = -\frac{c_3^{\text{eff}}}{32\pi^2} \left(\frac{c_1^{\text{eff}} + c_2^{\text{eff}}}{c_3^{\text{eff}}} - 2.03 \right),$$

$$C_{\gamma Z}^{\text{eff}} = -\frac{c_3^{\text{eff}}}{32\pi^2} \left(\frac{c_W^2 c_2^{\text{eff}} - c_W^2 c_1^{\text{eff}}}{c_3^{\text{eff}}} - 0.74/2 \right). \tag{58}$$

It is usual to define $\Lambda_{\text{PQ}} = |c_3^{\text{eff}}|f_a$.

$$\frac{E}{N} = \frac{c_1^{\text{eff}} + c_2^{\text{eff}}}{c_3^{\text{eff}}}. \tag{59}$$

The axion–photon interaction is given by

$$g_{a\gamma\gamma} = \frac{4e^2 C_{\gamma\gamma}^{\text{eff}}}{\Lambda_{\text{PQ}}} = -\frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - 2.03 \right) \tag{60}$$

where $\alpha = \frac{e^2}{4\pi}$. Due to the gluon-axion interaction, the axion gets a mass term, which is described at low energies as an axion–pion interaction [231]

$$m_a = 5.7(7)\mu\text{eV} \left(\frac{10^{12}\text{GeV}}{f_a} \right). \tag{61}$$

References

1. G. Aad et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. Phys. Lett. B **716**, 1–29 (2012)
2. S. Chatrchyan et al., Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC. Phys. Lett. B **716**, 30–61 (2012)
3. G.C. Branco, P.M. Ferreira, L. Lavoura, M.N. Rebelo, M. Sher, J.P. Silva, Theory and phenomenology of two-Higgs-doublet models. Phys. Rept. **516**, 1–102 (2012)
4. T.D. Lee, A Theory of Spontaneous T Violation. Phys. Rev. D **8**, 1226–1239 (1973)
5. H.E. Haber, G.L. Kane, The Search for Supersymmetry: Probing Physics Beyond the Standard Model. Phys. Rept. **117**, 75–263 (1985)
6. J.E. Kim, Light Pseudoscalars, Particle Physics and Cosmology. Phys. Rept. **150**, 1–177 (1987)
7. N. Turok, J. Zadrozny, Electroweak baryogenesis in the two doublet model. Nucl. Phys. B **358**, 471–493 (1991)
8. V. Barger, P. Langacker, M. McCaskey, M. Ramsey-Musolf, G. Shaughnessy, Complex Singlet Extension of the Standard Model. Phys. Rev. D **79**, 015018 (2009)
9. R.M. Schabinger, J.D. Wells, A Minimal spontaneously broken hidden sector and its impact on Higgs boson physics at the large hadron collider. Phys. Rev. D **72**, 093007 (2005)
10. C.-W. Chiang, M.J. Ramsey-Musolf, E. Senaha, Standard Model with a Complex Scalar Singlet: Cosmological Implications and Theoretical Considerations. Phys. Rev. D **97**(1), 015005 (2018)
11. J. McDonald, Gauge singlet scalars as cold dark matter. Phys. Rev. D **50**, 3637–3649 (1994)

12. L. Lopez Honorez, E. Nezri, J. F. Oliver, M. H. G. Tytgat, The Inert Doublet Model: An Archetype for Dark Matter. *JCAP* **02**, 028 (2007)
13. L. Lopez Honorez, C. E. Yaguna, The inert doublet model of dark matter revisited. *JHEP* **09**, 046 (2010)
14. M. Carena, H.E. Haber, I. Low, N.R. Shah, C.E.M. Wagner, Alignment limit of the NMSSM Higgs sector. *Phys. Rev. D* **93**(3), 035013 (2016)
15. S.L. Glashow, Partial Symmetries of Weak Interactions. *Nucl. Phys.* **22**, 579–588 (1961)
16. S. Weinberg, A Model of Leptons. *Phys. Rev. Lett.* **19**, 1264–1266 (1967)
17. A. Salam, Weak and Electromagnetic Interactions. *Conf. Proc. C* **680519**, 367–377 (1968)
18. R. Davis Jr., D.S. Harmer, K.C. Hoffman, Search for neutrinos from the sun. *Phys. Rev. Lett.* **20**, 1205–1209 (1968)
19. J. N. Bahcall, “Solving the mystery of the missing neutrinos,” *6* (2004)
20. B. Pontecorvo, Inverse beta processes and nonconservation of lepton charge. *Zh. Eksp. Teor. Fiz.* **34**, 247 (1957)
21. L. Wolfenstein, Neutrino Oscillations in Matter. *Phys. Rev. D* **17**, 2369–2374 (1978)
22. S.P. Mikheyev, A.Y. Smirnov, Resonance amplification of oscillations in matter and spectroscopy of solar neutrinos. *Sov. J. Nucl. Phys.* **42**, 913–917 (1985)
23. F. Kaether, W. Hampel, G. Heusser, J. Kiko, T. Kirsten, Reanalysis of the GALLEX solar neutrino flux and source experiments. *Phys. Lett. B* **685**, 47–54 (2010)
24. B.T. Cleveland, T. Daily, R. Davis Jr., J.R. Distel, K. Lande, C.K. Lee, P.S. Wildenhain, J. Ullman, Measurement of the solar electron neutrino flux with the Homestake chlorine detector. *Astrophys. J.* **496**, 505–526 (1998)
25. B. Aharmim et al., Combined analysis of all three phases of solar neutrino data from the sudbury neutrino observatory. *Phys. Rev. C* **88**, 025501 (2013)
26. G. Bellini et al., Measurement of the solar 8B neutrino rate with a liquid scintillator target and 3 MeV energy threshold in the Borexino detector. *Phys. Rev. D* **82**, 033006 (2010)
27. G. Bellini, J. Benziger, D. Bick, G. Bonfini, D. Bravo, B. Caccianiga, L. Cadonati, F. Calaprice, A. Caminata, P. Cavalcante, A. Chavarria, A. Chepurinov, D. D’Angelo, S. Davini, A. Derbin, A. Empl, A. Etenko, K. Fomenko, D. Franco, F. Gabriele, C. Galbiati, S. Gazzana, C. Ghiano, M. Giammarchi, M. Göger-Neff, A. Goretti, M. Gromov, C. Hagner, E. Hungerford, A. Ianni, A. Ianni, V. Kobychhev, D. Korabely, G. Korga, D. Kryn, M. Laubenstein, B. Lehnert, T. Lewke, E. Litvinovich, F. Lombardi, P. Lombardi, L. Ludhova, G. Lukyanchenko, I. Machulin, S. Manecki, W. Maneschg, S. Marcocci, Q. Meindl, E. Meroni, M. Meyer, L. Miramonti, M. Misiaszek, M. Montuschi, P. Mosteiro, V. Muratova, L. Oberauer, M. Obolensky, F. Ortica, K. Otis, M. Pallavicini, L. Papp, L. Perasso, A. Pocar, G. Ranucci, A. Razeto, A. Re, A. Romani, N. Rossi, R. Saldanha, C. Salvo, S. Schönert, H. Simgen, M. Skorokhvatov, O. Smirnov, A. Sotnikov, S. Sukhotin, Y. Suvorov, R. Tartaglia, G. Testera, D. Vignaud, R.B. Vogelaar, F. von Feilitzsch, H. Wang, J. Winter, M. Wojcik, A. Wright, M. Wurm, O. Zaimidoroga, S. Zavatarelli, K. Zuber, G. Zuzel, B. Collaboration, Neutrinos from the primary proton-proton fusion process in the sun. *Nature* **512**, 383–386 (2014)
28. J. Hosaka et al., Solar neutrino measurements in super-Kamiokande-I. *Phys. Rev. D* **73**, 112001 (2006)
29. J. Hosaka et al., Three flavor neutrino oscillation analysis of atmospheric neutrinos in Super-Kamiokande. *Phys. Rev. D* **74**, 032002 (2006)
30. M.G. Aartsen et al., Determining neutrino oscillation parameters from atmospheric muon neutrino disappearance with three years of IceCube DeepCore data. *Phys. Rev. D* **91**(7), 072004 (2015)
31. A. Gando et al., Constraints on θ_{13} from A Three-Flavor Oscillation Analysis of Reactor Antineutrinos at KamLAND. *Phys. Rev. D* **83**, 052002 (2011)
32. M. Apollonio et al., Limits on neutrino oscillations from the CHOOZ experiment. *Phys. Lett. B* **466**, 415–430 (1999)
33. A. Piepke, Final results from the Palo Verde neutrino oscillation experiment. *Prog. Part. Nucl. Phys.* **48**, 113–121 (2002)
34. F.P. An et al., New Measurement of Antineutrino Oscillation with the Full Detector Configuration at Daya Bay. *Phys. Rev. Lett.* **115**(11), 111802 (2015)
35. S.-B. Kim, Measurement of neutrino mixing angle Θ_{13} and mass difference Δm_{ee}^2 from reactor antineutrino disappearance in the RENO experiment. *Nucl. Phys. B* **908**, 94–115 (2016)
36. J. Kopp, P.A.N. Machado, M. Maltoni, T. Schwetz, Sterile Neutrino Oscillations: The Global Picture. *JHEP* **05**, 050 (2013)
37. P. Adamson et al., Measurement of Neutrino and Antineutrino Oscillations Using Beam and Atmospheric Data in MINOS. *Phys. Rev. Lett.* **110**(25), 251801 (2013)
38. K. Abe et al., Observation of Electron Neutrino Appearance in a Muon Neutrino Beam. *Phys. Rev. Lett.* **112**, 061802 (2014)
39. P. Adamson et al., First measurement of electron neutrino appearance in NOvA. *Phys. Rev. Lett.* **116**(15), 151806 (2016)
40. J. Schechter, J.W.F. Valle, Neutrino Masses in SU(2) x U(1) Theories. *Phys. Rev. D* **22**, 2227 (1980)
41. E. Ma, U. Sarkar, Neutrino masses and leptogenesis with heavy Higgs triplets. *Phys. Rev. Lett.* **80**, 5716–5719 (1998)
42. R. Foot, H. Lew, X.G. He, G.C. Joshi, Seesaw Neutrino Masses Induced by a Triplet of Leptons. *Z. Phys. C* **44**, 441 (1989)
43. R.N. Mohapatra, J.W.F. Valle, Neutrino Mass and Baryon Number Nonconservation in Superstring Models. *Phys. Rev. D* **34**, 1642 (1986)
44. M.C. Gonzalez-Garcia, J.W.F. Valle, Fast Decaying Neutrinos and Observable Flavor Violation in a New Class of Majoron Models. *Phys. Lett. B* **216**, 360–366 (1989)
45. G.C. Branco, D. Emmanuel-Costa, M.N. Rebelo, P. Roy, Four Zero Neutrino Yukawa Textures in the Minimal Seesaw Framework. *Phys. Rev. D* **77**, 053011 (2008)
46. S. Choubey, W. Rodejohann, P. Roy, Phenomenological consequences of four zero neutrino Yukawa textures. *Nucl. Phys. B* **808**, 272–291 (2009). [Erratum: *Nucl. Phys. B* 818, 136–136 (2009)]
47. B. Adhikary, A. Ghosal, P. Roy, Neutrino Masses, Cosmological Bound and Four Zero Yukawa Textures. *Mod. Phys. Lett. A* **26**, 2427–2435 (2011)
48. B. Adhikary, A. Ghosal, P. Roy, Baryon asymmetry from leptogenesis with four zero neutrino Yukawa textures. *JCAP* **01**, 025 (2011)
49. G.C. Branco, M.N. Rebelo, J.I. Silva-Marcos, Leptogenesis, Yukawa textures and weak basis invariants. *Phys. Lett. B* **633**, 345–354 (2006)
50. J.-W. Mei, Running neutrino masses, leptonic mixing angles and CP-violating phases: From M(Z) to Lambda(GUT). *Phys. Rev. D* **71**, 073012 (2005)
51. C. Hagedorn, J. Kersten, M. Lindner, Stability of texture zeros under radiative corrections in see-saw models. *Phys. Lett. B* **597**, 63–72 (2004)
52. S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt, Running neutrino mass parameters in see-saw scenarios. *JHEP* **03**, 024 (2005)
53. P. O. Ludl, W. Grimus, A complete survey of texture zeros in the lepton mass matrices. *JHEP* **07**, 090 (2014). [Erratum: *JHEP* 10, 126 (2014)]
54. P.M. Ferreira, L. Lavoura, New textures for the lepton mass matrices. *Nucl. Phys. B* **891**, 378–400 (2015)
55. P.O. Ludl, W. Grimus, A complete survey of texture zeros in general and symmetric quark mass matrices. *Phys. Lett. B* **744**, 38–42 (2015)

56. W. Grimus, A.S. Joshipura, L. Lavoura, M. Tanimoto, Symmetry realization of texture zeros. *Eur. Phys. J. C* **36**, 227–232 (2004)
57. R.M. Fonseca, W. Grimus, Classification of lepton mixing matrices from finite residual symmetries. *JHEP* **09**, 033 (2014)
58. Y. Giraldo, R. Martinez, E. Rojas, J.C. Salazar, Flavored axions and the flavor problem. *Eur. Phys. J. C* **82**(12), 1131 (2022)
59. F. Björkeröth, L. Di Luzio, F. Mescia, E. Nardi, $U(1)$ flavour symmetries as Peccei-Quinn symmetries. *JHEP* **02**, 133 (2019)
60. T.P. Cheng, M. Sher, Mass Matrix Ansatz and Flavor Nonconservation in Models with Multiple Higgs Doublets. *Phys. Rev. D* **35**, 3484 (1987)
61. K. Matsuda, H. Nishiura, Can four-zero-texture mass matrix model reproduce the quark and lepton mixing angles and CP violating phases? *Phys. Rev. D* **74**, 033014 (2006)
62. A. E. Carcamo Hernandez, R. Martinez, J. A. Rodriguez, Different kind of textures of Yukawa coupling matrices in the two Higgs doublet model type III. *Eur. Phys. J. C* **50**, 935–948 (2007)
63. P. Langacker, M. Plumacher, Flavor changing effects in theories with a heavy Z' boson with family nonuniversal couplings. *Phys. Rev. D* **62**, 013006 (2000)
64. K. Leroux, D. London, Flavor changing neutral currents and leptophobic Z' gauge bosons. *Phys. Lett. B* **526**, 97–103 (2002)
65. S.F. King, S. Moretti, R. Nevzorov, Theory and phenomenology of an exceptional supersymmetric standard model. *Phys. Rev. D* **73**, 035009 (2006)
66. C. Alvarado, R. Martinez, F. Ochoa, Quark mass hierarchy in 3-3-1 models. *Phys. Rev. D* **86**, 025027 (2012)
67. L.E. Ibanez, G.G. Ross, Fermion masses and mixing angles from gauge symmetries. *Phys. Lett. B* **332**, 100–110 (1994)
68. P. Binetruy, P. Ramond, Yukawa textures and anomalies. *Phys. Lett. B* **350**, 49–57 (1995)
69. Y. Nir, Gauge unification, Yukawa hierarchy and the mu problem. *Phys. Lett. B* **354**, 107–110 (1995)
70. V. Jain, R. Shrock, Models of fermion mass matrices based on a flavor dependent and generation dependent $U(1)$ gauge symmetry. *Phys. Lett. B* **352**, 83–91 (1995)
71. E. Dudas, S. Pokorski, C.A. Savoy, Yukawa matrices from a spontaneously broken Abelian symmetry. *Phys. Lett. B* **356**, 45–55 (1995)
72. F. Pisano, V. Pleitez, An $SU(3) \times U(1)$ model for electroweak interactions. *Phys. Rev. D* **46**, 410–417 (1992)
73. P.H. Frampton, Chiral dilepton model and the flavor question. *Phys. Rev. Lett.* **69**, 2889–2891 (1992)
74. R. Foot, H.N. Long, T.A. Tran, $SU(3)_L \otimes U(1)_N$ and $SU(4)_L \otimes U(1)_N$ gauge models with right-handed neutrinos. *Phys. Rev. D* **50**(1), R34–R38 (1994)
75. H.N. Long, The 331 model with right handed neutrinos. *Phys. Rev. D* **53**, 437–445 (1996)
76. H.N. Long, $SU(3)_L \times U(1)_N$ model for right-handed neutrino neutral currents. *Phys. Rev. D* **54**, 4691–4693 (1996)
77. H.N. Long, Scalar sector of the 3 3 1 model with three Higgs triplets. *Mod. Phys. Lett. A* **13**, 1865–1874 (1998)
78. R.A. Diaz, R. Martinez, J.A. Rodriguez, A New supersymmetric $SU(3)_L \times U(1)_X$ gauge model. *Phys. Lett. B* **552**, 287–292 (2003)
79. R.A. Diaz, R. Martinez, F. Ochoa, The Scalar sector of the $SU(3)(c) \times SU(3)(L) \times U(1)(X)$ model. *Phys. Rev. D* **69**, 095009 (2004)
80. R.A. Diaz, R. Martinez, F. Ochoa, $SU(3)(c) \times SU(3)(L) \times U(1)(X)$ models for beta arbitrary and families with mirror fermions. *Phys. Rev. D* **72**, 035018 (2005)
81. F. Ochoa, R. Martinez, Family dependence in $SU(3)(c) \times SU(3)(L) \times U(1)(X)$ models. *Phys. Rev. D* **72**, 035010 (2005)
82. S.M. Barr, Light Fermion Mass Hierarchy and Grand Unification. *Phys. Rev. D* **21**, 1424 (1980)
83. T.R. Taylor, G. Veneziano, Quenching the Cosmological Constant. *Phys. Lett. B* **228**, 311–316 (1989)
84. S.M. Barr, A Simple and Predictive Model for Quark and Lepton Masses. *Phys. Rev. Lett.* **64**, 353 (1990)
85. S. Weinberg, Electromagnetic and weak masses. *Phys. Rev. Lett.* **29**, 388–392 (1972)
86. R.N. Mohapatra, Gauge Model for Chiral Symmetry Breaking and Muon electron Mass Ratio. *Phys. Rev. D* **9**, 3461 (1974)
87. S.M. Barr, A. Zee, A New Approach to the electron-Muon Mass Ratio. *Phys. Rev. D* **15**, 2652 (1977)
88. B.S. Balakrishna, Fermion Mass Hierarchy From Radiative Corrections. *Phys. Rev. Lett.* **60**, 1602 (1988)
89. S.M. Barr, Flavor without flavor symmetry. *Phys. Rev. D* **65**, 096012 (2002)
90. L. Ferretti, S.F. King, A. Romanino, Flavour from accidental symmetries. *JHEP* **11**, 078 (2006)
91. S.M. Barr, Doubly Lopsided Mass Matrices from Unitary Unification. *Phys. Rev. D* **78**, 075001 (2008)
92. P.H. Frampton, T.W. Kephart, Fermion Mixings in $SU(9)$ Family Unification. *Phys. Lett. B* **681**, 343–346 (2009)
93. K.S. Babu, S.M. Barr, Large neutrino mixing angles in unified theories. *Phys. Lett. B* **381**, 202–208 (1996)
94. J. Sato, T. Yanagida, Large lepton mixing in a coset space family unification on $E(7)/SU(5) \times U(1)^{*3}$. *Phys. Lett. B* **430**, 127–131 (1998)
95. N. Irges, S. Lavignac, P. Ramond, Predictions from an anomalous $U(1)$ model of Yukawa hierarchies. *Phys. Rev. D* **58**, 035003 (1998)
96. S.M. Barr, I. Dorsner, A General classification of three neutrino models and $U(e3)$. *Nucl. Phys. B* **585**, 79–104 (2000)
97. N. Haba, H. Murayama, Anarchy and hierarchy. *Phys. Rev. D* **63**, 053010 (2001)
98. X.-G. He, Y.-Y. Keum, R.R. Volkas, $A(4)$ flavor symmetry breaking scheme for understanding quark and neutrino mixing angles. *JHEP* **04**, 039 (2006)
99. Y.H. Ahn, S.K. Kang, C.S. Kim, Spontaneous CP Violation in A_4 Flavor Symmetry and Leptogenesis. *Phys. Rev. D* **87**(11), 113012 (2013)
100. R. González Felipe, H. Serôdio, J. P. Silva, Models with three Higgs doublets in the triplet representations of A_4 or S_4 . *Phys. Rev. D* **87**(5), 055010 (2013)
101. H. Ishimori, E. Ma, New Simple A_4 Neutrino Model for Nonzero θ_{13} and Large δ_{CP} . *Phys. Rev. D* **86**, 045030 (2012)
102. F. Gonzalez Canales, A. Mondragon, M. Mondragon, The S_3 Flavour Symmetry: Neutrino Masses and Mixings. *Fortsch. Phys.* **61**, 546–570 (2013)
103. R.N. Mohapatra, C.C. Nishi, S_4 Flavored CP Symmetry for Neutrinos. *Phys. Rev. D* **86**, 073007 (2012)
104. G.-J. Ding, S.F. King, C. Luhn, A.J. Stuart, Spontaneous CP violation from vacuum alignment in S_4 models of leptons. *JHEP* **05**, 084 (2013)
105. E. Ma, Neutrino Mixing and Geometric CP Violation with $\Delta(27)$ Symmetry. *Phys. Lett. B* **723**, 161–163 (2013)
106. C.C. Nishi, Generalized CP symmetries in $\Delta(27)$ flavor models. *Phys. Rev. D* **88**(3), 033010 (2013)
107. G. Altarelli, F. Feruglio, Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions. *Nucl. Phys. B* **720**, 64–88 (2005)
108. H. Ishimori, Y. Shimizu, M. Tanimoto, A. Watanabe, Neutrino masses and mixing from S_4 flavor twisting. *Phys. Rev. D* **83**, 033004 (2011)
109. A. Kadosh, E. Pallante, An $A(4)$ flavor model for quarks and leptons in warped geometry. *JHEP* **08**, 115 (2010)
110. G.-J. Ding, Y.-L. Zhou, Dirac Neutrinos with S_4 Flavor Symmetry in Warped Extra Dimensions. *Nucl. Phys. B* **876**, 418–452 (2013)

111. Y. Giraldo, Texture Zeros and WB Transformations in the Quark Sector of the Standard Model. *Phys. Rev. D* **86**, 093021 (2012)
112. G. C. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe, Texture zeros and weak basis transformations. *Phys. Lett. B* **477**, 147–155 (2000)
113. Y. Giraldo, E. Rojas, CKM mixings from mass matrices with five texture zeros. *Phys. Rev. D* **104**(7), 075009 (2021)
114. P. Ramond, R.G. Roberts, G.G. Ross, Stitching the Yukawa quilt. *Nucl. Phys. B* **406**, 19–42 (1993)
115. H. Fritzsch, Z.-Z. Xing, Mass and flavor mixing schemes of quarks and leptons. *Prog. Part. Nucl. Phys.* **45**, 1–81 (2000)
116. G.C. Branco, L. Lavoura, F. Mota, Nearest Neighbor Interactions and the Physical Content of Fritzsch Mass Matrices. *Phys. Rev. D* **39**, 3443 (1989)
117. X.-G. He, W.-S. Hou, Relating the Long B Lifetime to a Very Heavy Top Quark. *Phys. Rev. D* **41**, 1517 (1990)
118. H. Fritzsch, Weak Interaction Mixing in the Six - Quark Theory. *Phys. Lett. B* **73**, 317–322 (1978)
119. H. Fritzsch, Calculating the Cabibbo Angle. *Phys. Lett. B* **70**, 436–440 (1977)
120. H. Fritzsch, Quark Masses and Flavor Mixing. *Nucl. Phys. B* **155**, 189–207 (1979)
121. D.-S. Du, Z.-Z. Xing, A Modified Fritzsch ansatz with additional first order perturbation. *Phys. Rev. D* **48**, 2349–2352 (1993)
122. H. Fritzsch, Z.-Z. Xing, Four zero texture of Hermitian quark mass matrices and current experimental tests. *Phys. Lett. B* **555**, 63–70 (2003)
123. R. L. Workman et al., Review of Particle Physics. *PTEP* **2022**, 083C01 (2022)
124. Z.-Z. Xing, H. Zhang, Complete parameter space of quark mass matrices with four texture zeros. *J. Phys. G* **30**, 129–136 (2004)
125. Y.-F. Zhou, Textures and hierarchies in quark mass matrices with four texture zeros 9 (2003)
126. R. H. Benavides, Y. Giraldo, L. Mu noz, W. A. Ponce, E. Rojas, Five Texture Zeros for Dirac Neutrino Mass Matrices. *J. Phys. G*, **47**(11), 115002 (2020)
127. Y. Giraldo, Seeking Texture Zeros in the Quark Mass Matrix Sector of the Standard Model. *Nucl. Part. Phys. Proc.* **267–269**, 76–78 (2015)
128. F. Wilczek, Axions and family symmetry breaking. *Phys. Rev. Lett.* **49**, 1549–1552 (1982)
129. L. Calibbi, F. Goertz, D. Redigolo, R. Ziegler, J. Zupan, Minimal axion model from flavor. *Phys. Rev. D* **95**(9), 095009 (2017)
130. C.Q. Geng, J.N. Ng, Flavor Connections and Neutrino Mass Hierarchy Invisible Axion Models Without Domain Wall Problem. *Phys. Rev. D* **39**, 1449 (1989)
131. Z.G. Berezhiani, M.Y. Khlopov, Cosmology of Spontaneously Broken Gauge Family Symmetry. *Z. Phys. C* **49**, 73–78 (1991)
132. M. Hindmarsh, P. Moulatsiotis, Constraints on variant axion models. *Phys. Rev. D* **56**, 8074–8081 (1997)
133. Y. Giraldo, R. Martínez, E. Rojas, J. C. Salazar, A minimal axion model for mass matrices with five texture-zeros. **4** (2023)
134. F. Björkeroth, E.J. Chun, S.F. King, Flavourful Axion Phenomenology. *JHEP* **08**, 117 (2018)
135. M. Reig, J.W.F. Valle, F. Wilczek, SO(3) family symmetry and axions. *Phys. Rev. D* **98**(9), 095008 (2018)
136. M. Linster, R. Ziegler, A Realistic $U(2)$ Model of Flavor. *JHEP* **08**, 058 (2018)
137. Y.H. Ahn, Compact model for Quarks and Leptons via flavored-Axions. *Phys. Rev. D* **98**(3), 035047 (2018)
138. F. Arias-Aragon, L. Merlo, The minimal flavour violating axion. *JHEP* **10**, 168 (2017). [Erratum: *JHEP* **11**, 152 (2019)]
139. Y. Ema, K. Hamaguchi, T. Moroi, K. Nakayama, Flaxion: a minimal extension to solve puzzles in the standard model. *JHEP* **01**, 096 (2017)
140. T. Alanne, S. Blasi, F. Goertz, Common source for scalars: Flavored axion-Higgs unification. *Phys. Rev. D* **99**(1), 015028 (2019)
141. S. Bertolini, L. Di Luzio, H. Kolečová, M. Malinský, Massive neutrinos and invisible axion minimally connected. *Phys. Rev. D* **91**(5), 055014 (2015)
142. A. Celis, J. Fuentes-Martín, H. Serôdio, A class of invisible axion models with FCNCs at tree level. *JHEP* **12**, 167 (2014)
143. Y.H. Ahn, Flavored Peccei-Quinn symmetry. *Phys. Rev. D* **91**, 056005 (2015)
144. C. Cheung, Axion Protection from Flavor. *JHEP* **06**, 074 (2010)
145. M.E. Albrecht, T. Feldmann, T. Mannel, Goldstone Bosons in Effective Theories with Spontaneously Broken Flavour Symmetry. *JHEP* **10**, 089 (2010)
146. T. Appelquist, Y. Bai, M. Piai, SU(3) Family Gauge Symmetry and the Axion. *Phys. Rev. D* **75**, 073005 (2007)
147. R.D. Peccei, H.R. Quinn, CP Conservation in the Presence of Instantons. *Phys. Rev. Lett.* **38**, 1440–1443 (1977)
148. R.D. Peccei, H.R. Quinn, Constraints Imposed by CP Conservation in the Presence of Instantons. *Phys. Rev. D* **16**, 1791–1797 (1977)
149. S. Weinberg, A New Light Boson? *Phys. Rev. Lett.* **40**, 223–226 (1978)
150. F. Wilczek, Problem of Strong P and T Invariance in the Presence of Instantons. *Phys. Rev. Lett.* **40**, 279–282 (1978)
151. J.E. Kim, Weak Interaction Singlet and Strong CP Invariance. *Phys. Rev. Lett.* **43**, 103 (1979)
152. M.S. Turner, Windows on the Axion. *Phys. Rept.* **197**, 67–97 (1990)
153. G.G. Raffelt, Astrophysical methods to constrain axions and other novel particle phenomena. *Phys. Rept.* **198**, 1–113 (1990)
154. J. Preskill, M.B. Wise, F. Wilczek, Cosmology of the Invisible Axion. *Phys. Lett. B* **120**, 127–132 (1983)
155. L.F. Abbott, P. Sikivie, A Cosmological Bound on the Invisible Axion. *Phys. Lett. B* **120**, 133–136 (1983)
156. M. Dine, W. Fischler, The Not So Harmless Axion. *Phys. Lett. B* **120**, 137–141 (1983)
157. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Can Confinement Ensure Natural CP Invariance of Strong Interactions? *Nucl. Phys. B* **166**, 493–506 (1980)
158. M. Dine, W. Fischler, M. Srednicki, A Simple Solution to the Strong CP Problem with a Harmless Axion. *Phys. Lett. B* **104**, 199–202 (1981)
159. A.R. Zhitnitsky, On Possible Suppression of the Axion Hadron Interactions. (In Russian). *Sov. J. Nucl. Phys.* **31**, 260 (1980)
160. L. Di Luzio, M. Giannotti, E. Nardi, L. Visinelli, The landscape of QCD axion models. *Phys. Rept.* **870**, 1–117 (2020)
161. N. Viaux, M. Catelan, P.B. Stetson, G. Raffelt, J. Redondo, A.A.R. Valcarce, A. Weiss, Neutrino and axion bounds from the globular cluster M5 (NGC 5904). *Phys. Rev. Lett.* **111**, 231301 (2013)
162. E. Arik et al., Probing eV-scale axions with CAST. *JCAP* **02**, 008 (2009)
163. Y. Inoue, Y. Akimoto, R. Ohta, T. Mizumoto, A. Yamamoto, M. Minowa, Search for solar axions with mass around 1 eV using coherent conversion of axions into photons. *Phys. Lett. B* **668**, 93–97 (2008)
164. M. M. Miller Bertolami, B. E. Melendez, L. G. Althaus, J. Isern, Revisiting the axion bounds from the Galactic white dwarf luminosity function. *JCAP* **10**, 069 (2014)
165. M. Giannotti, I.G. Irastorza, J. Redondo, A. Ringwald, K. Saikawa, Stellar Recipes for Axion Hunters. *JCAP* **10**, 010 (2017)
166. J. Isern, E. García-Berro, S. Torres, R. Cojocaru, S. Catalán, Axions and the luminosity function of white dwarfs: the thin and thick discs, and the halo. *Mon. Not. R. Astron. Soc.* **478**, 2569–2575 (2018)

167. O. Straniero, I. Dominguez, M. Giannotti, and A. Mirizzi, Axion-electron coupling from the RGB tip of Globular Clusters (2018). arXiv e-prints, p. [arXiv:1802.10357](https://arxiv.org/abs/1802.10357)
168. R. Verma, Exploring the predictability of symmetric texture zeros in quark mass matrices. *Phys. Rev. D* **96**(9), 093010 (2017)
169. Z.-Z. Xing, Flavor structures of charged fermions and massive neutrinos. *Phys. Rept.* **854**, 1–147 (2020)
170. B.R. Desai, A.R. Vaucher, Quark mass matrices with four and five texture zeroes, and the CKM matrix, in terms of mass eigenvalues. *Phys. Rev. D* **63**, 113001 (2001)
171. W.A. Ponce, J.D. Gómez, R.H. Benavides, Five texture zeros and CP violation for the standard model quark mass matrices. *Phys. Rev. D* **87**(5), 053016 (2013)
172. Y. Giraldo, E. Rojas, Five Non-Fritzsch Texture Zeros for Quarks Mass Matrices in the Standard Model. in 38th International Symposium on Physics in Collision, 11 (2018)
173. C. Hagedorn, W. Rodejohann, Minimal mass matrices for Dirac neutrinos. *JHEP* **07**, 034 (2005)
174. G. Ahuja, M. Gupta, M. Randhawa, R. Verma, Texture specific mass matrices with Dirac neutrinos and their implications. *Phys. Rev. D* **79**, 093006 (2009)
175. X.-W. Liu, S. Zhou, Texture Zeros for Dirac Neutrinos and Current Experimental Tests. *Int. J. Mod. Phys. A* **28**, 1350040 (2013)
176. R. Verma, Lower bound on neutrino mass and possible CP violation in neutrino oscillations. *Phys. Rev. D* **88**, 111301 (2013)
177. R. Verma, Lepton textures and neutrino oscillations. *Int. J. Mod. Phys. A* **29**(21), 1444009 (2014)
178. P. Fakay, S. Sharma, G. Ahuja, M. Gupta, Leptonic mixing angle θ_{13} and ruling out of minimal texture for Dirac neutrinos. *PTEP* **2014**(2), 023B03 (2014)
179. L.M. Cebola, D. Emmanuel-Costa, R.G. Felipe, Confronting predictive texture zeros in lepton mass matrices with current data. *Phys. Rev. D* **92**(2), 025005 (2015)
180. R.R. Gautam, M. Singh, M. Gupta, Neutrino mass matrices with one texture zero and a vanishing neutrino mass. *Phys. Rev. D* **92**(1), 013006 (2015)
181. G. Ahuja, S. Sharma, P. Fakay, M. Gupta, General lepton textures and their implications. *Mod. Phys. Lett. A* **30**(34), 1530025 (2015)
182. M. Singh, Texture One Zero Dirac Neutrino Mass Matrix With Vanishing Determinant or Trace Condition. *Nucl. Phys. B* **931**, 446–468 (2018)
183. G. Ahuja, M. Gupta, Texture zero mass matrices and nature of neutrinos. *Int. J. Mod. Phys. A* **33**(31), 1844032 (2018)
184. J. Barranco, D. Delepine, L. Lopez-Lozano, Neutrino Mass Determination from a Four-Zero Texture Mass Matrix. *Phys. Rev. D* **86**, 053012 (2012)
185. Y.A. Garnica, S.F. Mantilla, R. Martinez, H. Vargas, From Peccei Quinn symmetry to mass hierarchy problem. *J. Phys. G* **48**(9), 095002 (2021)
186. A. Ringwald, K. Saikawa, Axion dark matter in the post-inflationary Peccei-Quinn symmetry breaking scenario. *Phys. Rev. D* **93**(8), 085031 (2016). [Addendum: *Phys.Rev.D* **94**, 049908 (2016)]
187. I. Brivio, M.B. Gavela, L. Merlo, K. Mimasu, J.M. No, R. del Rey, V. Sanz, ALPs Effective Field Theory and Collider Signatures. *Eur. Phys. J. C* **77**(8), 572 (2017)
188. R. H. Benavides, D. V. Forero, L. Mu noz, J. M. Mu noz, A. Rico, A. Tapia, Five texture zeros in the lepton sector and neutrino oscillations at DUNE. *Phys. Rev. D* **107**(3), 036008 (2023)
189. H. Georgi, D.B. Kaplan, L. Randall, Manifesting the Invisible Axion at Low-energies. *Phys. Lett. B* **169**, 73–78 (1986)
190. M.B. Gavela, R. Houtz, P. Quilez, R. Del Rey, O. Sumensari, Flavor constraints on electroweak ALP couplings. *Eur. Phys. J. C* **79**(5), 369 (2019)
191. M. Bauer, M. Neubert, A. Thamm, Collider Probes of Axion-Like Particles. *JHEP* **12**, 044 (2017)
192. A. Salvio, A. Strumia, W. Xue, Thermal axion production. *JCAP* **01**, 011 (2014)
193. E. Cortina Gil et al. Measurement of the very rare $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay. *JHEP* **06**, 093 (2021)
194. R.D. Bolton et al., Search for rare Muon decays with the crystal box detector. *Phys. Rev. D* **38**, 2077 (1988)
195. A. Jodidio, B. Balke, J. Carr, G. Gidal, K.A. Shinsky, H.M. Steiner, D.P. Stoker, M. Strovink, R.D. Tripp, B. Gobbi, C.J. Oram, Erratum: Search for right-handed currents in muon decay [phys. rev. d **34**, 1967 (1986)]. *Phys. Rev. D* **37**, 237–238 (1988)
196. R. Bayes et al., Search for two body muon decay signals. *Phys. Rev. D* **91**(5), 052020 (2015)
197. H. Albrecht et al., A Search for lepton flavor violating decays $\tau \rightarrow e \alpha$, $\tau \rightarrow \mu \alpha$. *Z. Phys. C* **68**, 25–28 (1995)
198. H. Georgi, D.V. Nanopoulos, Suppression of Flavor Changing Effects From Neutral Spinless Meson Exchange in Gauge Theories. *Phys. Lett. B* **82**, 95–96 (1979)
199. C. O’Hare, cajohare/axionlimits: Axionlimits. <https://cajohare.github.io/AxionLimits/> (2020)
200. S. Chaudhuri, P.W. Graham, K. Irwin, J. Mardon, S. Rajendran, Y. Zhao, Radio for hidden-photon dark matter detection. *Phys. Rev. D* **92**(7), 075012 (2015)
201. D. Alesini, D. Babusci, D. Di Gioacchino, C. Gatti, G. Lamanna, C. Ligi, The KLASH proposal. **7** (2017)
202. C. Gatti et al. The Klash Proposal: Status and Perspectives. in 14th Patras Workshop on Axions, WIMPs and WISPs, 11 (2018)
203. I. Stern, ADMX Status. *PoS ICHEP2016*, 198 (2016)
204. M. Lawson, A.J. Millar, M. Pancaldi, E. Vitagliano, F. Wilczek, Tunable axion plasma haloscopes. *Phys. Rev. Lett.* **123**(14), 141802 (2019)
205. S. Beurthey et al., “MADMAX Status Report,” 3 (2020)
206. E. Armengaud et al., Conceptual Design of the International Axion Observatory (IAXO). *JINST* **9**, T05002 (2014)
207. I. Shilon, A. Dudarev, H. Silva, U. Wagner, H.H.J. ten Kate, New Superconducting Toroidal Magnet System for IAXO, the International AXion Observatory. *AIP Conf. Proc.* **1573**(1), 1559–1566 (2015)
208. J.L. Ouellet et al., First Results from ABRACADABRA-10 cm: A Search for Sub- μeV Axion Dark Matter. *Phys. Rev. Lett.* **122**(12), 121802 (2019)
209. C. Bartram et al. Search for Invisible Axion Dark Matter in the 3.3–4.2 μeV Mass Range. *Phys. Rev. Lett.* **127**(26), 261803 (2021)
210. T. Braine et al., Extended Search for the Invisible Axion with the Axion Dark Matter Experiment. *Phys. Rev. Lett.* **124**(10), 101303 (2020)
211. C. Bartram et al., Axion dark matter experiment: Run 1B analysis details. *Phys. Rev. D* **103**(3), 032002 (2021)
212. V. Anastassopoulos et al., New CAST Limit on the Axion-Photon Interaction. *Nature Phys.* **13**, 584–590 (2017)
213. S. Andriamonje et al., An Improved limit on the axion-photon coupling from the CAST experiment. *JCAP* **04**, 010 (2007)
214. S. Lee, S. Ahn, J. Choi, B. R. Ko, Y. K. Semertzidis, Axion Dark Matter Search around 6.7 μeV . *Phys. Rev. Lett.* **124**(10), 101802 (2020)
215. J. Jeong, S. Youn, S. Bae, J. Kim, T. Seong, J.E. Kim, Y.K. Semertzidis, Search for Invisible Axion Dark Matter with a Multiple-Cell Haloscope. *Phys. Rev. Lett.* **125**(22), 221302 (2020)
216. O. Kwon et al., First Results from an Axion Haloscope at CAPP around 10.7 μeV . *Phys. Rev. Lett.* **126**(19), 191802 (2021)
217. K.M. Backes et al., A quantum-enhanced search for dark matter axions. *Nature* **590**(7845), 238–242 (2021)
218. L. Zhong et al., Results from phase 1 of the HAYSTAC microwave cavity axion experiment. *Phys. Rev. D* **97**(9), 092001 (2018)
219. P. Gondolo, G.G. Raffelt, Solar neutrino limit on axions and keV-mass bosons. *Phys. Rev. D* **79**, 107301 (2009)

220. A. Ayala, I. Domínguez, M. Giannotti, A. Mirizzi, O. Straniero, Revisiting the bound on axion-photon coupling from Globular Clusters. *Phys. Rev. Lett.* **113**(19), 191302 (2014)
221. M. Regis, M. Taoso, D. Vaz, J. Brinchmann, S.L. Zoutendijk, N.F. Bouché, M. Steinmetz, Searching for light in the darkness: Bounds on ALP dark matter with the optical MUSE-faint survey. *Phys. Lett. B* **814**, 136075 (2021)
222. D. Grin, G. Covone, J.-P. Kneib, M. Kamionkowski, A. Blain, E. Jullo, A Telescope Search for Decaying Relic Axions. *Phys. Rev. D* **75**, 105018 (2007)
223. V. Prasolov, *Problems and theorems in linear algebra* (American Mathematical Society, USA, 1944)
224. S. Hassani, *Mathematical Physics: A Modern Introduction to Its Foundations* (Springer International Publishing, USA, 2013)
225. Z.-Z. Xing, Z.-H. Zhao, On the four-zero texture of quark mass matrices and its stability. *Nucl. Phys. B* **897**, 302–325 (2015)
226. M. Gupta, G. Ahuja, Flavor mixings and textures of the fermion mass matrices. *Int. J. Mod. Phys. A* **27**, 1230033 (2012)
227. H. Fusaoka, Y. Koide, Updated estimate of running quark masses. *Phys. Rev. D* **57**, 3986–4001 (1998)
228. J. Cardozo, J. H. Mu noz, N. Quintero, E. Rojas, Analysing the charged scalar boson contribution to the charged-current B meson anomalies. *J. Phys. G* **48**(3), 035001 (2021)
229. I. Esteban, M.C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni, T. Schwetz, Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of θ_{23} , δ_{CP} , and the mass ordering. *JHEP* **01**, 106 (2019)
230. G. Alonso-Álvarez, M.B. Gavela, P. Quilez, Axion couplings to electroweak gauge bosons. *Eur. Phys. J. C* **79**(3), 223 (2019)
231. G. Grilli di Cortona, E. Hardy, J. Pardo Vega, G. Villadoro, The QCD axion, precisely. *JHEP* **01**, 034 (2016)

Alternative 3-3-1 models with exotic electric charges

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Abstract

We report the most general classification of 3-3-1 models with $\beta = \sqrt{3}$. We found several solutions where anomaly cancellation occurs among fermions of different families. These solutions are particularly interesting as they generate non-universal heavy neutral vector bosons. Non-universality in the standard model fermion charges under an additional gauge group generates charged lepton flavor violation and flavor changing neutral currents; we discuss under what conditions the new models can evade constraints coming from these processes. In addition, we also report the *Large Hadron Collider*-(LHC) constraints.

Keywords: 3-3-1 models, 3-3-1 models with exotic electric charges, 3-3-1 models with $\beta = \sqrt{3}$, non-universal models, Pleitez and Frampton non-universal 3-3-1 model, universal one-family models, non-universal heavy neutral vector bosons

1. Introduction

Models with exotic fermions based on the gauge group symmetry $SU(3) \otimes SU(3) \otimes U(1)$ (hereafter 3-3-1 models for short) have been proposed since the early 1970s [1–11]; however,

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many of these models lacked important properties of what is known nowadays as 3-3-1 models. For a model to be interesting from a modern perspective [12], it must be chiral, the triangle anomalies must be canceled out only with a number of generations multiple of 3, and most importantly, it must contain the standard model (SM).

In the 1990s, non-universal models without exotic leptons gained popularity as they were very convenient in addressing flavor problems [13, 14]. These models have also been helpful in explaining neutrino masses [15–24], dark matter [25–35], charge quantization [36], strong CP violation [37, 38], muon anomalous magnetic moment ($g-2$ muon anomaly) [39–41] and flavor anomalies [42–45].

Pleitez and Frampton proposed the non-universal 3-3-1 models [13, 14] as examples of electroweak extensions with lepton number violation, where the number of families is determined by anomaly cancellation. In the literature, there are many examples of models without exotic electric charges, these models have been appropriately classified, and their phenomenology is well known [46–48]. The original model of Pleitez and Frampton has exotic electric charges in the quark sector and corresponds to what is known in the literature as $\beta = \sqrt{3}$ [12]. As far as we know, an exhaustive classification of models with this β does not exist in the literature, and therefore a work in this line is necessary. It is important to notice that there are solutions for arbitrary β [49]; however, this solution does not account for all the possible models for a given β . As we will see, the parameter β cannot be arbitrarily large, from the matching conditions $|\beta| \lesssim \cot \theta_W \sim 1.8$. This condition constitutes a very important restriction regarding the possible realizations of the 3-3-1 symmetry at low energies as it limits the number of possible non-trivial cases to a countable set.

In section 2, we review the basics of the 3-3-1 models. In section 3, we propose sets of fermions corresponding to families of quarks and leptons with the left-handed triplets, anti-triplets, and singlets of $SU(3)_L$. In section 4, we show the anomaly-free sets (AFSs) that constitute the basis for model building. This section lists all possible 3-3-1 models with $\beta = \sqrt{3}$ modulo lepton vector arrays. Finally, in section 5, we show the collider constraints and the conditions the models must satisfy to avoid flavor changing neutral currents and charged lepton flavor violation (CLFV) restrictions.

2. 3-3-1 models

In the subsequent discussion, we work the electroweak gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, expanding the electroweak sector of the SM, $SU(2)_L \otimes U(1)_Y$, to $SU(3)_L \otimes U(1)_X$. Furthermore, we assume that, similar to the SM, the color group $SU(3)_c$ is vector-like (i.e. anomaly-free). Left-handed quarks (color triplets) and left-handed leptons (color singlets) transform under the two fundamental representations of $SU(3)_L$ (i.e. 3 and 3^*).

Two categories of models will emerge: universal single-family models, where anomalies cancel within each family similar to the SM, and family models, where anomalies are canceled through interactions among multiple families.

In the context of 3-3-1 models, the most complete electric charge operator for this electroweak sector is

$$Q = \alpha T_{L3} + \beta T_{L8} + X\mathbf{1}, \quad (1)$$

here, $T_{La} = \lambda_a/2$, where λ_a ; $a = 1, 2, \dots, 8$ represents the Gell-Mann matrices for $SU(3)_L$ normalized as $\text{Tr}(\lambda_a \lambda_b) = 2\delta_{ab}$, and $\mathbf{1} = \text{Diag}(1, 1, 1)$ is the diagonal 3×3 unit matrix. Assuming $\alpha = 1$, the $SU(2)_L$ isospin group of the SM is fully covered in $SU(3)_L$. The parameter $\beta = \frac{2b}{\sqrt{3}}$ is a free parameter that defines the model (β is proportional to b present in

Table 1. Z' chiral charges for the SM leptons and the right-handed neutrino when embedded in S_{L1} . Here, θ_W is the electroweak mixing angle.

$\ell = (\nu_L, e_L)^T \subset 3, e_R \subset 1$ (as in S_{L1})		
Fields	$g_{Z'} \epsilon_L^{Z'}$	$g_{Z'} \epsilon_R^{Z'}$
ν_e	$\frac{g_L}{\cos \theta_W} \frac{2 \cos^2 \theta_W - 3}{\sqrt{3(1 - 4 \sin^2 \theta_W)}}$	0
e	$\frac{g_L}{\cos \theta_W} \frac{2 \cos^2 \theta_W - 3}{\sqrt{3(1 - 4 \sin^2 \theta_W)}}$	$\frac{g_L}{\cos \theta_W} \frac{\sqrt{3} \sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}$

the electric charge of the exotic vector boson K_μ). The X values are determined through anomaly cancellation. The 8 gauge fields A_μ^a of $SU(3)_L$ can be expressed as [46, 47]

$$\sum_a \lambda_a A_\mu^a = \sqrt{2} \begin{pmatrix} D_{1\mu}^0 & W_\mu^+ & K_\mu^{(b+1/2)} \\ W_\mu^- & D_{2\mu}^0 & K_\mu^{(b-1/2)} \\ K_\mu^{-(b+1/2)} & K_\mu^{-(b-1/2)} & D_{3\mu}^0 \end{pmatrix}, \quad (2)$$

here, $D_{1\mu}^0 = A_\mu^3/\sqrt{2} + A_\mu^8/\sqrt{6}$, $D_{2\mu}^0 = -A_\mu^3/\sqrt{2} + A_\mu^8/\sqrt{6}$, and $D_{3\mu}^0 = -2A_\mu^8/\sqrt{6}$. The superscripts on the gauge bosons in equation (2) indicate the electric charge of the particles, some of which are functions of the parameter b .

2.1. The minimal model

In references [14, 50], it was demonstrated that, for $b = 3/2$ (i.e. $\beta = \sqrt{3}$), the following fermion structure is free of all gauge anomalies: $\psi_{lL}^T = (l^-, \nu_l^0, l^+)_L \sim (1, 3^*, 0)$, $Q_{iL}^T = (u_i, d_i, X_i)_L \sim (3, 3, -1/3)$, and $Q_{3L}^T(d_3, u_3, Y) \sim (3, 3^*, 2/3)$, where $l = e, \mu, \tau$ represents the lepton family index, $i = 1, 2$ for the first two quark families, and the quantum numbers after the tilde (\sim) denote the 3-3-1 representation. The right-handed fields are $u_{aL}^c \sim (3^*, 1, -2/3)$, $d_{aL}^c \sim (3^*, 1, 1/3)$, $X_{iL}^c \sim (3^*, 1, 4/3)$, and $Y_L^c \sim (3^*, 1, -5/3)$, where $a = 1, 2, 3$ is the quark family index, and there are three exotic quarks with electric charges: $-4/3$ and $5/3$. This version is referred to as *minimal* in the literature because it avoids the use of exotic leptons, including possible right-handed neutrinos.

3. Lepton and quark generations

In what follows, we will propose sets of leptons S_{Li} and quarks S_{Qi} containing triplets (anti-triplets) and singlets of $SU(3)$. These sets must contain at least one SM generation of SM fermions. From equation (1), for $\beta = \sqrt{3}$, the electric charges of the 3 and 3^* triplets are: $Q_{\text{QED}}(3) = \text{Diag}(1 + X, X, -1 + X)$ and $Q_{\text{QED}}(3^*) = \text{Diag}(-1 + X, X, 1 + X)$, respectively. The general expressions for the Z' charges, with the $Z - Z'$ mixing angle equals to zero, are shown in appendix A. For the SM fields embedded in the sets: $S_{L1}, S_{L2}, S_{L3}, S_{Q1}$ and S_{Q2} , the Z' charges are shown in tables 1–5, respectively.

- Lepton generation $S_{L1} = [(\nu_e^0, e^-, E_2^{--}) \oplus e^+ \oplus E_2^{++}]_L$ with quantum numbers (1, 3, -1); (1, 1, 1) and (1, 1, 2) respectively. The Z' charges for the SM fields are shown in table 1:
- Set $S_{L2} = [(e^-, \nu_e^0, E_1^+) \oplus e^+ \oplus E_1^-]_L$ with quantum numbers (1, 3^* , 0); (1, 1, 1) and (1, 1, -1), respectively. The Z' charges for the SM fields are shown in table 2:

Table 2. Z' chiral charges for the SM leptons and the right-handed neutrino when embedded in S_{L2} . Here, θ_W is the electroweak mixing angle.

$$\ell = (\nu_L, e_L)^T \subset 3^*, e_R \subset 1 \text{ (as in } S_{L2}\text{)}$$

Fields	$g_{Z'} \epsilon_L^{Z'}$	$g_{Z'} \epsilon_R^{Z'}$
ν_e	$\frac{g_L}{\cos \theta_W} \frac{\sqrt{1-4\sin^2 \theta_W}}{2\sqrt{3}}$	0
e	$\frac{g_L}{\cos \theta_W} \frac{\sqrt{1-4\sin^2 \theta_W}}{2\sqrt{3}}$	$\frac{g_L}{\cos \theta_W} \frac{\sqrt{3} \sin^2 \theta_W}{\sqrt{1-4\sin^2 \theta_W}}$

Table 3. Z' chiral charges for the SM leptons and the right-handed neutrino when embedded in S_{L3} . Here, θ_W is the electroweak mixing angle.

$$\ell = (\nu_L, e_L)^T, e_R \subset 3^* \text{ (as in } S_{L3}\text{)}$$

Fields	$g_{Z'} \epsilon_L^{Z'}$	$g_{Z'} \epsilon_R^{Z'}$
ν_e	$\frac{g_L}{\cos \theta_W} \frac{\sqrt{1-4\sin^2 \theta_W}}{2\sqrt{3}}$	0
e	$\frac{g_L}{\cos \theta_W} \frac{\sqrt{1-4\sin^2 \theta_W}}{2\sqrt{3}}$	$\frac{g_L}{\cos \theta_W} \frac{\sqrt{1-4\sin^2 \theta_W}}{\sqrt{3}}$

Table 4. Z' chiral charges for the SM quarks when they are embedded in S_{Q1} . Here, θ_W is the electroweak mixing angle.

$$q = (u_L, d_L)^T \subset 3^*, u_R, d_R \subset 1 \text{ (as in } S_{Q1}\text{)}$$

Fields	$g_{Z'} \epsilon_L^{Z'}$	$g_{Z'} \epsilon_R^{Z'}$
u	$\frac{g_L}{\cos \theta_W} \frac{1}{2\sqrt{3(1-4\sin^2 \theta)}}$	$-\frac{g_L}{\cos \theta_W} \frac{2\sin^2 \theta_W}{\sqrt{3(1-4\sin^2 \theta)}}$
d	$\frac{g_L}{\cos \theta_W} \frac{1}{2\sqrt{3(1-4\sin^2 \theta)}}$	$\frac{g_L}{\cos \theta_W} \frac{\sin^2 \theta_W}{\sqrt{3(1-4\sin^2 \theta)}}$

Table 5. Z' chiral charges for the SM quarks when they are embedded in S_{Q2} . Here, θ_W is the electroweak mixing angle.

$$q = (u_L, d_L)^T \subset 3, u_R, d_R \subset 1 \text{ (as in } S_{Q2}\text{)}$$

Fields	$g_{Z'} \epsilon_L^{Z'}$	$g_{Z'} \epsilon_R^{Z'}$
u	$\frac{g_L}{\cos \theta_W} \frac{1-2\sin^2 \theta_W}{2\sqrt{3(1-4\sin^2 \theta)}}$	$-\frac{g_L}{\cos \theta_W} \frac{2\sin^2 \theta_W}{\sqrt{3(1-4\sin^2 \theta)}}$
d	$\frac{g_L}{\cos \theta_W} \frac{1-2\sin^2 \theta_W}{2\sqrt{3(1-4\sin^2 \theta)}}$	$\frac{g_L}{\cos \theta_W} \frac{\sin^2 \theta_W}{\sqrt{3(1-4\sin^2 \theta)}}$

- Set $S_{L3} = [(e^-, \nu_e^0, e^+)]_L$ with quantum numbers $(1, 3^*, 0)$. The Z' charges for the SM fields are shown in table 3:
- Set $S_{Q1} = [(d, u, Q_1^{5/3}) \oplus u^c \oplus d^c \oplus Q_1^c]_L$ with quantum numbers $(3, 3^*, 2/3); (3^*, 1, -2/3); (3^*, 1, 1/3)$ and $(3^*, 1, -5/3)$, respectively. The Z' for the SM fields are shown in table 4:
- Set $S_{Q2} = [(u, d, Q_2^{-4/3}) \oplus u^c \oplus d^c \oplus Q_2^c]_L$ with quantum numbers $(3, 3, -1/3); (3^*, 1, -2/3); (3^*, 1, 1/3)$ and $(3^*, 1, 4/3)$, respectively. The Z' charges for the SM fields are shown in table 5:

Table 6. Contribution to the anomalies for each family of quarks S_{Q_i} , leptons S_{L_i} and exotics S_{E_i} , for 3-3-1 models with $\beta = \sqrt{3}$.

Anomalías	S_{L1}	S_{L2}	S_{L3}	S_{Q1}	S_{Q2}	S_{E1}	S_{E2}
$[SU(3)_C]^2 U(1)_X$	0	0	0	0	0	0	0
$[SU(3)_L]^2 U(1)_X$	-1	0	0	2	-1	1	0
$[\text{Grav}]^2 U(1)_X$	0	0	0	0	0	0	0
$[U(1)_X]^3$	6	0	0	-12	6	-6	0
$[SU(3)_L]^3$	1	-1	-1	-3	3	-1	1

Table 7. AFSs for $\beta = \sqrt{3}$. We have classified the AFS according to the content of quark families, i.e. Q_i^I , Q_i^{II} , and Q_i^{III} . Combinations of these sets with three SM quark and three SM lepton families can be considered as 3-3-1 models.

i	Vector-like lepton set (L_i)	One quark set (Q_i^I)	Two quarks set (Q_i^{II})	Three quarks set (Q_i^{III})
1	$S_{E2} + S_{L2}$	$S_{E2} + 2S_{L1} + S_{Q1}$	$S_{L1} + S_{L2} + S_{Q1} + S_{Q2}$	$3S_{L1} + 2S_{Q1} + S_{Q2}$
2	$S_{E1} + S_{L1}$	$S_{E1} + 2S_{L2} + S_{Q2}$	$S_{L1} + S_{L3} + S_{Q1} + S_{Q2}$	$3S_{L2} + S_{Q1} + 2S_{Q2}$
3	$S_{E2} + S_{L3}$	$S_{E1} + S_{L2} + S_{L3} + S_{Q2}$		$3S_{L3} + S_{Q1} + 2S_{Q2}$
4		$S_{E1} + 2S_{L3} + S_{Q2}$		$2S_{L2} + S_{L3} + S_{Q1} + 2S_{Q2}$
5				$S_{L2} + 2S_{L3} + S_{Q1} + 2S_{Q2}$

- To cancel anomalies, it is advantageous introducing triplets and anti-triplets of exotic leptons; for example, $S_{E1} = [(N_1^0, E_4^+, E_3^{++}) \oplus E_4^- \oplus E_3^{--}]_L$ with quantum numbers (1, 3^* , 1); (1, 1, -1) and (1, 1, -2), respectively. We do not report the Z' charges of exotic fermion fields because we assume they have a very high mass.
- Additional exotic lepton sets. $S_{E2} = [(E_5^+, N_2^0, E_6^-) \oplus E_5^- \oplus E_6^+]_L$ with quantum numbers (1, 3, 0); (1, 1, -1) and (1, 1, 1), respectively. A more economical set is $S_{E3} = [(E_5^+, N_2^0, E_5^-)]$ which has identical contributions to the anomalies as S_{E2} but different Z' charges. However, these details are irrelevant for the low energy phenomenology, so we do not include S_{E3} in table 6.

4. Irreducible anomaly free sets and models

Table 6 shows the contribution of each set to the anomalies. From table 6, it is possible to obtain the irreducible AFSs [48], shown in table 7. The irreducible AFSs Q_i^I , Q_i^{II} and Q_i^{III} in table 7 correspond to fermion sets with one quark family, two quark families, or three quark families, respectively. These sets can be combined to build three family models as shown in table 8. There are 33 different models (without considering all the possible embeddings). These models can also be extended by adding vector-like lepton sets, L_i , indicated in the second column of table 7. To exemplify the possible embeddings we show some cases in table 10. The choice of models in table 10 show how the phenomenology depends on the SM fermion embedding in the model. For example, in the case of M10, the embedding determines whether it is strongly coupled. M17 was chosen because it had several embeddings. M3 is the minimal model. M4 is similar to the minimal model but is not universal in the lepton sector.

In general, we obtain three classes of models as we can see below:

Table 8. Three-family models built from the irreducible anomaly-free sets (table 7). It is possible to obtain (trivially) new models by adding vector-like lepton sets; we are not considering these possibilities in our counting unless they are necessary to complete the lepton families.

Models	
M1	Q_1^{III} $3S_{L1} + 2S_{Q1} + S_{Q2}$
M2	Q_2^{III} $3S_{L2} + S_{Q1} + 2S_{Q2}$
M3	Q_3^{III} $3S_{L3} + S_{Q1} + 2S_{Q2}$
M4	Q_4^{III} $2S_{L2} + S_{L3} + S_{Q1} + 2S_{Q2}$
M5	Q_5^{III} $S_{L2} + 2S_{L3} + S_{Q1} + 2S_{Q2}$
M6	$Q_1^{\text{II}} + Q_1^{\text{I}}$ $3S_{L1} + S_{L2} + S_{E2} + 2S_{Q1} + S_{Q2}$
M7	$Q_1^{\text{II}} + Q_2^{\text{I}}$ $S_{L1} + 3S_{L2} + S_{E1} + S_{Q1} + 2S_{Q2}$
M8	$Q_1^{\text{II}} + Q_3^{\text{I}}$ $S_{L1} + 2S_{L2} + S_{L3} + S_{E1} + S_{Q1} + 2S_{Q2}$
M9	$Q_1^{\text{II}} + Q_4^{\text{I}}$ $S_{L1} + S_{L2} + 2S_{L3} + S_{E1} + S_{Q1} + 2S_{Q2}$
M10	$Q_2^{\text{II}} + Q_1^{\text{I}}$ $3S_{L1} + S_{L3} + S_{E2} + 2S_{Q1} + S_{Q2}$
M11	$Q_2^{\text{II}} + Q_2^{\text{I}}$ $S_{L1} + 2S_{L2} + S_{L3} + S_{E1} + S_{Q1} + 2S_{Q2}$
M12	$Q_2^{\text{II}} + Q_3^{\text{I}}$ $S_{L1} + S_{L2} + 2S_{L3} + S_{E1} + S_{Q1} + 2S_{Q2}$
M13	$Q_2^{\text{II}} + Q_4^{\text{I}}$ $S_{L1} + 3S_{L3} + S_{E1} + S_{Q1} + 2S_{Q2}$
M14	$Q_1^{\text{I}} + Q_2^{\text{I}} + Q_3^{\text{I}}$ $2S_{L1} + 3S_{L2} + S_{L3} + 2S_{E1} + S_{E2} + S_{Q1} + 2S_{Q2}$
M15	$Q_1^{\text{I}} + Q_2^{\text{I}} + Q_4^{\text{I}}$ $2S_{L1} + 2S_{L2} + 2S_{L3} + 2S_{E1} + S_{E2} + S_{Q1} + 2S_{Q2}$
M16	$Q_1^{\text{I}} + Q_3^{\text{I}} + Q_4^{\text{I}}$ $2S_{L1} + S_{L2} + 3S_{L3} + 2S_{E1} + S_{E2} + S_{Q1} + 2S_{Q2}$
M17	$Q_2^{\text{I}} + Q_3^{\text{I}} + Q_4^{\text{I}}$ $3S_{L2} + 3S_{L3} + 3S_{E1} + 3S_{Q2}$
M18	$3Q_1^{\text{I}}$ $6S_{L1} + 3S_{E2} + 3S_{Q1}$
M19	$2Q_1^{\text{I}} + Q_2^{\text{I}}$ $4S_{L1} + 2S_{L2} + 2S_{E2} + S_{E1} + 2S_{Q1} + S_{Q2}$
M20	$2Q_1^{\text{I}} + Q_3^{\text{I}}$ $4S_{L1} + S_{L2} + S_{L3} + S_{E1} + 2S_{E2} + 2S_{Q1} + S_{Q2}$
M21	$2Q_1^{\text{I}} + Q_4^{\text{I}}$ $4S_{L1} + 2S_{L3} + S_{E1} + 2S_{E2} + 2S_{Q1} + S_{Q2}$
M22	$3Q_2^{\text{I}}$ $6S_{L2} + 3S_{E1} + 3S_{Q2}$
M23	$2Q_2^{\text{I}} + Q_1^{\text{I}}$ $2S_{L1} + 4S_{L2} + 2S_{E1} + S_{E2} + S_{Q1} + 2S_{Q2}$
M24	$2Q_2^{\text{I}} + Q_3^{\text{I}}$ $5S_{L2} + S_{L3} + 3S_{E1} + 3S_{Q2}$
M25	$2Q_2^{\text{I}} + Q_4^{\text{I}}$ $4S_{L2} + 2S_{L3} + 3S_{E1} + 3S_{Q2}$
M26	$3Q_3^{\text{I}}$ $3S_{L2} + 3S_{L3} + 3S_{E1} + 3S_{Q2}$
M27	$2Q_3^{\text{I}} + Q_1^{\text{I}}$ $2S_{L1} + 2S_{L2} + 2S_{L3} + 2S_{E1} + S_{E2} + S_{Q1} + 2S_{Q2}$
M28	$2Q_3^{\text{I}} + Q_2^{\text{I}}$ $4S_{L2} + 2S_{L3} + 3S_{E1} + 3S_{Q2}$
M29	$2Q_3^{\text{I}} + Q_4^{\text{I}}$ $2S_{L2} + 4S_{L3} + 3S_{E1} + 3S_{Q2}$
M30	$3Q_4^{\text{I}}$ $6S_{L3} + 3S_{E1} + 3S_{Q2}$
M31	$2Q_4^{\text{I}} + Q_1^{\text{I}}$ $2S_{L1} + 4S_{L3} + 2S_{E1} + S_{E2} + S_{Q1} + 2S_{Q2}$
M32	$2Q_4^{\text{I}} + Q_2^{\text{I}}$ $2S_{L2} + 4S_{L3} + 3S_{E1} + 3S_{Q2}$
M33	$2Q_4^{\text{I}} + Q_3^{\text{I}}$ $S_{L2} + 5S_{L3} + 3S_{E1} + 3S_{Q2}$

- Completely non-universal models: this happens if we embed each of the SM families in different sets; for example, one of the possible embeddings for the M12 model in table 8 is to put the first lepton family in S_{L3} and the remaining lepton families in S_{L1} and S_{L2} . This class of models usually has very strong restrictions from FCNC and CLFV.
- Universal Models: in several AFSs, there are embeddings with the three families of SM leptons in sets with the same quantum numbers; the same applies for the three families of the SM quarks. For example, in the M26 model in table 8, it is possible to embed all the three SM families in the sets $3S_{L3} + 3S_{Q2}$. The remaining fields are considered exotic fermions and are necessary to cancel anomalies.

Table 9. The lepton families S_{L_1} and S_{L_2} are strongly coupled (For S_{L_1} and S_{L_2} the left-handed lepton doublet ℓ and the right-handed charged lepton singlet e_R have couplings greater than 1, respectively). Therefore only S_{L_3} is phenomenologically viable for the first family. Depending on the quark content, i.e. S_{Q_1} or S_{Q_2} , we have two different constraints.

Particle content first generation	LHC-lower limit in TeV
$S_{L_3} + S_{Q_1}$	7.3
$S_{L_3} + S_{Q_2}$	6.4

- The 2 + 1 models: most AFSs have embeddings where two families are in sets with the same quantum numbers, and the third family is a different set. To avoid the strongest FCNC restrictions, it is necessary that the left-handed doublets of the first two SM quark families have identical quantum numbers. This condition is also desirable for Lepton families, although some models could avoid the FCNC constraints without satisfying this condition. A typical example of these models is the Pisano–Pleitez–Frampton minimal model [13, 14], $3S_{L_3} + S_{Q_1} + 2S_{Q_2}$ (the M3 model in table 8). This model is universal in the lepton sector and non-universal in the quark sector.

5. LHC and low energy constraints

We consider the ATLAS search for high-mass dilepton resonances in the mass range of 250 GeV to 6 TeV in proton–proton collisions at a center-of-mass energy of $\sqrt{s} = 13$ TeV during Run 2 of the LHC with an integrated luminosity of 139 fb^{-1} [51]. This data was collected from searches of Z' bosons decaying dileptons. We obtain the lower limit on the Z' mass from the intersection of the theoretical predictions for the cross-section with the corresponding upper limit reported by ATLAS at a 95% confidence level. We use the expressions given in [52–54] to calculate the theoretical cross-section. We assume that the $Z - Z'$ mixing angle θ (see appendix D) equals zero for these bounds. In table 9, the LHC constraints for some models are presented. It is important to stress that the leptons of the first family, i.e. the electron and its neutrino, should be embedded in S_{L_3} since it is the only scenario where the right-handed electron has Z' couplings less than 1. In table 9, this is the best option for models with the first two lepton generations embedded in S_{L_3} , as it happens for the minimal model (M3), since having identical quantum numbers for the first and second lepton families avoids possible issues with CLFV and FCNC. To avoid the strongest FCNC constraints in the quark sector, the charges of the left-handed quarks of the first two families should be identical [55]; this feature is assumed to calculate the lower mass limits in table 9. It is important to stress the non-universal Z' couplings modify processes such as [56]: coherent $\mu - e$ conversion in a muon atom, $K^0 - \bar{K}^0$ and $B - \bar{B}$ mixing, ϵ , and ϵ'/ϵ , lepton, and semileptonic decays (e.g. $\mu \rightarrow e\gamma$) which, if observed in the future, the Non-universal Models will be favored over the universal ones. For models with a Z' boson coupling in a different way to the third family, there are different predictions for the branching ratios $B(t \rightarrow Hu)$ and $B(t \rightarrow Hc)$. These predictions are strongly constrained by colliders [57]. In table 10, SC stands for strongly coupled, indicating that in the sets S_{L_1} and S_{L_2} , the coupling of the right-handed electron is greater than one, and therefore, the collider constraints are very strong. Even though Z' with couplings greater than one to the SM fields of the first generation are quite

Table 10. Alternative embeddings of the SM fields for some of the models in table 8. The lepton sets in square brackets contain the SM fields. The superscripts correspond to the particle content of the SM, where ℓ ($\bar{\ell}$) stands for a left-handed lepton doublet embedded in a $SU(3)_L$ triplet (anti-triplet), and $e^{'+}$ (e^+) is the right-handed charged lepton embedded in a $SU(3)_L$ triplet (singlet). The check mark \checkmark means that at least two (2+1) or three (universal) families have the same charges under the gauge symmetry. The cross \times stands for the opposite. LHC constraints are obtained from table 9 for embeddings in which we can choose the same Z' charges for the first two families, otherwise, we leave the space blank. To avoid a strongly coupled model in the Lepton sector, it is necessary to embed the first Lepton family (electron and electron neutrino) in S_{L3} . This feature will be helpful to distinguish between the different embeddings. The embedding also defines the content of exotic particles in each case.

Model	j	SM Lepton embeddings	Universal	2 + 1	Quark configuration	LHC-lower limit
$M3 = Q_3^{\text{III}}$ (Minimal)	—	$[3S_{L3}^{\ell+e'+}]$	\checkmark	\times	$2S_{Q2} + S_{Q1}$	6.4 TeV
$M4 = Q_4^{\text{III}}$	—	$[2S_{L2}^{\ell+e'+} + S_{L3}^{\ell+e'+}]$	\times	\checkmark	$2S_{Q2} + S_{Q1}$	6.4 TeV
$M6 = (Q_1^{\text{I}} + Q_1^{\text{II}})^j$	1	$[3S_{L1}^{\ell+e'+}] + S_{L2} + S_{E2}$	\checkmark	\times	$2S_{Q1} + S_{Q2}$	SC
	2	$[2S_{L1}^{\ell+e'+} + S_{L2}^{\ell+e'+}] + S_{L1} + S_{E2}$	\times	\checkmark	$2S_{Q1} + S_{Q2}$	SC
$M17 = (Q_2^{\text{I}} + Q_3^{\text{I}} + Q_4^{\text{I}})^j$	1	$[3S_{L2}^{\ell+e'+}] + 3S_{L3} + 3S_{E1}$	\checkmark	\times	$3S_{Q2}$	SC
	2	$[3S_{L3}^{\ell+e'+}] + 3S_{L2} + 3S_{E1}$	\checkmark	\times	$3S_{Q2}$	6.4 TeV
	3	$[2S_{L2}^{\ell+e'+} + S_{L3}^{\ell+e'+}] + S_{L2} + 2S_{L3} + 3S_{E1}$	\times	\checkmark	$3S_{Q2}$	6.4 TeV
	4	$[S_{L2}^{\ell+e'+} + 2S_{L3}^{\ell+e'+}] + 2S_{L2} + S_{L3} + 3S_{E1}$	\times	\checkmark	$3S_{Q2}$	6.4 TeV
$M10 = (Q_1^{\text{I}} + Q_2^{\text{II}})^j$	1	$[3S_{L1}^{\ell+e'+}] + S_{L3} + S_{E2}$	\checkmark	\times	$2S_{Q1} + S_{Q2}$	SC
	2	$[2S_{L1}^{\ell+e'+} + S_{L3}^{\ell+e'+}] + S_{L1} + S_{E2}$	\times	\checkmark	$2S_{Q1} + S_{Q2}$	7.3 TeV

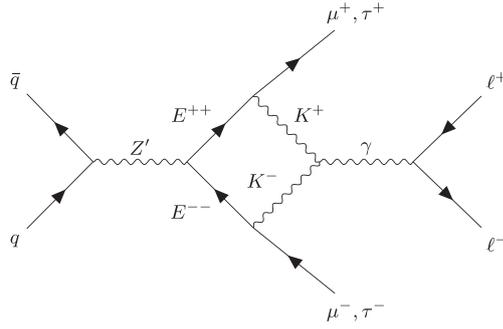


Figure 1. Doubly charged exotic lepton contribution to the process $q\bar{q} \rightarrow Z' \rightarrow E^{++}E^{--} \rightarrow \ell^+\ell^-\gamma \rightarrow \ell^+\ell^-\mu^+\mu^-(\tau^+\tau^-)$.

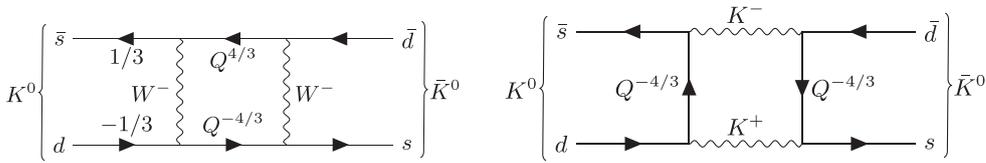


Figure 2. Exotic quark contribution to the $K^0 - \bar{K}^0$ mixing.

disfavored by colliders [52], strongly coupled models are also attractive in several phenomenological approaches [55, 58]; for this reason, it is important to realize the existence of these models, which naturally appear in 3-3-1 models with large β values. Regarding constraints on exotic particles, the restrictions on the mass of a sequential heavy lepton are above 100 GeV [59]. For exotic quarks t' and b' , the allowed mass ranges are above 1370 GeV and 1570 GeV, respectively [59]. The restrictions on fields with exotic electric charges are weaker because the identification algorithms assume the charges are proportional to the charge of the electron [60]. The presence of doubly charged exotic leptons can generate new decay channels in proton–proton collisions at very high energies. In figure 1, the Feynman diagram for the process $q\bar{q} \rightarrow Z' \rightarrow E^{++}E^{--} \rightarrow \ell^+\ell^-\gamma \rightarrow \ell^+\ell^-\mu^+\mu^-(\tau^+\tau^-)$, generating four boosted leptons in the final state (the doubly charged exotic lepton appears in S_{Li} , which strongly couples the Z' ; for this reason, to avoid collider constraints, we restrict to leptons of the second or third family). On the other hand, exotic quarks modify the $K^0 - \bar{K}^0$ mixing, as shown in figure 2. Fermions with exotic electric charges can contribute to several processes; however, an exhaustive study of these processes is beyond the purpose of this work.

6. Conclusions

Since that for 3-3-1 models, the absolute value of the parameter β must be less than $\beta \lesssim \cot \theta_W = 1.8$ (for $\sin^2 \theta_W = 0.231$ in the $\overline{\text{MS}}$ renormalization scheme at the Z-pole energy scale), and the values of β are further limited by the Requirement that the vector boson charges be integers, the possible values of this parameter are reduced to a few cases. For a realistic model, the maximum possible value corresponds to $\beta = \sqrt{3} \sim 1.73$. This case is important since it contains the Pleitez-Frampton minimal model. We have constructed three sets of lepton families, S_{Li} , two quark families, S_{Qi} , and two exotic lepton families S_{Ei} , and we

calculated their contribution to anomalies. In our analysis, we obtained 14 irreducible AFSs, from which we built 33 non-trivial 3-3-1 models (without considering the different embeddings) with at least three quark and three lepton families for each case. Each of these embeddings constitutes a phenomenologically distinguishable model; however, we limited our analysis of the possible embeddings to a few cases. In the same way, from our analysis of the 3-3-1 models with $\beta = \sqrt{3}$ we report the couplings of the SM fields to the Z' boson for all the possible quark and lepton families and the corresponding lower limits on the Z' mass. We also discuss the conditions under which the reported models avoid FCNC and CLFV. We also observed that strongly coupled models appear naturally and require a high value for the Z' mass. They can be helpful in specific phenomenological approaches based on models with strong dynamics. In the future, a detailed analysis of each model will be necessary; however, this is beyond the scope of the present work.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A. Z' charges for a general 3-3-1 model

At low energy, the 3-3-1 models, i.e. the gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ reduces to the low energy effective theory $SU(3)_C \otimes SU(2)_L \otimes U(1)_{8L} \otimes U(1)_X \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. From the covariant derivatives, for the neutral currents, we obtain the interaction Lagrangian

$$-\mathcal{L} \supset g_L J_{3L}^\mu A_{3L\mu} + g_L J_{8L}^\mu A_{8L\mu} + g_X J_X^\mu A_{X\mu}, \quad (\text{A1})$$

which can be written as

$$\begin{aligned} -\mathcal{L}_{NC} &= g_i J_{i\mu} A_i^\mu = g_j J_{j\mu} O_{jk} O_{kl}^T A_l^\mu, \\ &= \tilde{g}_k \tilde{J}_{k\mu} \tilde{A}_k^\mu, \end{aligned} \quad (\text{A2})$$

where $\tilde{A}_k^\mu = O_{kl}^T A_l^\mu$, then $(A_1^\mu, A_2^\mu) = (A_{8L}^\mu, A_X^\mu)$, $(\tilde{A}_1^\mu, \tilde{A}_2^\mu) = (B^\mu, Z'^\mu)$, $(g_1 J_1^\mu, g_2 J_2^\mu) = (g_L A_{8L}^\mu, g_X A_X^\mu)$ and $(\tilde{g}_1 \tilde{A}_1^\mu, \tilde{g}_2 \tilde{A}_2^\mu) = (g_Y J_Y^\mu, g_{Z'} J_{Z'}^\mu)$. At high energies, the symmetry is broken following the breaking chain $SU(3)_C \otimes SU(3)_L \otimes U_X(1) \rightarrow SU(3)_C \otimes SU(2)_L \otimes U_{8L}(1) \otimes U_X(1) = SU(3)_C \otimes SU(2)_L \otimes U_Y(1) \otimes U'(1)$, i.e.

$$\begin{pmatrix} A_{3L} \\ B^\mu \\ Z'^\mu \end{pmatrix} = \begin{pmatrix} 1 & 0_{1 \times 2} \\ 0_{2 \times 1} & O_{2 \times 2}^T \end{pmatrix} \begin{pmatrix} A_{3L} \\ A_{8L}^\mu \\ A_X^\mu \end{pmatrix}. \quad (\text{A3})$$

Next step $SU(3)_C \otimes SU(2)_L \otimes U_Y(1) \otimes U'(1) \rightarrow SU(3)_C \otimes U_{\text{QED}}(1)$, i.e.

$$\begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix} = \begin{pmatrix} \sin \theta_W & \cos \theta_W & 0 \\ \cos \theta_W & -\sin \theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_{3L}^\mu \\ B^\mu \\ Z'^\mu \end{pmatrix} \quad (\text{A4})$$

where the fields correspond to the SM photon A^μ and the Z^μ boson, and a heavy vector-boson Z' . Proceeding similarly for the currents, and limiting ourselves to the fields on which the orthogonal submatrix $Q_{2 \times 2}$ acts, from equation (A2) we obtain $\tilde{g}_k \tilde{J}_k^\mu = g_j J_j^\mu O_{jk}$, i.e.

$$\begin{aligned} \tilde{g}_k \tilde{J}_{k\mu} &= (g_Y J_Y^\mu, g_{Z'} J_{Z'}^\mu) = (g_L J_{L8}^\mu, g_X J_X^\mu) \begin{pmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{pmatrix}, \\ &= (g_L J_{L8}^\mu O_{11} + g_X J_X^\mu O_{21}, g_L J_{L8}^\mu O_{12} + g_X J_X^\mu O_{22}). \end{aligned} \quad (\text{A5})$$

Without further assumption

$$\begin{pmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{pmatrix} = \begin{pmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{pmatrix}, \quad (\text{A6})$$

so that

$$\begin{aligned} g_Y J_Y^\mu &= g_L J_{L8}^\mu \cos \omega + g_X J_X^\mu \sin \omega, \\ g_{Z'} J_{Z'}^\mu &= -g_L J_{L8}^\mu \sin \omega + g_X J_X^\mu \cos \omega. \end{aligned} \quad (\text{A7})$$

The charge operator in a three-dimensional representation is given by

$$Q_{\text{QED}} = T_{L3} + \beta T_{L8} + X \mathbf{1}, \quad (\text{A8})$$

hence

$$Y = \beta T_{L8} + X. \quad (\text{A9})$$

From this expression, it is possible to obtain a relation between the currents (the currents are proportional to the charges)

$$J_Y^\mu = \beta J_{L8}^\mu + J_X^\mu. \quad (\text{A10})$$

Comparing this result with (A7)

$$\beta = \frac{g_L \cos \omega}{g_Y}, \quad 1 = \frac{g_X \sin \omega}{g_Y}. \quad (\text{A11})$$

From $\cos^2 \omega + \sin^2 \omega = 1$, we obtain

$$\left(\frac{\beta}{g_L} \right)^2 + \left(\frac{1}{g_X} \right)^2 = \frac{1}{g_Y^2}. \quad (\text{A12})$$

In the SM, $g_L \approx 0.652$ and $g_Y = g_L \tan \theta_W$,

$$g_X = \frac{g_L \tan \theta_W}{\sqrt{1 - \beta^2 \tan^2 \theta_W}}. \quad (\text{A13})$$

This expression shows that the parameter β cannot be arbitrarily large from the matching conditions $\beta \lesssim \cot \theta_W$; some care must be taken on this approximation since this is a renormalization-scheme dependent inequality. From these expressions, we obtain

$$\cos \omega = \frac{\beta}{g_L} g_Y = \beta \tan \theta_W, \quad \sin \omega = \sqrt{1 - \beta^2 \tan^2 \theta_W}. \quad (\text{A14})$$

From equation (A7), $g_{Z'} \epsilon_{Z'} = -g_L T_{8L} \sin \omega + g_X X_X \cos \omega$, we obtain

$$\begin{aligned} g_{Z'} \epsilon_{Z'} &= -g_L T_{8L} \sqrt{1 - \beta^2 \tan^2 \theta_W} + \beta \frac{g_L \tan^2 \theta_W X}{\sqrt{1 - \beta^2 \tan^2 \theta_W}}, \\ &= g_L \left(-T_{8L} \tilde{\alpha} + \beta \frac{\tan^2 \theta_W X}{\tilde{\alpha}} \right), \end{aligned} \quad (\text{A15})$$

where $\tilde{\alpha} = \sqrt{1 - \beta^2 \tan^2 \theta_W} = \frac{1}{\cos \theta_W} \sqrt{1 - 4 \sin^2 \theta_W}$ for $\beta = \sqrt{3}$.

Appendix B. Chiral charges for the 3 representation

In what follows, we propose sets of fermions representing the particle content of a generation of leptons or quarks, for left-handed triplets 3, and for right-handed fermions in an $SU(3)_L$ singlet, in general we have

$$g_{Z'} \epsilon_L^{Z'}(3) = g_L \begin{pmatrix} -\frac{1}{2\sqrt{3}} \tilde{\alpha} + \beta \frac{\tan^2 \theta_W X}{\tilde{\alpha}} & 0 & 0 \\ 0 & -\frac{1}{2\sqrt{3}} \tilde{\alpha} + \beta \frac{\tan^2 \theta_W X}{\tilde{\alpha}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} \tilde{\alpha} + \beta \frac{\tan^2 \theta_W X}{\tilde{\alpha}} \end{pmatrix}, \quad \epsilon_R^{Z'} = g_L \beta \frac{\tan^2 \theta_W X_R}{\tilde{\alpha}}. \quad (\text{B1})$$

Here we add the subindex R to the X -charge of the right-handed singlet to emphasize that it differs from the quantum number of the left-handed triplet, i.e. X . If the charge conjugate of the right-handed fermion is identified with the third component of a triplet, then $\epsilon_R^{Z'} = -g_L \left(\frac{1}{\sqrt{3}} \tilde{\alpha} + \beta \frac{\tan^2 \theta_W X}{\tilde{\alpha}} \right)$.

Appendix C. The conjugate representation 3^*

To cancel the anomalies of $SU(3)_L$, triplets must be put in the conjugate representation. In general, for any set of generators T^a of an $SU(N)$ symmetry with $N \leq 3$ there exists another set of generators $-T^a$, which satisfy the same algebra. This set of generators spawns the so-called conjugate representation of $SU(N)$. With these generators, we can build charge operators and multiplets containing the SM particles. To compare with the conjugate representation, we use the projectors

$$p_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tilde{p}_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (\text{C1})$$

They should not be confused with permutation operators, as the purpose of these operators is to compare only the first two rows of the charge operators. \tilde{p}_{12} also permutes the first two eigenvalues to make a proper comparison with the conjugate operator. We can obtain the X^C , i.e. the charge of the triplet 3^* in the conjugate representation, from the equation

$$\begin{aligned} & \tilde{p}_{12}(T_{L3} + \beta T_{L8} + X\mathbf{1})\tilde{p}_{12}^T \\ & = p_{12}(-T_{L3} - \beta T_{L8} + X^C\mathbf{1})p_{12}^T, \end{aligned} \quad (\text{C2})$$

only the signs of the $SU(3)$ generators were changed. This matrix equation is equivalent to a couple of linear equations. These equations have the solution $X^C = \left(\frac{\beta}{\sqrt{3}} + X\right) = (1 + X)$. An equivalent treatment is to obtain the conjugate representation from $T_{3L} - \beta T_{8L} + X^c$, which generates the exact electric charges but in a different order. We verify that both approaches contribute identically to the anomalies, developing the same particle content and models. For left-handed triplets in the conjugate representation 3^* , and right-handed fermions in an $SU(3)_L$ singlet, we have, in general,

$$g_{Z'}\epsilon_L^{Z'}(3^*) = g_L \begin{pmatrix} +\frac{1}{2\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C & 0 & 0 \\ 0 & +\frac{1}{2\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C & 0 \\ 0 & 0 & -\frac{1}{\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C \end{pmatrix}, \quad \epsilon_R^{Z'} = g_L\beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C. \quad (\text{C3})$$

If the charge conjugate of the right-handed fermion is identified with the third component of a triplet, then $\epsilon_R^{Z'} = -g_L\left(-\frac{1}{\sqrt{3}}\tilde{\alpha} + \beta\frac{\tan^2\theta_W}{\tilde{\alpha}}X^C\right)$.

Appendix D. Z-Z' mixing

Mixing angle θ between Z and Z' is tightly constrained [61], i.e. $\theta < 10^{-3}$; however, in several phenomenological analyzes, it is still useful delivering expressions for the mass eigenstates.

$$\begin{aligned} Z_1^\mu &= Z^\mu \cos \theta + Z'^\mu \sin \theta, \\ Z_2^\mu &= -Z^\mu \sin \theta + Z'^\mu \cos \theta. \end{aligned} \quad (\text{D1})$$

At low energies, Z_1 is identified with the SM Z boson. In order to keep the Lagrangian invariant, this field rotation must be compensated by the corresponding rotation of the currents

$$\begin{aligned} g_1 J_1^\mu &= g_Z J_Z^\mu \cos \theta + g_{Z'} J_{Z'}^\mu \sin \theta, \\ g_2 J_2^\mu &= -g_Z J_Z^\mu \sin \theta + g_{Z'} J_{Z'}^\mu \cos \theta. \end{aligned} \quad (\text{D2})$$

From which we get

$$\begin{aligned} g_1 Q_1 &= g_Z Q_Z \cos \theta + g_{Z'} Q_{Z'} \sin \theta, \\ g_2 Q_2 &= -g_Z Q_Z \sin \theta + g_{Z'} Q_{Z'} \cos \theta. \end{aligned} \quad (\text{D3})$$

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References

- [1] Schechter J and Ueda Y 1973 Unified weak-electromagnetic gauge schemes based on the three-dimensional unitary group *Phys. Rev. D* **8** 484
- [2] Albright C H, Jarlskog C and Tjia M O 1975 Implications of gauge theories for heavy leptons *Nucl. Phys. B* **86** 535
- [3] Fayet P 1974 A gauge theory of weak and electromagnetic interactions with spontaneous parity breaking *Nucl. Phys. B* **78** 14
- [4] Fritzsch H and Minkowski P 1976 SU(3) as gauge group of the vector-like weak and electromagnetic interactions *Phys. Lett. B* **63** 99
- [5] Segre G, Weyers J and Vector-Like A 1976 Theory with a parity violating neutral current and natural symmetry *Phys. Lett. B* **65** 243
- [6] Mohapatra R N and Pati J C 1976 Essential restriction on the symmetry of a unified theory for the case of massive gluons *Phys. Lett. B* **63** 204
- [7] Lee B W and Weinberg S 1977 SU(3) \times U(1) gauge theory of the weak and electromagnetic interactions *Phys. Rev. Lett.* **38** 1237
- [8] Lee B W and Shrock R E 1978 An su(3) \times U(1) theory of weak and electromagnetic interactions *Phys. Rev. D* **17** 2410
- [9] Langacker P, Segre G and Golshani M 1978 Gauge theory of weak and electromagnetic interactions with an SU(3) \times U(1) symmetry *Phys. Rev. D* **17** 1402
- [10] Georgi H and Pais A 1979 Generalization of gim: horizontal and vertical flavor mixing *Phys. Rev. D* **19** 2746
- [11] Singer M, Valle J W F and Schechter J 1980 Canonical neutral current predictions from the weak electromagnetic gauge group su(3) \times u(1) *Phys. Rev. D* **22** 738
- [12] Pleitez V 2021 Challenges for the 3-3-1 models *5th Colombian Meeting on High Energy Physics* arXiv:2112.10888
- [13] Pisano F and Pleitez V 1992 An SU(3) \times U(1) model for electroweak interactions *Phys. Rev. D* **46** 410
- [14] Frampton P H 1992 Chiral dilepton model and the flavor question *Phys. Rev. Lett.* **69** 2889
- [15] Queiroz F, C A de, Pires S and da Silva P S R 2010 A minimal 3-3-1 model with naturally sub-eV neutrinos *Phys. Rev. D* **82** 065018
- [16] Caetano W, Cogollo D, Pires C A de S and Rodrigues da Silva P S 2012 Combining type I and type II seesaw mechanisms in the minimal 3-3-1 model *Phys. Rev. D* **86** 055021
- [17] Ferreira J G Jr, Pinheiro P R D, Pires C A de S and da Silva P S R 2011 The minimal 3-3-1 model with only two higgs triplets *Phys. Rev. D* **84** 095019
- [18] Mohapatra R N and Senjanovic G 1980 Neutrino mass and spontaneous parity nonconservation *Phys. Rev. Lett.* **44** 912
- [19] Dias A G, Pires C A de S and Rodrigues da Silva P S 2005 Naturally light right-handed neutrinos in a 3-3-1 model *Phys. Lett. B* **628** 85
- [20] Dias A G, Pires C A de S and Rodrigues da Silva P S 2010 The left-right SU(3)(l) \times SU(3)(r) \times U(1)(x) model with light, kev and heavy neutrinos *Phys. Rev. D* **82** 035013
- [21] Cogollo D, Diniz H and Pires C A de S 2009 Kev right-handed neutrinos from type ii seesaw mechanism in a 3-3-1 model *Phys. Lett. B* **677** 338
- [22] Ky N A and Van N T H 2005 Scalar sextet in the 331 model with right-handed neutrinos *Phys. Rev. D* **72** 115017
- [23] Palcu A 2006 Implementing canonical seesaw mechanism in the exact solution aa 3-3-1 gauge model without exotic electric charges *Mod. Phys. Lett. A* **21** 2591
- [24] Dias A G, Pires C A de S and da Silva P S R 2011 How the inverse see-saw mechanism can reveal itself natural, canonical and independent of the right-handed neutrino mass *Phys. Rev. D* **84** 053011
- [25] Fregolente D and Tonasse M D 2003 Selfinteracting dark matter from an SU(3)(l) \times U(1)(N) electroweak model *Phys. Lett. B* **555** 7
- [26] Long H N and Lan N Q 2003 Selfinteracting dark matter and higgs bosons in the SU(3)(C) \times SU(3)(L) \times U(1)(N) model with right-handed neutrinos *EPL* **64** 571
- [27] Filippi S, Ponce W A and Sanchez L A 2006 Dark matter from the scalar sector of 3-3-1 models without exotic electric charges *Europhys. Lett.* **73** 142
- [28] Mizukoshi J K, Pires C A de S, Queiroz F S and Rodrigues da Silva P S 2011 Wimps in a 3-3-1 model with heavy sterile neutrinos *Phys. Rev. D* **83** 065024

- [29] Profumo S and Queiroz F S 2014 Constraining the Z' mass in 331 models using direct dark matter detection *Eur. Phys. J. C* **74** 2960
- [30] Kelso C, Pires C A de S, Profumo S, Queiroz F S and Rodrigues da Silva P S 2014a A 331 wimpy dark radiation model *Eur. Phys. J. C* **74** 2797
- [31] Rodrigues da Silva P S and Brief A 2016 Review on wimps in 331 electroweak gauge models *Phys. Int.* **7** 15
- [32] Queiroz F S 2015 Non-thermal wimps as dark radiation *AIP Conf. Proc.* vol 1604 p 83 arXiv:1310.3026
- [33] Cogollo D, Gonzalez-Morales A X, Queiroz F S and Teles P R 2014 Excluding the light dark matter window of a 331 model using lhc and direct dark matter detection data *J. Cosmol. Astropart. Phys.* **JCAP11(2014)002**
- [34] Kelso C, Long H N, Martinez R and Queiroz F S 2014b Connection of $g - 2_\mu$, electroweak, dark matter, and collider constraints on 331 models *Phys. Rev. D* **90** 113011
- [35] Dong P V, Ngan N T K and Soa D V 2014 Simple 3-3-1 model and implication for dark matter *Phys. Rev. D* **90** 075019
- [36] de Sousa Pires C A and Ravinez O P 1998 Charge quantization in a chiral bilepton gauge model *Phys. Rev. D* **58** 035008
- [37] Pal P B 1995 The strong cp question in $SU(3)(C) \times SU(3)(L) \times U(1)(N)$ models *Phys. Rev. D* **52** 1659
- [38] Dias A G, Pires C A de S and Rodrigues da Silva P S 2003 Discrete symmetries, invisible axion and lepton number symmetry in an economic 3-3-1 model *Phys. Rev. D* **68** 115009
- [39] de Jesus A S, Kovalenko S, Queiroz F S, Pires C A de S and Villamizar Y S 2020 Dead or alive? Implications of the muon anomalous magnetic moment for 3-3-1 models *Phys. Lett. B* **809** 135689
- [40] Hue L T, Thanh P N and Tham T D 2020 Anomalous magnetic dipole moment $(g - 2)_\mu$ in 3-3-1 model with inverse seesaw neutrinos *Commun. Phys.* **30** 221
- [41] Hue L T, Phan K H, Nguyen T P, Long H N and Hung H T 2022 An explanation of experimental data of $(g - 2)_{e,\mu}$ in 3-3-1 models with inverse seesaw neutrinos *Eur. Phys. J. C* **82** 722
- [42] Buras A J, De Fazio F, Girrbach J and Carlucci M V 2013 The anatomy of quark flavour observables in 331 models in the flavour precision era *J. High Energy Phys.* **JHEP02(2013)023**
- [43] Addazi A, Ricciardi G, Scarlatella S, Srivastava R and Valle J W F 2022 Interpreting B anomalies within an extended 331 gauge theory *Phys. Rev. D* **106** 035030
- [44] Descotes-Genon S, Moscati M and Ricciardi G 2018 Nonminimal 331 model for lepton flavor universality violation in $b \rightarrow s\ell\ell$ decays *Phys. Rev. D* **98** 115030
- [45] Wei M and Chong-Xing Y 2017 Charged higgs bosons from the 3-3-1 models and the $\mathcal{R}(D^{(*)})$ anomalies *Phys. Rev. D* **95** 035040
- [46] Ponce W A, Florez J B and Sanchez L A 2002 Analysis of $SU(3)(c) \times SU(3)(L) \times U(1)(X)$ local gauge theory *Int. J. Mod. Phys. A* **17** 643
- [47] Ponce W A, Giraldo Y and Sanchez L A 2003 Minimal scalar sector of 3-3-1 models without exotic electric charges *Phys. Rev. D* **67** 075001
- [48] Benavides R H, Giraldo Y, Muñoz L, Ponce W A and Rojas E 2022 Systematic study of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ local gauge symmetry *J. Phys. G* **49** 105007
- [49] Diaz R A, Martinez R and Ochoa F 2005 $SU(3)(c) \times SU(3)(L) \times U(1)(X)$ models for beta arbitrary and families with mirror fermions *Phys. Rev. D* **72** 035018
- [50] Valle J W F and Singer M 1983 Lepton number violation with quasi Dirac neutrinos *Phys. Rev. D* **28** 540
- [51] Aad G(ATLAS) *et al* 2019 Search for high-mass dilepton resonances using 139 fb⁻¹ of pp collision data collected at $\sqrt{s} = 13$ TeV with the ATLAS detector *Phys. Lett. B* **796** 68
- [52] Erler J, Langacker P, Munir S and Rojas E 2011 Z' Bosons at colliders: a Bayesian viewpoint *J. High Energy Phys.* **JHEP11(2011)076**
- [53] Salazar C, Benavides R H, Ponce W A and Rojas E 2015 LHC constraints on 3-3-1 models *J. High Energy Phys.* **JHEP07(2015)096**
- [54] Benavides R H, Muñoz L, Ponce W A, Rodríguez O and Rojas E 2018 Electroweak couplings and LHC constraints on alternative Z' models in E_6 *Int. J. Mod. Phys. A* **33** 1850206
- [55] Langacker P 2009 The physics of heavy Z' Gauge Bosons *Rev. Mod. Phys.* **81** 1199
- [56] Langacker P and Plumacher M 2000 Flavor changing effects in theories with a heavy Z' boson with family nonuniversal couplings *Phys. Rev. D* **62** 013006

- [57] Tumasyan A(CMS) *et al* 2022 Search for flavor-changing neutral current interactions of the top quark and the Higgs boson decaying to a bottom quark-antiquark pair at $\sqrt{s} = 13$ TeV *J. High Energy Phys.* [JHEP02\(2022\)169](#)
- [58] Hill C T and Simmons E H 2003 Strong dynamics and electroweak symmetry breaking *Phys. Rep.* **381** 235
Hill C T and Simmons E H 2004 *Phys. Rep.* **390** 553–554
- [59] Workman R L *et al* (Particle Data Group) 2022 Review of particle physics *PTEP* **2022** 083C01
- [60] Romero Abad D, Portales J R and Ramirez Barreto E 2020 Electric charge quantisation in 331 models with exotic charges *Pramana* **94** 84
- [61] Erler J, Langacker P, Munir S and Rojas E 2009 Improved constraints on z-prime bosons from electroweak precision data *J. High Energy Phys.* [JHEP08\(2009\)017](#)

The standard model of particle physics as an effective theory from two non-universal $U(1)$'s

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Abstract

We study the possibility of obtaining the Standard Model (SM) of particle physics as an effective theory of a more fundamental one, whose electroweak sector includes two non-universal local $U(1)$ gauge groups, with the chiral anomaly cancellation taking place through an interplay among families. As a result of the spontaneous symmetry breaking, a massive gauge boson Z' arises, which couples differently to the third family of fermions (by assumption, we restrict ourselves to the scenario in which the Z' couples in the same way to the first two families). Two Higgs doublets and one scalar singlet are necessary to generate the SM fermion masses and break the gauge symmetries. We show that in our model, the flavor-changing neutral currents (FCNC) of the Higgs sector are identically zero if each right-handed SM fermion is only coupled with a single Higgs doublet. This result represents a FCNC cancellation mechanism different from the usual procedure in Two-Higgs Doublet Models. The non-universal nature of our solutions Requires the presence of three right-handed neutrino fields, one for each family. Our model generates all elements of the Dirac mass matrix for quarks and leptons, which is quite non-trivial for non-universal models. Thus, we can fit all the masses and mixing angles with two scalar doublets. Finally, we show the distribution of solutions for the scalar boson masses in our model by scanning well-motivated intervals for the

model parameters. We consider two possibilities for the scalar potential and compare these results with the Higgs-like resonant signals recently reported by the ATLAS and CMS experiments at the LHC. Finally, we also report collider, electroweak, and flavor constraints on the model parameters.

Keywords: non-universality, right-handed neutrinos, flavor physics

1. Introduction

The Standard Model of particle physics (SM) based on the local gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ [1] has been very successful so far, in the sense that its predictions are in good agreement with the present experimental results, including the latest discovery of the Higgs boson [2–4], a fundamental ingredient of the model that contributes to our understanding of the origin of mass for the subatomic particles. However, the SM fails short in explaining things as: hierarchical charged fermion masses, fermion mixing angles, charge quantization, strong CP violation, replication of families, neutrino masses and oscillations, and the matter-antimatter asymmetry of the Universe. Besides, gravity is excluded from the context of the model and good candidates for dark matter and dark energy present in the Universe are not provided [5–12].

Replication of families, also known as the ‘family problem’, refers to the fact that the SM is not able to predict the number N of fermion families existing in nature, something related with the universality of the model, which means that the gauge anomalies, in particular those associated with the $U(1)_Y$ hypercharge, cancel out exactly for each family; the only restriction, $N \leq 8$, comes from the asymptotic freedom of $SU(3)_C$ also known as quantum chromodynamics or QCD [13]. Experimental results at the CERN-LEP facilities early in the 1990s implied the existence of at least three families, each one having a neutral lepton with a mass less than half the mass of the neutral Z gauge boson [14]; this result was initially interpreted as an exact value for the total number of families in nature, which is not quite correct. As a matter of fact, the LEP data does not exclude the existence of additional families having heavy neutrinos.

Therefore, it is widely believed that the SM is not truly fundamental, with the prevailing view that the model is just a low-energy effective description of a more complete theory. There are several good candidates for this, all of them grouped in what is now known as ‘the physics beyond the Standard Model’ (BSM) [15–17]. Thus, there are numerous works with gauge extensions of the $U(1)$ type, either to explain neutrino masses or dark matter, etc. see references: [18–23] as to show some of them. However, our goal is to introduce two non-universal $U(1)$ symmetry gauge to SM to obtain it as an effective model. The consideration of Z' bosons with non-universal couplings is justified for theoretical and experimental reasons. From a theoretical perspective, these models arise naturally in several scenarios, for example in string models [24–27] and 3-3-1 models [28–32]. However, from a phenomenological point of view, they are convenient for studying experimental anomalies at low energies, for example: anomalous decays of B -mesons [33–37], Cabibbo angle anomaly [38], muon anomalous magnetic moment (or muon $g-2$) [39–41] and rare charm decays [42]. Recently CMS reported for the first time searches for neutral vector bosons with non-universal couplings [43] due to the multiple applications of this class of models. Thus, searching for signals associated with these models remains a relevant task in exploring physics BSM [44].

Table 1. Here, i runs over the number of families ($i = 1, 2, 3$), and $a = 1, 2$.

	$\ell_i \equiv (\nu_{iL}, e_{iL})^T$	ν_{iR}	e_{iR}	$q_i \equiv (u_{iL}, d_{iL})^T$	u_{iR}	d_{iR}	$\Phi_a = (\phi_a^+, \phi_a^0)^T$	σ
\hat{T}_3	$(\frac{1}{2}, -\frac{1}{2})^T$	0	0	$(\frac{1}{2}, -\frac{1}{2})^T$	0	0	$(\frac{1}{2}, -\frac{1}{2})^T$	0
\hat{Y}	-1	0	-2	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	1	0
\hat{Q}	$(0, -1)^T$	0	-1	$(\frac{2}{3}, -\frac{1}{3})^T$	$\frac{2}{3}$	$-\frac{1}{3}$	$(1, 0)^T$	0
$\hat{\alpha}$	α_{ji}	α_{ν_i}	α_{ei}	α_{qi}	α_{ui}	α_{di}	α_a	α_σ
$\hat{\beta}$	β_{ji}	β_{ν_i}	β_{ei}	β_{qi}	β_{ui}	β_{di}	β_a	β_σ

In what follows, and in order to shed some light on the shortcomings of the SM, we propose an extension of it; that is, a new model for three families based on the local gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_\alpha \otimes U(1)_\beta$, where the charges associated with the two Abelian factors are non-universal, in the sense that they are not the same for the three assumed families. The fermion content of our model is the same as that of the SM, extended with three right-handed neutrinos ν_{iR} ($i = 1, 2, 3$), one for each family.

2. The model

In this section, we elaborate on the mathematical aspects of the new model in consideration, which is a minimal extension of the SM, both in its gauge sector and in its fermion sector. As a consequence, the scalar sector must also be enlarged, something we are going to do in the most economical possible way.

As mentioned above, the model to be considered here is based on the local gauge symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_\alpha \otimes U(1)_\beta$, where $SU(3)_C$ and $SU(2)_L$ are the same as in the SM, and the two Abelian factors are non-universal, capable of projecting the SM $U(1)_Y$ hypercharge to a lower energy scale. So, as a result of the spontaneous symmetry breaking, a new gauge boson associated with a non-universal neutral weak current appears.

The fermion fields in our model are the same as in the SM, together with three neutral Weyl states associated with the three right-handed neutrino components, one for each family. This popular fermion extension of the SM has been used to explain neutrino masses and oscillations, the baryon asymmetry of the Universe, dark matter and dark radiation, and in our approach, it has the peculiarity that, unlike what happens in the SM, the three new fields have non-vanishing charges under both $U(1)$ factors.

As for the scalar sector, we first introduce a SM singlet field σ able to spontaneously break the $U(1)_\alpha \otimes U(1)_\beta$ symmetry down to $U(1)_Y$. To break the remaining symmetry and at the same time implement the Higgs mechanism, at least one $SU(2)_L$ scalar doublet Φ_2 (developing a vacuum expectation value (VEV) at an energy scale v_2) must be introduced, in such a way that the remaining symmetry $SU(3)_C \otimes U(1)_Q$ survives down to laboratory energies. We choose the quantum numbers of this doublet such that it only provides tree-level masses to the third fermion family. To generate (at tree level) the other fermion masses and the mixing matrices, at least one more $SU(2)_L$ scalar doublet Φ_1 must be included. This doublet develops a VEV at an energy scale $v_1 < v_2$. Table 1 shows the fermion and scalar content of our model, along with the notation used for the different Abelian charges, as well as the weak-isospin T_3 ,

hypercharge Y , and electric charge Q of the particles. In our analysis, we will assume that $\chi_{f_i} = \chi_{f_2} \neq \chi_{f_3}$, where χ_{f_i} stands for the Abelian α , β charges, $f = q, u, d, l, \nu, e$ and $i = 1, 2, 3$, that is, we consider a model with universal couplings for the first two fermion families, but not for the third one, a convenient condition in the implementation of models with minimal flavor violation, that in turn provides a way to distinguish the third family from the first two ones. In this way, our model is characterized by 24 parameters associated with the fermion sector and 6 more with the scalar one, for a total of 30 free parameters which can be fixed by demanding a renormalizable model, reproducing the SM hypercharges, and appropriate Yukawa couplings to provide fermion masses.

2.1. Cancellation of chiral anomalies

Regarding the renormalizability of the theory, we must ensure an anomaly-free scenario, which is achieved by imposing the following relations among the $U(1)$ fermion charges:

$$\begin{aligned}
[SU(3)_C]^2 \otimes U(1)_\alpha: \sum_i (2\alpha_{qi} - \alpha_{ui} - \alpha_{di}) &= 0, \\
[SU(2)_L]^2 \otimes U(1)_\alpha: \sum_i (3\alpha_{qi} + \alpha_{li}) &= 0, \\
[\text{grav}]^2 \otimes U(1)_\alpha: \sum_i (6\alpha_{qi} - 3\alpha_{ui} - 3\alpha_{di} + 2\alpha_{li} - \alpha_{\nu i} - \alpha_{ei}) &= 0, \\
[U(1)_\alpha]^2 U(1)_\beta: \sum_i (6\alpha_{qi}^2 \beta_{qi} - 3\alpha_{ui}^2 \beta_{ui} - 3\alpha_{di}^2 \beta_{di} + 2\alpha_{li}^2 \beta_{li} - \alpha_{\nu i}^2 \beta_{\nu i} - \alpha_{ei}^2 \beta_{ei}) &= 0, \\
[U(1)_\alpha]^3: \sum_i (6\alpha_{qi}^3 - 3\alpha_{ui}^3 - 3\alpha_{di}^3 + 2\alpha_{li}^3 - \alpha_{\nu i}^3 - \alpha_{ei}^3) &= 0,
\end{aligned} \tag{1}$$

together with the five corresponding equations for the $U(1)_\beta$ group. These are obtained from the previous ones via the $\alpha \leftrightarrow \beta$ exchanging for a total of 10 equations. Given that the number of involved unknowns is greater (24 assuming universality in the first two fermion families), the number of possible solutions is infinite, so, just like in the SM, chiral anomaly cancellation is not sufficient to explain the charge quantization [13].

2.2. The Lagrangian of the Model

In our model, the covariant derivative D_μ for the electroweak (EW) sector is given by

$$D^\mu = \partial^\mu + ig_L A_j^\mu \hat{T}_j + i \frac{g_\alpha}{2} B_\alpha^\mu \hat{\alpha} + i \frac{g_\beta}{2} B_\beta^\mu \hat{\beta}, \tag{2}$$

where \hat{T}_j , A_j^μ (with $j = 1, 2, 3$) and g_L denote, respectively, the generators, the gauge fields, and the coupling constant associated with the weak isospin gauge group $SU(2)_L$, while $\hat{\alpha}$, B_α^μ and g_α , with $\alpha = \alpha, \beta$, are the corresponding quantities related with the two Abelian $U(1)$ factors. The terms in the Lagrangian describing the relevant interactions in our analysis are then:

$$\begin{aligned}
\mathcal{L} \supset & -V(\Phi_1, \Phi_2, \sigma) \\
& + |D^\mu \Phi_1|^2 + |D^\mu \Phi_2|^2 + |D^\mu \sigma|^2 \\
& + i\bar{q}_j \not{D} q_j + i\bar{u}_{jR} \not{D} u_{jR} + i\bar{d}_{jR} \not{D} d_{jR} + i\bar{l}_j \not{D} l_j + i\bar{\nu}_{jR} \not{D} \nu_{jR} + i\bar{e}_{jR} \not{D} e_{jR} \\
& - Y_{jk}^e \bar{l}_j \Phi_1 e_{kR} - Y_{jk}^\nu \bar{l}_j \tilde{\Phi}_1 \nu_{kR} - Y_{jk}^d \bar{q}_j \Phi_1 d_{kR} - Y_{jk}^u \bar{q}_j \tilde{\Phi}_1 u_{kR} \\
& - Y_{j3}^e \bar{l}_j \Phi_2 e_{3R} - Y_{j3}^\nu \bar{l}_j \tilde{\Phi}_2 \nu_{3R} - Y_{j3}^d \bar{q}_j \Phi_2 d_{3R} - Y_{j3}^u \bar{q}_j \tilde{\Phi}_2 u_{3R} + \text{h.c.}, \tag{3}
\end{aligned}$$

where sum over repeated indices is implied, with j and k taking the values $\{1, 2, 3\}$ and $\{1, 2\}$, respectively. The term in the first line denotes the scalar potential. Due to the non-universal character of our model, a single scalar doublet Φ_1 is not enough to provide masses to all fermion particles and, simultaneously, to generate realistic mixing matrices. To this end, at least another Higgs doublet Φ_2 developing a VEV is required. Additionally, a scalar singlet must be introduced to break the abelian symmetries. The symmetry group $SU(2) \otimes U(1) \otimes U(1)$ has five generators, four of which are broken, such that at low energies only the electromagnetic gauge group $U(1)_{\text{QED}}$ survives. For a model with just two Higgs doublets, by applying the Higgs mechanism we obtain, in addition to the SM fields, two exotic fields: a CP even neutral scalar field and a charged one, the remaining ones are absorbed as goldstone bosons by the vector fields. To accommodate the experimental anomalies it is necessary to include a scalar singlet to break the $U(1) \otimes U(1)$ at high energies such that, in addition to the SM fields we get two CP even scalar fields and a pseudoscalar. This is also convenient if we want the Z' scale to be larger than the electroweak scale since the quadratic sum of the VEVs of the doublets must equal $v_{\text{SM}} = 246.24$ GeV. The scalar potential is analyzed in appendix A. The terms in the second line correspond to the scalar-gauge interactions responsible for the masses and mixings in the gauge sector (see appendix B). Terms in the third line give rise to fermion-gauge interactions, as discussed in section 3, and the Yukawa couplings present in the model are shown in the fourth and fifth lines. The invariance of the Yukawa interaction terms under the $U(1)_\alpha \otimes U(1)_\beta$ gauge symmetry implies the following relations between the $\chi(\alpha, \beta)$ charges:

$$\begin{aligned}
\chi_{lj} - \chi_1 - \chi_{ea} &= 0, \\
\chi_{lj} - \chi_2 - \chi_{e3} &= 0, \\
\chi_{lj} + \chi_1 - \chi_{\nu a} &= 0, \\
\chi_{lj} + \chi_2 - \chi_{\nu 3} &= 0, \\
\chi_{qj} - \chi_1 - \chi_{da} &= 0, \\
\chi_{qj} - \chi_2 - \chi_{d3} &= 0, \\
\chi_{qj} + \chi_1 - \chi_{ua} &= 0, \\
\chi_{qj} + \chi_2 - \chi_{u3} &= 0. \tag{4}
\end{aligned}$$

2.3. Spontaneous symmetry breaking

Our aim is to break the gauge symmetry of the model in two steps, namely,

$$SU(2)_L \otimes U(1)_\alpha \otimes U(1)_\beta \xrightarrow{\langle \sigma \rangle} SU(2)_L \otimes U(1)_Y \xrightarrow{\langle \Phi_a \rangle} U(1)_Q \tag{5}$$

where $a = 1, 2$. To achieve this, we allow the SM scalar singlet σ (charged under both $U(1)$'s factors) to acquire a VEV at a high energy scale, inducing a mixing between the B_χ fields that give rise to both: the SM gauge boson B associated with the $U(1)_Y$ hypercharge symmetry and a new massive gauge boson Z' with non-universal couplings to fermions. If θ is the angle

parameterizing this mixing, then

$$\begin{pmatrix} B_\alpha^\mu \\ B_\beta^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} B^\mu \\ Z'^\mu \end{pmatrix}. \quad (6)$$

Finally, at a lower energy scale (the EW one), the neutral components of the scalar doublets Φ_1 and Φ_2 develop VEVs inducing the last breaking. Consequently, the B and A_3 fields mix, giving rise to the massless photon A^μ and the massive SM neutral gauge boson Z . The corresponding mixing angle is the well-known Weinberg angle θ_W :

$$\begin{pmatrix} A_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \sin \theta_W & \cos \theta_W \\ \cos \theta_W & -\sin \theta_W \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}. \quad (7)$$

The unbroken electric charge generator \hat{Q} can be expressed as a linear combination of the three diagonal (broken) generators of the gauge group after the spontaneous symmetry breaking, that is

$$\hat{Q} = \hat{T}_{3L} + \frac{1}{2}(a_Y \hat{\alpha} + b_Y \hat{\beta}), \quad (8)$$

from which it follows that the SM hypercharge \hat{Y} can be identified as

$$\hat{Y} = a_Y \hat{\alpha} + b_Y \hat{\beta}, \quad (9)$$

where a_Y and b_Y are two non-vanishing free parameters. However, these parameters turn be useless for our purposes, so they will be set to 1 for simplicity⁵ In accordance with equation (9), the $U(1)$ charges displayed in table 1 must satisfy the following relations:

$$\begin{aligned} \alpha_{li} + \beta_{li} &= -1, \\ \alpha_{vi} + \beta_{vi} &= 0, \\ \alpha_{ei} + \beta_{ei} &= -2, \\ \alpha_{qi} + \beta_{qi} &= 1/3, \\ \alpha_{ui} + \beta_{ui} &= 4/3, \\ \alpha_{di} + \beta_{di} &= -2/3, \\ \alpha_a + \beta_a &= 1, \\ \alpha_\sigma + \beta_\sigma &= 0, \end{aligned} \quad (10)$$

for $i = 1, 2, 3$ and $a = 1, 2$. Thus, the breaking induced by the singlet σ at an energy scale v_σ allows to reproduce the SM hypercharges correctly.

2.4. Mass and mixing matrices for fermions

Let's now consider the generation of fermion mass, which takes place when Φ_2 induces the breaking that gives rise to the local gauge $SU(3)_C \otimes U(1)_Q$ symmetry conserved at low energies. As mentioned, for non-universal models, at least two scalar doublets are needed to provide masses to all the fermion particles and generate the mixing matrices. As usual, the VEV of the scalar doublets are given by

$$\langle \Phi_a \rangle = \begin{pmatrix} 0 \\ \frac{v_a}{\sqrt{2}} \end{pmatrix}, \quad (a = 1, 2). \quad (11)$$

⁵ From equation (10), one of them can be absorbed in a redefinition of the scalar singlet hypercharges ($a_Y = -b_Y \beta_\sigma \alpha_\sigma$).

Table 2. Here $i = 1, 2, 3$ and $a = 1, 2$. The corresponding $U(1)_\beta$ charges can be easily obtained by replacing β instead of α .

Field	$U(1)_\alpha$	Field	$U(1)_\alpha$	Field	$U(1)_\alpha$
u_{iL}	α_{q1}	u_{aR}	$\alpha_{\nu 1} + 4\alpha_{q1}$	Φ_1	$\alpha_{\nu 1} + 3\alpha_{q1}$
		u_{3R}	$\alpha_{\nu 3} + 4\alpha_{q1}$		
d_{iL}		d_{aR}	$-\alpha_{\nu 1} - 2\alpha_{q1}$	Φ_2	$\alpha_{\nu 3} + 3\alpha_{q1}$
		d_{3R}	$-\alpha_{\nu 3} - 2\alpha_{q1}$		
ν_{iL}	$-3\alpha_{q1}$	ν_{aR}	$\alpha_{\nu 1}$		
		ν_{3R}	$\alpha_{\nu 3}$		
e_{iL}		e_{aR}	$-\alpha_{\nu 1} - 6\alpha_{q1}$	σ	α_σ
		e_{3R}	$-\alpha_{\nu 3} - 6\alpha_{q1}$		

In this model, it is possible to generate Dirac masses for all the SM fermions including the SM neutrinos. In this case, the smallness of the neutrino masses relies on the Yukawa couplings as it does happen in the SM. The tree-level Dirac masses come from the Lagrangian equation (3). The resulting mass matrices take the form

$$M^f = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 Y_{11}^f & v_1 Y_{12}^f & v_2 Y_{13}^f \\ v_1 Y_{21}^f & v_1 Y_{22}^f & v_2 Y_{23}^f \\ v_1 Y_{31}^f & v_1 Y_{32}^f & v_2 Y_{33}^f \end{pmatrix}, \quad (12)$$

for $f = u, d, \nu, e$. From here, we see that despite the non-universality of the model, it is possible to have saturated mass matrices for leptons and quarks, i.e. with all the matrix elements different from zero, which is a fairly non-trivial result. As a consequence, the CKM and PMNS mixing matrices can be easily generated, with the mixing between the first two fermion families induced by Φ_1 , while both Φ_1 and Φ_2 contribute to all the mixing elements involving the third family.

Our model is capable of reproducing all the elements of the Dirac mass matrix (therefore, it has no texture zeros) so that it is always possible to reproduce the values of the masses and mixing angles, for both quarks and leptons [45–50].

It is important to emphasize that all elements of the mass matrix are generated for each type of SM fermion, such that for each flavor there are 9 complex parameters, which is equivalent to $2 \times 18 = 36$ real parameters for both the up-type and the down-type quark mass matrices. Since the CKM matrix and the quark masses amount to 9 parameters and a phase, the number of free parameters exceeds the number of physical parameters to be fitted [45]. In the lepton sector, the neutrino masses are not known, and just two square mass differences are experimentally available [51]; that is: $\delta m_{21} = m_2^2 - m_1^2$, $\delta m_{31} = m_3^2 - m_1^2$ (where $\{m_i\}_{i=1,2,3}$ are the neutrino masses), in this case there are only eight real parameters and a phase, but again the number of free parameters in the Yukawa couplings are enough to fit masses and mixing [50]. This freedom allows for the fitting of the quark and lepton masses and the CKM and PMNS mixing matrices with the most recent data from the literature [51].

2.5. Non-universal $U(1)$ charges

By solving the system of equations formed by equations (1), (4) and (10), we obtain a unique solution for the $U(1)$ fermion and scalar charges. The resulting expressions, shown in table 2, are given in terms of just three parameters, namely: α_{q1} , $\alpha_{\nu 1}$ and $\alpha_{\nu 3}$ ⁶. From this it follows

⁶ The α charge of the singlet σ , α_σ , remains as a free parameter, but it does not affect the fermion charges, as can be seen in table 2.

that the non-universality of the solution depends exclusively on the right-handed neutrino charges; so, in what follows, we will assume that $\alpha_{\nu 1} \neq \alpha_{\nu 3}$. Under this condition, the cancellation of chiral anomalies takes place among different families, and not family by family as it does in the SM.

As is well known from the literature on FCNCs, the strongest constraints on tree-level flavor couplings come usually from $F^0 - \bar{F}^0$ mixing processes ($F = K, B_d, D$) [52], to avoid this problem with neutral scalar currents, in most models with two Higgs doublets, discrete symmetries are proposed to cancel the scalar currents with flavor changes. Four 2HDMs are known in the literature, Type-I, Type-II, Type-X, and Type-Y [53], in these models each doublet is charged differently under the discrete symmetry such that one type of particle receives its mass from only one doublet. For example, type-up quarks receive the mass from the same scalar doublet. This is impossible for non-universal models since the charges $U(1)$ of the same type of quarks are different because the model is not universal. In our model, the up-type quarks of the first two families couple with one Higgs doublet while the up-type quark of the third family must couple with the other doublet. This result is guaranteed since the charges of the Higgs doublets are not equal. This is a different mechanism compared to the one in the models mentioned above; however, it can be considered as a particular case of condition III of the general theorem proved in reference [54].

3. EW currents and Z' couplings

Ignoring the kinetic terms, the part of the Lagrangian equation (3) describing the interactions between fermions and gauge bosons can be written as

$$-\mathcal{L} \supset \frac{g_L}{\sqrt{2}}(J_{W^+}^\mu W_\mu^+ + \text{h.c.}) + \frac{g_L}{2}J_3^\mu A_{3,\mu} + \frac{g_\alpha}{2}J_\alpha^\mu B_{\alpha,\mu} + \frac{g_\beta}{2}J_\beta^\mu B_{\beta,\mu}, \quad (13)$$

where the W_μ^+ field has been defined as $W^{+\mu} = (A_1^\mu - iA_2^\mu)/\sqrt{2}$ and the currents are given by

$$\begin{aligned} J_W^\mu &= \bar{\nu}_{iL}\gamma^\mu e_{iL} + \bar{u}_{iL}\gamma^\mu d_{iL}, \\ J_3^\mu &= \bar{u}_{iL}\gamma^\mu u_{iL} + \bar{\nu}_{iL}\gamma^\mu \nu_{iL} - \bar{d}_{iL}\gamma^\mu d_{iL} - \bar{e}_{iL}\gamma^\mu e_{iL}, \\ J_\chi^\mu &= \chi_{qi}(\bar{u}_{iL}\gamma^\mu u_{iL} + \bar{d}_{iL}\gamma^\mu d_{iL}) + \chi_{li}(\bar{\nu}_{iL}\gamma^\mu \nu_{iL} + \bar{e}_{iL}\gamma^\mu e_{iL}) \\ &\quad + \chi_{ui}\bar{u}_{iR}\gamma^\mu u_{iR} + \chi_{di}\bar{d}_{iR}\gamma^\mu d_{iR} + \chi_{\nu i}\bar{\nu}_{iR}\gamma^\mu \nu_{iR} + \chi_{ei}\bar{e}_{iR}\gamma^\mu e_{iR}, \end{aligned} \quad (14)$$

with a sum over the i index is implied and $\chi = \alpha, \beta$. In the basis defined by equation (6), the Lagrangian in equation (13) can be expressed as

$$-\mathcal{L} \supset \frac{g_L}{\sqrt{2}}(J_{W^+}^\mu W_\mu^+ + \text{h.c.}) + \frac{g_L}{2}J_3^\mu W_{3\mu} + \frac{g_Y}{2}J_Y^\mu B_\mu + g_{Z'}J_{Z'}^\mu Z'_\mu, \quad (15)$$

where the interactions of fermions with the Z' boson are given by

$$\begin{aligned} g_{Z'}J_{Z'}^\mu &= g_\beta J_\beta^\mu \cos \theta - g_\alpha J_\alpha^\mu \sin \theta, \\ &= g_{Z'} \sum_f \bar{f}_i \gamma^\mu (\tilde{\epsilon}_{fi}^L P_L + \tilde{\epsilon}_{fi}^R P_R) f_i, \\ &= g_{Z'} \sum_f \bar{f}_i \gamma^\mu (\tilde{g}_{fi}^V - \tilde{g}_{fi}^A \gamma^5) f_i. \end{aligned} \quad (16)$$

Here, f runs over $\{u, d, \nu, e\}$, $i = 1, 2, 3$ (corresponding with the SM family), $P_{L,R} = (1 \mp \gamma^5)/2$ are the chirality projectors and

$$g_{Z'} \tilde{\epsilon}_{f_i}^{L,R} = \frac{1}{2} [g_\beta \hat{\beta}(f_{iL,R}) \cos \theta - g_\alpha \hat{\alpha}(f_{iL,R}) \sin \theta], \quad (17)$$

$$\tilde{g}_{f_i}^{V,A} = \frac{1}{2} (\tilde{\epsilon}_{f_i}^L \pm \tilde{\epsilon}_{f_i}^R). \quad (18)$$

In equation (17), $\tilde{\epsilon}_{f_i}^{L(R)}$ denotes the left(right)-handed chiral coupling of the f_i fermion to the Z' boson, while in equation (18), $\tilde{g}_{f_i}^{V(A)}$ represents the corresponding vector (axial-vector) coupling. As for the couplings to the B field, we have that

$$\begin{aligned} g_Y J_Y^\mu &= g_\alpha J_\alpha^\mu \cos \theta + g_\beta J_\beta^\mu \sin \theta, \\ &= g_Y \sum_f \bar{f}_{iL} \gamma^\mu \hat{Y}(f_{iL}) f_{iL} + \bar{f}_{iR} \gamma^\mu \hat{Y}(f_{iR}) f_{iR}, \end{aligned} \quad (19)$$

with

$$g_Y \hat{Y}(f_{iL,R}) = g_\alpha \hat{\alpha}(f_{iL,R}) \cos \theta + g_\beta \hat{\beta}(f_{iL,R}) \sin \theta. \quad (20)$$

By comparing equations (9) and (20), taking into account our choice $a_Y = b_Y = 1$, we get the following relations among the coupling constants g_α , g_β , g_Y and the mixing angle θ :

$$g_\alpha \cos \theta = g_\beta \sin \theta = \frac{g_Y}{\sqrt{2}}, \quad (21)$$

from which it follows that

$$\tan \theta = \frac{g_\alpha}{g_\beta} \quad \text{and} \quad \frac{1}{g_\alpha^2} + \frac{1}{g_\beta^2} = \frac{2}{g_Y^2}. \quad (22)$$

By changing to the basis defined by equation (7), the Lagrangian in equation (15) can be rewritten as

$$-\mathcal{L} \supset e J_\gamma^\mu A_\mu + g_Z J_Z^\mu Z_\mu + g_{Z'} J_{Z'}^\mu Z'_\mu + g_W J_{W^+}^\mu W_\mu^+ + \text{h.c.}, \quad (23)$$

where

$$\begin{aligned} g_W &= \frac{g_L}{\sqrt{2}}, \\ e J_\gamma^\mu &= e \sum_f \bar{f}_i \gamma^\mu \hat{Q}(f_i) f_i, \\ g_Z J_Z^\mu &= \frac{g_L}{2 \cos \theta_W} \sum_f \bar{f}_i \gamma^\mu (\epsilon_{f_i}^L P_L + \epsilon_{f_i}^R P_R) f_i, \end{aligned} \quad (24)$$

with the chiral couplings to the Z boson defined as

$$\epsilon_{f_i}^{L,R} = 2 \left\{ \hat{T}_3(f_{iL,R}) - \sin^2 \theta_W \left[\hat{T}_3(f_{iL,R}) + \frac{1}{2} \hat{Y}(f_{iL,R}) \right] \right\}. \quad (25)$$

To obtain these expressions, the identification $e = g_L \sin \theta_W = g_Y \cos \theta_W$ was made, which implies the well-known relation

$$g_Y = g_L \tan \theta_W. \quad (26)$$

Taking into account the relations in equations (21) and (26), as well as the charges reported in table 2, and the parameters x , y , z and w defined as

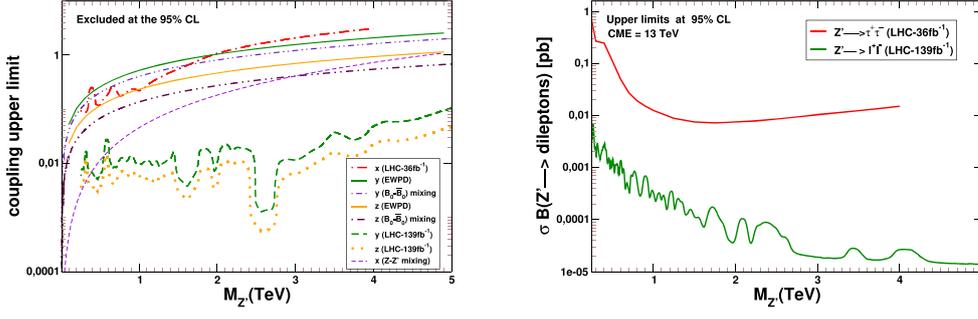


Figure 1. Left: Upper limit on the model parameters x, y, z . Right: 95% CL upper limits on the fiducial Z' production cross-section times the $Z' \rightarrow \ell^+\ell^-$ branching [59] (green continuous line) and the corresponding upper limits on the Z' decaying to $\tau\bar{\tau}$ pairs [60] (red continuous line).

Table 3. Chiral couplings between the fermion sector and the Z' gauge boson.

f	$\nu_{1,2}$	ν_3	$e_{1,2}$	e_3	$u_{1,2}$	u_3	$d_{1,2}$	d_3
$g_{Z'}\tilde{\epsilon}_f^L$	$-z$	$-z$	$-z$	$-z$	$\frac{1}{3}z$	$\frac{1}{3}z$	$\frac{1}{3}z$	$\frac{1}{3}z$
$g_{Z'}\tilde{\epsilon}_f^R$	$-y$	$-x$	$y - 2z$	$x - 2z$	$-y + \frac{4}{3}z$	$-x + \frac{4}{3}z$	$y - \frac{2}{3}z$	$x - \frac{2}{3}z$

$$\begin{aligned}
 x &\equiv \frac{g_L \tan \theta_W}{2\sqrt{2}} (\cot \theta + \tan \theta) \alpha_{\nu 3}, \\
 y &\equiv \frac{g_L \tan \theta_W}{2\sqrt{2}} (\cot \theta + \tan \theta) \alpha_{\nu 1}, \\
 z &\equiv \frac{g_L \tan \theta_W}{2\sqrt{2}} [\cot \theta - 3(\cot \theta + \tan \theta) \alpha_{q1}], \\
 w &\equiv \frac{g_L \tan \theta_W}{2\sqrt{2}} (\cot \theta + \tan \theta) \alpha_{\sigma},
 \end{aligned} \tag{27}$$

the Z' chiral couplings given by equation (17) can be expressed as indicated in table 3⁷. These charges are best suited for a phenomenological analysis of the new neutral vector boson, as it will be explained in the next section. Regarding the scalar fields Φ_1 and Φ_2 , their Z' couplings are given by $z - y$ and $z - x$, respectively.

4. Low energy and collider constraints

For the process $\bar{q}q \rightarrow Z' \rightarrow \ell^+\ell^-$, ATLAS reports upper limits on the fiducial cross-section times the $Z' \rightarrow \ell^+\ell^-$ branching from searches of high-mass dilepton resonances (dielectron and dimuon) during Run 2 of the Large Hadron Collider (LHC) at a center-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 139 fb^{-1} . From these constraints, we obtain upper limits on the y and z couplings corresponding to the green dashed and orange dotted

⁷ From here, we see that there are two families with identical charges and a different third one. However, universal models are still possible, for example, setting $z = x = y = 1$ yields the well-known expressions for the $B - L$ charges.

lines in the left-handed plot in figure 1. These limits are obtained from the intersection of the theoretical cross-section (for further details see our previous publications [55–58]) with the 95% CL upper limit on the cross-section reported by the ATLAS collaboration [59] (the green continuous line in the right plot figure 1). For the upper limits on the x parameter (red dot-dashed line in the left plot Figure 1), we use the ATLAS 95% upper limits on the production cross-section times branching fraction for a Z' boson decaying to a $\tau\bar{\tau}$ pair (the green continuous line in the right plot figure 1). This data was collected by ATLAS in searches of Z' bosons using a data sample corresponding to an integrated luminosity of 36.1 fb^{-1} from proton–proton collisions at a center of mass energy of 13 TeV [60].

Constraints on a parameter are obtained by marginalizing the other parameters. In the case of the parameter x , which represents the coupling strength between Z' and the fermions of the third family, the Z' is produced from an annihilation $b\bar{b} \rightarrow Z'$. Due to the strong collider constraints on the first two families, only Z' couplings with the third family are possible at low energies. In our model, this implies that $y, z \ll x$, as we can see from table 3. This implies that at low energies, the unique generator with unsuppressed coupling strengths is $T_{3R}(3)$. This symmetry is a well-known EW extension of the SM. The argument ‘3’ refers to the third family, and the subscript $3R$ refers to the generator $\sigma_3/2$, whose representation in the third family of the SM is $(b_R, t_R)^T$. If we allow right-handed neutrinos, as is, in fact, the case in our model, a lepton representation $(\tau_R, \nu_{\tau R})^T$ is also possible.

From reference [61], the $Z - Z'$ mixing angle Θ is restricted to be less than 10^{-3} , which holds true for most models. Based on this result, we can assume Θ identically zero, which is a typical assumption in collider constraints [62].

We also report Electroweak Precision data constraints on the y and z parameters (green and orange continuous lines in the left panel of figure 1), obtained using the GAPP package [61, 63], which includes low-energy weak neutral current experiments (this includes weak charges of the cesium atom and electron, as well as the constraints coming from cross-section ratios of neutrinos and antineutrino deep inelastic scattering. Measurements of the top and W masses are also in this set of observables) and Z -pole observables.

As our model is non-universal, it has two possible sources of FCNC: the non-universal couplings of the Z' and the couplings of the SM fermions to two scalar doublets. Since the charges of the first two families are equal, we can ignore constraints from observables with flavor changes between quarks and leptons of the first two families, such as: K^0 -mixing, $\mu-e$ conversion, etc. In our case, one of the strongest constraints on the parameters comes from $B^0-\bar{B}^0$ -mixing. figure 1 shows the upper limits on the y and z parameters at a 95% confidence level. An extended Higgs sector generates a mixture of Z and Z' that is proportional to the couplings of the Higgs to Z' and to the expectation values of the neutral components of the scalar doublets. As shown in appendix D, the $Z - Z'$ constraints are relevant for the x parameter because z and y are strongly constrained for colliders. As can be seen from the purple dashed line in figure 1, this is the most restrictive constraint on the x parameter.

In two Higgs doublet models, FCNC can be avoided if the mass matrix for SM fermions with the same electric charge and isospin is generated from a single Higgs doublet. As we show in the appendix C, if the right-handed SM fermion is a singlet under the gauge group and if each right-handed SM singlet fermion couples to only one Higgs doublet (there is no problem if the scalar doublet has non-zero couplings to several right-handed fermions.), then there are no FCNC for the scalar sector; this is the case in our model.

5. Analysis of Higgs-like resonant signals

Recently, several anomalies have been reported in searches of high-mass scalar resonances in proton–proton collisions at the LHC. The 2HDMs are the most straightforward extensions of the SM that can explain these observations. Additionally, our model includes a scalar singlet σ that gives mass to the Z' . The Higgs mechanism requires at least two CP-odd bosons to provide mass to the Z and the Z' and one charged scalar boson to give mass to the SM W boson, which leaves us with three CP even scalar bosons, one CP-odd scalar boson and one charged scalar. Our analysis aims to determine the typical masses for these bosons in the best-motivated parameter space and compare them with the experimental anomalies reported in the literature. As explained in detail in the appendix A, of the three neutral scalar fields in the interaction space, h_1, h_2 , and ξ , we can obtain using a unitary transformation, the neutral states in the mass space, H_1, H_2, H_3 . Great interest has generated an anomaly that can be explained by a light neutral scalar Higgs with a mass $M_{H1} \approx 95$ GeV [64] and a charged Higgs around $M_{C^\pm} \approx 130$ GeV [65]. For the charged Higgs, in [66] a detailed analysis of the phenomenological implications of a new resonance with a three sigma significance was studied. On a mass basis, we will denote as M_A the only CP-odd field that is not absorbed as a Goldstone boson. An excess of events was also found in channels involving the productions of SM gauge bosons, $\gamma\gamma$ and $Z\gamma$ (for further analysis, look in [67] and references therein). This analysis provides a good indication of new scalar resonances decaying into two photons with invariant masses of 95 [68] and 152 GeV [67]. Other excesses over the expected value in the SM for dibosons are reported at 680 GeV [69], which are compatible with the excess in $\gamma\gamma$ and $b\bar{b}$ reported by the CMS collaboration [70]. A more complete review of these anomalies and additional references can be found at [71]. In this reference, they also mention an excess reported by the ATLAS collaboration that can be interpreted as a pseudoscalar with a mass of 650 GeV produced in association with a scalar with a mass of 450 GeV.

Recently, a deviation from the background-only expectation occurred for high scalar resonances with masses (575 200) GeV and a local (global) significance of 3.5 (2.0) standard deviations, as reported by the ATLAS collaboration [72]. It is important to stress that this analysis shows good agreement with the background-only hypothesis for the masses (65 090) GeV, where CMS reported an excess with a local (global) significance of 3.8 (2.8) standard deviations [70].

To account for these experimental anomalies from the scalar potential of our model (see equation 32), we consider two possible assignments for the charges of the scalar singlet σ . One of them leads to a cubic coupling among the scalar fields, while the other to quartic term (the remaining terms in the scalar potential 32 are always present regardless of the σ charge). If the Z' coupling of σ is $x - y$, that is, $\alpha_\sigma = \alpha_{\nu 3} - \alpha_{\nu 1}$, then the following term is allowed:

$$\mu[(\Phi_1^\dagger \Phi_2)\sigma + \text{h.c.}].$$

In this case, the coupling constant μ has dimensions of mass, and in order to have a consistent mass spectrum, its values must be in the range $-77.3 \text{ GeV} \leq \mu < 0$. Similarly, if the coupling of σ to the Z' boson is $\frac{1}{2}(x - y)$, or equivalently $\alpha_\sigma = (\alpha_{\nu 3} - \alpha_{\nu 1})/2$, it is possible to form the term

$$\lambda[(\Phi_1^\dagger \Phi_2)\sigma^2 + \text{h.c.}], \quad (28)$$

where the constant λ is dimensionless, and restricted to the range $(-0.44, 0)$. According to our scalar sector (whose scalar potential we show in appendix A), to reproduce part of the spectrum of anomalies in the scalar sector (determined by the VEVs and coupling constants) we must identify the middle-mass neutral scalar boson as the SM Higgs boson to which we

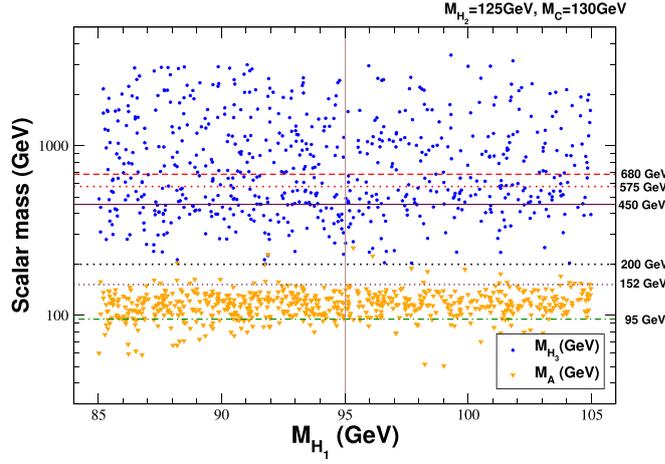


Figure 2. Distribution of the scalar mass M_{H_3} (the blue round points) and the pseudoscalar M_A (orange triangle points) for a scalar potential including the cubic term $\mu\Phi_1^\dagger\Phi_2\sigma + \text{h.c.}$. In this term, μ has mass dimensions and takes values in the range $(-77.3, 0)$ GeV. We vary the other dimensionless parameters of the scalar potential in the range $[-1.5, 1.5]$, and take the VEV of the scalar singlet v_σ as a free parameter ranging from 250 to 2000 GeV. The VEV's v_1 and v_2 vary subject to the conditions $\sqrt{v_1^2 + v_2^2} = v = 246.24$ GeV and $v_2 \gg v_1$. The masses of the scalars and pseudo-scalars: M_{H_1} , M_{H_3} and M_A are determined by the tadpole equations (33). See the text for further details (see appendix A).

assign its well-known mass of $M_{H_2} = 125$ GeV. In our model, we ensure that only the massive charged scalar field coincides with the anomaly $M_{C^\pm} = 130$ GeV. This happens while the SM vector boson W absorbs the other massless-charged field through the Higgs mechanism. Similarly, the Higgs mechanism requires a pseudoscalar field from one of the scalar doublets to give mass to the Z boson, and the pseudoscalar field of the scalar singlet to give mass to the Z' . The mass of the remaining scalar fields (M_{H_1} , M_{H_3} and M_A) are free parameters. For the other dimensionless parameters of the potential pot, their values are assumed to be in the range $[-1.5, 1.5]$. Regarding the VEVs of the Φ_1 and Φ_2 , they are chosen such that $v_1^2 + v_2^2 = (246.24 \text{ GeV})^2$ with $v_2 \gg v_1$. This hierarchy between VEVs is necessary to align the Higgs doublet Φ_2 with that of the SM. We take the VEV of the scalar singlet $\langle\sigma\rangle = v_\sigma/\sqrt{2}$ as a free parameter varying between 250 and 2000 GeV. Finally, in order to satisfy the collider constraints, we require $z, y \ll x$, and take $x \gtrsim 1$ for Z' masses above 2 TeV, as explained in section appendix B.

To illustrate the density of solutions, figures 2 and 3 display a total of 640 solutions spread across the M_{H_1} versus M_{H_3} and M_{H_1} versus M_A axes. It is important to emphasize our identification of the lightest CP-even Higgs scalar H_1 with the anomaly at 95 GeV. Therefore, we have considered exploring the mass interval 95 ± 10 GeV. In these figures, we can see that many of the experimental anomalies coincide with the regions with the highest density of solutions. This coincidence is important since we have made the free parameters of the theory vary in intervals that we consider natural. It is very important that in our analysis we imposed the hierarchy $m_{H_1} < m_{H_2} < m_{H_3}$. The results are identical if we choose $m_{H_2} < m_{H_1}$, and identify the Higgs with M_{H_1} . From the density plot, we see that the highest density of solutions for the pseudo-scalar mass is between 100 and 200 GeV. This result holds for both cubic and

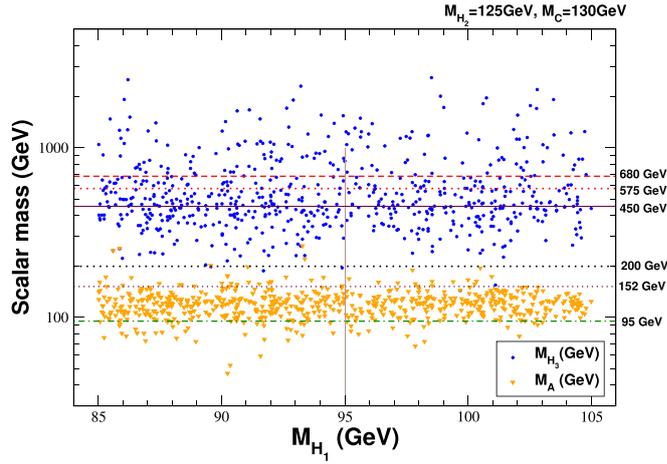


Figure 3. Distribution of the scalar mass M_{H_3} (the blue round points) and the pseudo-scalar M_A (orange triangle points) for a scalar potential including the quartic term $\lambda\Phi_1^\dagger\Phi_2\sigma^2+\text{h.c.}$ In this term, λ is dimensionless and takes values in the interval $(-0.44, 0)$. We vary the other dimensionless parameters of the scalar potential in the range $[-1.5, 1.5]$, and take the VEV of the scalar singlet v_σ as a free parameter ranging from 250 to 2000 GeV. The VEV's v_1 and v_2 vary subject to the conditions $\sqrt{v_1^2 + v_2^2} = v = 246.24$ GeV and $v_2 \gg v_1$. The masses of the scalars and pseudoscalars: M_{H_1} , M_{H_3} and M_A are determined by the tadpole equations (33). See the text for further details.

quartic potentials. In this range, we have three experimental anomalies (the one at 95 GeV, the one at 152 GeV, and the one at 200 GeV). If any of these anomalies accumulate statistics, this strongly suggests that a pseudoscalar particle could explain the resonance.

For the quartic potential (see Figure 3), the highest density of solutions is found below 1000 GeV, while for the cubic potential (see figure 2), there is a high density of solutions up to 3000 GeV. Below 1000 GeV we have three experimental anomalies with masses: 450 GeV, 575 GeV, and 680. From the density plots we see that for a quartic potential, it is more probable to have solutions in this range when compared to the cubic potential.

6. Conclusions

In this work, we assume that the SM is a low-energy effective theory of a more fundamental theory characterized by a gauge symmetry of the form $SU(3)_C \otimes SU(2)_L \otimes U(1)_\alpha \otimes U(1)_\beta$, and whose particle content is that of the SM extended with three right-handed neutrinos, a second Higgs doublet and a scalar singlet. Additionally, we impose that both $U(1)$ charges are non-universal and contribute non-trivially to the SM hypercharge, i.e. they are not inert charges. Under these assumptions, we showed that the most general expression for the Z' chiral couplings is as those shown in table tab3. In this model, it is possible to generate all the mass matrix elements of with only two Higgs doublets. From this, it is possible to adjust the model to reproduce the CKM and PMNS mixing matrices. This feature is highly non-trivial for non-universal scenarios and represents a great advantage of this model. It is important to mention that to maintain the non-universality condition, it was preferable to avoid Majorana mass terms (if we want to reproduce the electric charges of the SM particles (which are

universal) from non-universal $U(1)$ charges, in most of the cases studied, we must avoid introducing neutrino Majorana masses).

From the assumptions of our work, as well as the collider, electroweak and flavor constraints, we also conclude that for a model with two non-inert Abelian symmetries at low energies ($M_{Z'} < 5$ TeV), only the residual symmetry $T_{3R}(3)$, in addition to the SM gauge symmetry, has an unsuppressed coupling strength. The argument [3] says that only couplings to the third family are possible. Models with couplings to the first and second families are strongly constrained, so that only Z' couplings below 0.1 are possible, i.e. $g_{Z'} \tilde{\epsilon}_{L,R} < 0.1$. For a Z' coupling to the third family, it is possible to have Z' charges such that $g_{Z'} \tilde{\epsilon}_{L,R} \sim 1$ for Z' masses above 2 TeV.

Our work analyzes some Higgs-like anomalies recently reported by the ATLAS and CMS collaborations [67]. To this end, we show the distribution of 400 solutions in the M_{H_1} , M_{H_3} and M_{H_2} , M_A planes. These results are shown in figures 2 and 3. This analysis concludes that explaining some of the observed anomalies within the model is possible.

We show that the scalar sector FCNC cancel if each right-handed fermion couples only to a single Higgs doublet (although the scalar doublet can have non-zero couplings with several right-handed fermions). This will be the case as long as the right-handed fermions are singlets of the gauge group.

Acknowledgments

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A. Scalar potential

Our model contains two scalar doublets, Φ_1 and Φ_2 , and a scalar singlet σ . In general, these fields can be expressed as

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + h_1 + i \eta_1}{\sqrt{2}} \end{pmatrix}, \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + h_2 + i \eta_2}{\sqrt{2}} \end{pmatrix}, \sigma = \frac{v_\sigma + \xi + i \zeta}{\sqrt{2}}, \quad (29)$$

where $\langle \Phi_1 \rangle = (0, v_1/\sqrt{2})^T$, $\langle \Phi_2 \rangle = (0, v_2/\sqrt{2})^T$ and $v_\sigma = \sqrt{2} \langle \sigma \rangle$. For the doublet Φ_2 (which is close to H_1 in the Georgi basis) to be aligned with the Higgs of the SM we impose the hierarchy

$$v_\sigma > v_2 \gg v_1. \quad (30)$$

Since the Higgs doublet is a linear combination of the two scalar doublets, then

$$\sqrt{v_1^2 + v_2^2} = v = 246.24 \text{ GeV}, \quad (31)$$

The most general scalar potential consistent with the gauge symmetry $SU(2)_L \otimes U(1)_\alpha \otimes U(1)_\beta$ is [68]:

$$\begin{aligned} V(\Phi_1, \Phi_2, \sigma) = & \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 + \mu_\sigma^2 |\sigma|^2 + \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_\sigma |\sigma|^4 \\ & + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \lambda_{1\sigma} |\Phi_1|^2 |\sigma|^2 + \lambda_{2\sigma} |\Phi_2|^2 |\sigma|^2 \\ & + \text{linear term in } \sigma \text{ (or quadratic term in } \sigma), \end{aligned} \quad (32)$$

where a linear interaction term in σ (which we will denote as the cubic term) of the form

$$\mu [(\Phi_1^\dagger \Phi_2) \sigma + \text{h.c.}]$$

is possible if α_σ in table 2 is taken to be $\alpha_{\nu 3} - \alpha_{\nu 1}$. Here μ is a coupling with mass dimensions. On the other hand, if α_σ is equal to $\frac{1}{2}(\alpha_{\nu 3} - \alpha_{\nu 1})$, then the quadratic term in σ (which we will denote as the quartic term),

$$\lambda [(\Phi_1^\dagger \Phi_2) \sigma^2 + \text{h.c.}],$$

is the one that is present. In this case, the coupling λ is dimensionless. By minimizing the potential in equation pot, we then obtain that

$$\begin{aligned} \mu_1^2 = & -\frac{\sqrt{2}}{2} \frac{\mu v_2 v_\sigma}{v_1} - \lambda_1 v_1^2 - \frac{\lambda_3 + \lambda_4}{2} v_2^2 - \frac{\lambda_{1\sigma}}{2} v_\sigma^2, \\ \mu_2^2 = & -\frac{\sqrt{2}}{2} \frac{\mu v_1 v_\sigma}{v_2} - \lambda_2 v_2^2 - \frac{\lambda_3 + \lambda_4}{2} v_1^2 - \frac{\lambda_{2\sigma}}{2} v_\sigma^2, \\ \mu_\sigma^2 = & -\frac{\sqrt{2}}{2} \frac{\mu v_1 v_2}{v_\sigma} - \lambda_\sigma v_\sigma^2 - \frac{\lambda_{1\sigma}}{2} v_1^2 - \frac{\lambda_{2\sigma}}{2} v_2^2. \end{aligned} \quad (33)$$

in the cubic case, while in the quartic one, the corresponding expressions can be obtained from the previous ones by making the substitution $\sqrt{2}\mu \rightarrow \lambda v_\sigma$.

A.1. Mass spectrum of the neutral scalar sector

From the potential equation (32) and the previous minimization conditions, we can build the mass matrices from the fields defined in equation (29). For the CP-even scalar field basis (h_1, h_2, ξ) , the mass matrix is given in the cubic case [73] by:

$$\begin{pmatrix} 2\lambda_1 v_1^2 - \frac{\mu v_2 v_\sigma}{\sqrt{2} v_1} & \frac{\mu v_\sigma}{\sqrt{2}} + v_1 v_2 (\lambda_3 + \lambda_4) & \frac{\mu v_2}{\sqrt{2}} + \lambda_{1\sigma} v_1 v_\sigma \\ \frac{\mu v_\sigma}{\sqrt{2}} + v_1 v_2 (\lambda_3 + \lambda_4) & 2\lambda_2 v_2^2 - \frac{\mu v_1 v_\sigma}{\sqrt{2} v_2} & \frac{\mu v_1}{\sqrt{2}} + \lambda_{2\sigma} v_2 v_\sigma \\ \frac{\mu v_2}{\sqrt{2}} + \lambda_{1\sigma} v_1 v_\sigma & \frac{\mu v_1}{\sqrt{2}} + \lambda_{2\sigma} v_2 v_\sigma & 2\lambda_\sigma v_\sigma^2 - \frac{\mu v_1 v_2}{\sqrt{2} v_\sigma} \end{pmatrix}, \quad (34)$$

while in the quartic case, it corresponds to:

$$\begin{pmatrix} 2\lambda_1 v_1^2 - \frac{\lambda v_2 v_\sigma^2}{2 v_1} & \frac{\lambda v_\sigma^2}{2} + v_1 v_2 (\lambda_3 + \lambda_4) & v_\sigma (\lambda v_2 + \lambda_{1\sigma} v_1) \\ \frac{\lambda v_\sigma^2}{2} + v_1 v_2 (\lambda_3 + \lambda_4) & 2\lambda_2 v_2^2 - \frac{\lambda v_1 v_\sigma^2}{2 v_2} & v_\sigma (\lambda v_1 + \lambda_{2\sigma} v_2) \\ v_\sigma (\lambda v_2 + \lambda_{1\sigma} v_1) & v_\sigma (\lambda v_1 + \lambda_{2\sigma} v_2) & 2\lambda_\sigma v_\sigma^2 \end{pmatrix}. \quad (35)$$

These are square mass matrices of rank three with mass eigenvalues M_{H_1} , M_{H_2} and M_{H_3} , corresponding to the mass eigenstates H_1 , H_2 and H_3 , respectively. We will identify the states according to the mass hierarchy:

$$M_{H_1} < M_{H_2} < M_{H_3}.$$

The intermediate-mass scalar state, H_2 , can be identified as the SM Higgs, while the light mass scalar state H_1 and the heavy mass scalar state H_3 are new scalar fields that, in principle, can be observed in the LHC experiments. The hierarchy equation (30) causes the scalar H_2 to align with h_2 .

A.1.1. Mass spectrum of the neutral pseudoscalar sector. In the (η_1, η_2, ζ) basis, the pseudoscalar squared mass matrix takes the following form for the cubic case:

$$\frac{\mu}{\sqrt{2}} \begin{pmatrix} -\frac{v_2 v_\sigma}{v_1} & v_\sigma & v_2 \\ v_\sigma & -\frac{v_1 v_\sigma}{v_2} & -v_1 \\ v_2 & -v_1 & -\frac{v_1 v_2}{v_\sigma} \end{pmatrix}. \quad (36)$$

The corresponding mass matrix for the quartic case is

$$\frac{\lambda v_\sigma}{2} \begin{pmatrix} -\frac{v_2 v_\sigma}{v_1} & v_\sigma & 2v_2 \\ v_\sigma & -\frac{v_1 v_\sigma}{v_2} & -2v_1 \\ 2v_2 & -2v_1 & -4\frac{v_1 v_2}{v_\sigma} \end{pmatrix}. \quad (37)$$

In both cases, these mass matrices have rank 1. The two zero eigenvalues correspond to the two Goldstone bosons that give mass to the Z and Z' bosons after the spontaneous symmetry breaking. The non-zero eigenvalue corresponds to a measurable pseudoscalar with mass equal to:

$$M_A^2 = \begin{cases} -\frac{\mu(v_1^2 v_2^2 + v^2 v_\sigma^2)}{\sqrt{2} v_1 v_2 v_\sigma} & \text{(cubic case),} \\ -\frac{\lambda(4v_1^2 v_2^2 + v^2 v_\sigma^2)}{2v_1 v_2} & \text{(quartic case),} \end{cases} \quad (38)$$

whose mixing comes mainly from η_1 .

A.1.2. Mass spectrum of the charged scalar sector. In the (ϕ_1^\pm, ϕ_2^\pm) basis, the squared mass matrix for charged scalar particles is

$$\frac{1}{2} \begin{pmatrix} -\sqrt{2} \frac{\mu v_2 v_\sigma}{v_1} - \lambda_4 v_2^2 & \sqrt{2} \mu v_\sigma + \lambda_4 v_1 v_2 \\ \sqrt{2} \mu v_\sigma + \lambda_4 v_1 v_2 & -\sqrt{2} \frac{\mu v_1 v_\sigma}{v_2} - \lambda_4 v_1^2 \end{pmatrix}, \quad (39)$$

for the cubic case, and

$$\frac{1}{2} \begin{pmatrix} -\frac{\lambda v_2 v_\sigma^2}{v_1} - \lambda_4 v_2 & \lambda v_\sigma^2 + \lambda_4 v_1 v_2 \\ \lambda v_\sigma^2 + \lambda_4 v_1 v_2 & -\frac{\lambda v_1 v_\sigma^2}{v_2} - \lambda_4 v_1 \end{pmatrix}, \quad (40)$$

for the quartic case. As before, these mass matrices have rank 1, with the only zero eigenvalue corresponding to the Goldstone boson giving mass to the charged W boson. The remaining charged scalar acquires a mass equal to

$$M_{C^\pm}^2 = \begin{cases} -\frac{v^2}{2} \left(\sqrt{2} \frac{\mu v_\sigma}{v_1 v_2} + \lambda_4 \right), & \text{(cubic case);} \\ -\frac{v^2}{2} \left(\frac{\lambda v_\sigma^2}{v_1 v_2} + \lambda_4 \right), & \text{(quartic case).} \end{cases} \quad (41)$$

Appendix B. The gauge boson masses

Let us now determine the mass of the neutral gauge bosons. These are obtained from the scalar-gauge couplings introduced by the covariant derivatives of the scalar fields in the Lagrangian terms

$$\mathcal{L} \supset |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 + |D_\mu \sigma|^2, \quad (42)$$

where

$$D^\mu = \partial^\mu + \frac{i}{2} g_L A_j^\mu \hat{T}_j + \frac{i}{2} g_\alpha B_\alpha^\mu \hat{\alpha} + \frac{i}{2} g_\beta B_\beta^\mu \hat{\beta}, \quad (43)$$

with \hat{T}_j , A_j^μ ($j = 1, 2, 3$) and g_L denoting, respectively, the generators, the gauge fields and the coupling constant associated with the weak isospin gauge group $SU(2)_L$ ⁸, while $\hat{\alpha}$, B_α^μ and g_α , with $\alpha = \alpha, \beta$, are the corresponding quantities related with the two Abelian $U(1)$ factors. For the Higgs doublets Φ_a ($a = 1, 2$) and the singlet σ , we have

$$\begin{aligned} D^\mu \Phi_a &= \left[\partial^\mu + \frac{i}{2} g_L \begin{pmatrix} A_3^\mu & \sqrt{2} W^\mu \\ \sqrt{2} W^{\mu\dagger} & -A_3^\mu \end{pmatrix} + \frac{i}{2} g_\alpha \alpha_a B_\alpha^\mu + \frac{i}{2} g_\beta \beta_a B_\beta^\mu \right] \Phi_a, \\ D^\mu \sigma &= \left(\partial^\mu + \frac{i}{2} g_\alpha \alpha_\sigma B_\alpha^\mu + \frac{i}{2} g_\beta \beta_\sigma B_\beta^\mu \right) \sigma, \end{aligned} \quad (44)$$

Here α_a (β_a) and α_σ (β_σ) denote, respectively, the $U(1)_{\alpha(\beta)}$ charges for Φ_a and σ given in table 2. Additionally, the W field has been defined as

$$W^\mu = \frac{A_1^\mu - i A_2^\mu}{\sqrt{2}}. \quad (45)$$

Taking into account the definition of Φ_a and σ given in equation (29), as well as the basis changes defined in equations (6) and (7), which imply

⁸ The $SU(2)_L$ generators are defined in terms of the Pauli matrices according to $T_i = \frac{1}{2} \sigma_i$.

$$\begin{aligned} B_\chi^\mu &= a_\chi A^\mu + b_\chi Z^\mu + c_\chi Z'^\mu, & (\chi = \alpha, \beta); \\ A_3^\mu &= \sin \theta_W A^\mu + \cos \theta_W Z^\mu, \end{aligned} \quad (46)$$

with

$$\begin{aligned} a_\alpha &= \cos \theta \cos \theta_W, & b_\alpha &= -\cos \theta \sin \theta_W, & c_\alpha &= -\sin \theta, \\ a_\beta &= \sin \theta \cos \theta_W, & b_\beta &= -\sin \theta \sin \theta_W, & c_\beta &= \cos \theta, \end{aligned} \quad (47)$$

it can be shown that the mass terms for the Z^μ , Z'^μ and W^μ gauge bosons are

$$\begin{aligned} \mathcal{L} \supset & \frac{1}{2}(g_Z^2 v^2) Z_\mu Z^\mu + \frac{1}{2}[g_{Z'}^2(\gamma_1'^2 v_1^2 + \gamma_2'^2 v_2^2 + \gamma_\sigma'^2 v_\sigma^2)] Z'_\mu Z'^\mu \\ & - \frac{1}{2}[2g_Z g_{Z'}(\gamma_1' v_1^2 + \gamma_2' v_2^2)] Z_\mu Z'^\mu + g_W^2 v^2 W_\mu^\dagger W^\mu, \end{aligned} \quad (48)$$

the coupling constants g_Z and g_W are defined as in the SM, i.e.

$$g_Z = \frac{g_L}{2 \cos \theta_W}, \quad g_W = \frac{g_L}{2}, \quad (49)$$

while $g_{Z'}$ is defined through the following relations:

$$\begin{aligned} g_{Z'} \gamma'_a &= -\frac{1}{2}(g_\alpha \alpha_a \sin \theta - g_\beta \beta_a \cos \theta), & (a = 1, 2.); \\ g_{Z'} \gamma'_\sigma &= -\frac{1}{2}(g_\alpha \alpha_\sigma \sin \theta - g_\beta \beta_\sigma \cos \theta). \end{aligned} \quad (50)$$

In terms of the x , y , z and w parameters defined in equation (27), these couplings can be expressed as

$$g_{Z'} \gamma'_1 = z - y, \quad g_{Z'} \gamma'_2 = z - x, \quad g_{Z'} \gamma'_\sigma = -w. \quad (51)$$

Writing the $Z - Z'$ mixing matrix as

$$M_{Z-Z'}^2 = \begin{bmatrix} g_Z^2 v^2 & -g_Z g_{Z'} \gamma'_a v_a^2 \\ -g_Z g_{Z'} \gamma'_a v_a^2 & g_{Z'}^2 (\gamma_a'^2 v_a^2 + \gamma_\sigma'^2 v_\sigma^2) \end{bmatrix} \equiv \begin{pmatrix} \mathcal{A} & -\mathcal{C} \\ -\mathcal{C} & \mathcal{B} \end{pmatrix}, \quad (52)$$

with a sum over the a index implied, then the square masses of the physical neutral gauge bosons Z_1 and Z_2 are given by

$$m_{Z_{1,2}}^2 = \frac{1}{2}[\mathcal{A} + \mathcal{B} \mp \sqrt{(\mathcal{A} - \mathcal{B})^2 + 4\mathcal{C}^2}]. \quad (53)$$

If \mathcal{O} is the diagonalizing orthogonal matrix defining the mass basis, i.e.

$$\begin{pmatrix} Z^\mu \\ Z'^\mu \end{pmatrix} = \mathcal{O} \begin{pmatrix} Z_1^\mu \\ Z_2^\mu \end{pmatrix} = \begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} Z_1^\mu \\ Z_2^\mu \end{pmatrix}, \quad (54)$$

then the mixing angle Θ can be determined from

$$\tan 2\Theta = \frac{2\mathcal{C}}{\mathcal{B} - \mathcal{A}}. \quad (55)$$

From this expression, it is possible to obtain

$$\Theta = \frac{1}{2} \arctan \left\{ \frac{g_L [(z-y)v_1^2 + (z-x)v_2^2]}{2 \cos \theta_W [(z-y)^2 v_1^2 + (z-x)^2 v_2^2 + w^2 v_\sigma^2] - \frac{g_L^2 v^2}{4 \cos^2 \theta_W}} \right\}. \quad (56)$$

To satisfy the current constraint on the mixing angle [61] it is necessary to keep this angle below 10^{-3} , which is possible in two scenarios: (1) a light Z' mass, i.e. $M_{Z'} \ll M_Z$ or a heavy Z' mass, i.e. $M_{Z'} \gg M_Z$, which requires $x \gtrsim 1$, $z \sim y \ll 1$ and $v < v_\sigma$. As usual in calculating collider constraints [62] we assume $\theta_{Z-Z'} = 0$. In analyzing the scalar anomalies $w = x - y$ in the cubic case, or $w = \frac{x-y}{2}$ for a potential with quartic coupling term (as explained in Appendix A). For that analysis, assuming a heavy Z' mass is more convenient.

Appendix C. Analysis of scalar FCNCs

The Yukawa interactions are described by the general Lagrangian

$$-\mathcal{L}_Y = \bar{q}'_{iL} y_{ij}^{ad} \Phi_a d'_{jR} + \bar{q}'_{iL} y_{ij}^{au} \tilde{\Phi}_a u'_{jR} + \bar{l}'_{iL} y_{ij}^{ae} \Phi_a e'_{jR} + \bar{l}'_{iL} y_{ij}^{av} \tilde{\Phi}_a \nu'_{jR} + \text{h.c.}, \quad (57)$$

with $i, j = 1, 2, 3$, and $a = 1, 2$. Here

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ \frac{v_a + h_a + i\eta_a}{\sqrt{2}} \end{pmatrix}, \quad \tilde{\Phi}_a = i\sigma_2 \Phi_a^*. \quad (58)$$

According to the non-universal $U(1)$ charges, the Yukawa couplings present in our model are

$$-\mathcal{L}_Y = \bar{q}'_{iL} y_{ia}^{1d} \Phi_1 d'_{aR} + \bar{q}'_{iL} y_{ia}^{1u} \tilde{\Phi}_1 u'_{aR} + \bar{l}'_{iL} y_{ia}^{1e} \Phi_1 e'_{aR} + \bar{l}'_{iL} y_{ia}^{1\nu} \tilde{\Phi}_1 \nu'_{aR} \\ + \bar{q}'_{iL} y_{i3}^{2d} \Phi_2 d'_{3R} + \bar{q}'_{iL} y_{i3}^{2u} \tilde{\Phi}_2 u'_{3R} + \bar{l}'_{iL} y_{i3}^{2e} \Phi_2 e'_{3R} + \bar{l}'_{iL} y_{i3}^{2\nu} \tilde{\Phi}_2 \nu'_{3R} + \text{h.c.} \quad (59)$$

So, the Yukawa matrices have the following structure

$$Y^{1f} = \begin{pmatrix} y_{11}^{1f} & y_{12}^{1f} & 0 \\ y_{21}^{1f} & y_{22}^{1f} & 0 \\ y_{31}^{1f} & y_{32}^{1f} & 0 \end{pmatrix}, \quad Y^{2f} = \begin{pmatrix} 0 & 0 & y_{13}^{2f} \\ 0 & 0 & y_{23}^{2f} \\ 0 & 0 & y_{33}^{2f} \end{pmatrix}. \quad (60)$$

To analyze the FCNC, it is convenient to rotate the scalar doublets to the Georgi basis where only one of the CP neutral even components of the doublets acquires VEV while the remaining ones are zero. Explicitly this corresponds to

$$\begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \rightarrow \mathcal{H}_\alpha = R_{\alpha\beta} \Phi_\beta. \quad (61)$$

In the unitary gauge

$$\mathcal{H}_1 = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}, \quad \mathcal{H}_2 = \begin{pmatrix} \mathcal{H}^+ \\ \frac{\mathcal{H}^0 + iA^0}{\sqrt{2}} \end{pmatrix}. \quad (62)$$

The Georgi basis should not be confused with the mass states of the scalar bosons. As discussed in [74], in this basis, the \mathcal{H}_1 boson gives mass to the SM fermions (the scalar singlet does not couple to SM fermions) and does not generate FCNC. Therefore, it is not convenient

to use the mass eigenstates, a mixture of the scalar singlet and the doublets, when studying the interactions between the scalar sector and the SM fermions. In most observables, the scalar boson is a virtual particle, and the boson that interacts with the SM fermions is the projection onto the subspace formed by the two doublets. In 2HDM, this feature is very useful since FCNC in the scalar sector can only be generated by \mathcal{H}_2 . Therefore, in this work, we focus on the CP-even neutral component of this doublet.

In terms of the new basis,

$$\begin{aligned} y_{ij}^{\text{of}} \Phi_\alpha &= x_{ij}^{\text{of}} \mathcal{H}_\alpha, \\ y_{ij}^{\text{of}} \tilde{\Phi}_\alpha &= x_{ij}^{\text{of}} \tilde{\mathcal{H}}_\alpha, \end{aligned} \quad (63)$$

the rotated yukawa couplings are

$$\begin{aligned} x_{ij}^{1f} &= \cos \beta \ y_{ij}^{1f} + \sin \beta \ y_{ij}^{2f}, \\ x_{ij}^{2f} &= -\sin \beta \ y_{ij}^{1f} + \cos \beta \ y_{ij}^{2f}. \end{aligned} \quad (64)$$

Thus

$$\begin{aligned} \mathcal{L}_Y &= -\bar{q}'_{iL} x_{ij}^{\text{od}} \mathcal{H}_\alpha d'_{jR} - \bar{q}'_{iL} x_{ij}^{\text{ou}} \tilde{\mathcal{H}}_\alpha u'_{jR} - \bar{l}'_{iL} x_{ij}^{\text{oe}} \mathcal{H}_\alpha e'_{jR} - \bar{l}'_{iL} x_{ij}^{\text{ov}} \tilde{\mathcal{H}}_\alpha \nu'_{jR} + h.c., \\ &= -\bar{q}'_{iL} x_{ij}^{1d} \mathcal{H}_1 d'_{jR} - \bar{q}'_{iL} x_{ij}^{2d} \mathcal{H}_2 d'_{jR} - \bar{q}'_{iL} x_{ij}^{1u} \tilde{\mathcal{H}}_1 u'_{jR} - \bar{q}'_{iL} x_{ij}^{2u} \tilde{\mathcal{H}}_2 u'_{jR} \\ &\quad - \bar{l}'_{iL} x_{ij}^{1e} \mathcal{H}_1 e'_{jR} - \bar{l}'_{iL} x_{ij}^{2e} \mathcal{H}_2 e'_{jR} - \bar{l}'_{iL} x_{ij}^{1\nu} \tilde{\mathcal{H}}_1 \nu'_{jR} - \bar{l}'_{iL} x_{ij}^{2\nu} \tilde{\mathcal{H}}_2 \nu'_{jR} + h.c., \\ &= -\frac{1}{\sqrt{2}}(v+h)(\bar{d}'_{iL} x_{ij}^{1d} d'_{jR} + \bar{u}'_{iL} x_{ij}^{1u} u'_{jR} + \bar{e}'_{iL} x_{ij}^{1e} e'_{jR} + \bar{\nu}'_{iL} x_{ij}^{1\nu} \nu'_{jR}) \\ &\quad - \frac{1}{\sqrt{2}}(\mathcal{H}^0 + i\mathcal{A}^0)(\bar{d}'_{iL} x_{ij}^{2d} d'_{jR} + \bar{e}'_{iL} x_{ij}^{2e} e'_{jR}) - \frac{1}{\sqrt{2}}(\mathcal{H}^0 - i\mathcal{A}^0)(\bar{u}'_{iL} x_{ij}^{2u} u'_{jR} + \bar{\nu}'_{iL} x_{ij}^{2\nu} \nu'_{jR}) \\ &\quad - \mathcal{H}^+(\bar{u}'_{iL} x_{ij}^{2d} d'_{jR} + \bar{\nu}'_{iL} x_{ij}^{2e} e'_{jR}) + \mathcal{H}^-(\bar{d}'_{iL} x_{ij}^{2u} u'_{jR} + \bar{e}'_{iL} x_{ij}^{2\nu} \nu'_{jR}) + h.c., \end{aligned}$$

where we have taken into account that $\tilde{\mathcal{H}}_2 = \left(\frac{\mathcal{H}^0 - i\mathcal{A}^0}{\sqrt{2}}, -\mathcal{H}^- \right)^T$. Next, we define the fermion mass eigenstates as:

$$f_L \equiv V_L^{f\dagger} f'_L, \quad f_R \equiv V_R^{f\dagger} f'_R, \quad (65)$$

with $V_{L(R)}^f$ are appropriated unitary matrices. In terms of the non-prime fields, we get

$$\begin{aligned} \mathcal{L}_Y &= -\frac{1}{\sqrt{2}}(v+h)(\bar{d}_{iL} z_{ij}^{1d} d_{jR} + \bar{e}_{iL} z_{ij}^{1e} e_{jR} + \bar{u}_{iL} z_{ij}^{1u} u_{jR} + \bar{\nu}_{iL} z_{ij}^{1\nu} \nu_{jR}) \\ &\quad - \frac{1}{\sqrt{2}}(\mathcal{H}^0 + i\mathcal{A}^0)(\bar{d}_{iL} z_{ij}^{2d} d_{jR} + \bar{e}_{iL} z_{ij}^{2e} e_{jR}) - \frac{1}{\sqrt{2}}(\mathcal{H}^0 - i\mathcal{A}^0)(\bar{u}_{iL} z_{ij}^{2u} u_{jR} + \bar{\nu}_{iL} z_{ij}^{2\nu} \nu_{jR}) \\ &\quad - \mathcal{H}^+(\bar{u}_{iL} z_{ij}^{2ud} d_{jR} + \bar{\nu}_{iL} z_{ij}^{2ve} e_{jR}) + \mathcal{H}^-(\bar{d}_{iL} z_{ij}^{2du} u_{jR} + \bar{e}_{iL} z_{ij}^{2ev} \nu_{jR}) + h.c., \end{aligned} \quad (66)$$

where

$$z^{\text{of}} \equiv V_L^{f\dagger} x^{\text{of}} V_R^f, \quad z^{2gf} \equiv V_L^{g\dagger} x^{2f} V_R^f. \quad (67)$$

In this way, from equation (64), we get

$$z^{1f} = \cos \beta (V_L^{f\dagger} y^{1f} V_R^f) + \sin \beta (V_L^{f\dagger} y^{2f} V_R^f), \quad (68)$$

$$z^{2f} = -\sin \beta (V_L^{f\dagger} y^{1f} V_R^f) + \cos \beta (V_L^{f\dagger} y^{2f} V_R^f), \quad (69)$$

$$z^{2gf} = -\sin \beta (V_L^{g\dagger} y^{1f} V_R^f) + \cos \beta (V_L^{g\dagger} y^{2f} V_R^f). \quad (70)$$

In the Georgi basis, only the CP-odd component of \mathcal{H}_1 acquires VEV, and therefore, the interaction of the SM fermions with this doublet generates the masses of quarks and leptons; consequently, in the mass eigenstates the matrix z^{1f} must be diagonal, i.e.

$$z^{1f} = \begin{pmatrix} \frac{\sqrt{2}m_1^f}{v} & 0 & 0 \\ 0 & \frac{\sqrt{2}m_2^f}{v} & 0 \\ 0 & 0 & \frac{\sqrt{2}m_3^f}{v} \end{pmatrix}, \quad (71)$$

where m_i^f corresponds to the mass of the fermion f_i . Because in our model, the right-handed quarks and leptons are singlets under the gauge group we can define the right-handed fermions as: $f_j'' = (V_R^f)_{ij} f_j$ for all f , such that V_R^f completely disappears from the Lagrangian. This transformation leaves all the terms invariant under the gauge group since the gauge singlets are of the form $\bar{f}_{Ri} f_i$ or $\bar{f}_{Ri} \partial_\mu f_i$ and V_R^f is a global transformation. We are not modifying the Yukawa interaction terms since we are only redefining them. In this way, the coupling of fermions to scalar bosons is

$$z^{\alpha f} \equiv V_L^{f\dagger} x^{\alpha f} V_R^f \rightarrow V_L^{f\dagger} x^{\alpha f}.$$

A consequence of this result is that if the Yukawa coupling of a scalar boson to one of the right-handed fermions is zero in the interaction space, i.e. $y_{ij}^{\alpha f} = 0$ for all j , then in mass eigenstates, the corresponding Yukawa coupling is also identically zero, i.e. $V_{ik}^f y_{kj}^{\alpha f} = 0$ for all j . That is, if the diagonalization matrix V_L^f of the left-handed fermions f is given by

$$V_L^f = \begin{pmatrix} v_{11}^f & v_{12}^f & v_{13}^f \\ v_{21}^f & v_{22}^f & v_{23}^f \\ v_{31}^f & v_{32}^f & v_{33}^f \end{pmatrix}, \quad (72)$$

the equation (60) implies that

$$V_L^{f\dagger} y^{1f} = \begin{pmatrix} v_{11}^* y_{11}^{1f} + v_{21}^* y_{21}^{1f} + v_{31}^* y_{31}^{1f} & v_{11}^* y_{12}^{1f} + v_{21}^* y_{22}^{1f} + v_{31}^* y_{32}^{1f} & 0 \\ v_{12}^* y_{11}^{1f} + v_{22}^* y_{21}^{1f} + v_{32}^* y_{31}^{1f} & v_{12}^* y_{12}^{1f} + v_{22}^* y_{22}^{1f} + v_{32}^* y_{32}^{1f} & 0 \\ v_{13}^* y_{11}^{1f} + v_{23}^* y_{21}^{1f} + v_{33}^* y_{31}^{1f} & v_{13}^* y_{12}^{1f} + v_{23}^* y_{22}^{1f} + v_{33}^* y_{32}^{1f} & 0 \end{pmatrix},$$

$$V_L^{f\dagger} y^{2f} = \begin{pmatrix} 0 & 0 & v_{11}^* y_{13}^{2f} + v_{21}^* y_{23}^{2f} + v_{31}^* y_{33}^{2f} \\ 0 & 0 & v_{12}^* y_{13}^{2f} + v_{22}^* y_{23}^{2f} + v_{32}^* y_{33}^{2f} \\ 0 & 0 & v_{13}^* y_{13}^{2f} + v_{23}^* y_{23}^{2f} + v_{33}^* y_{33}^{2f} \end{pmatrix}. \quad (73)$$

Since the coupling z^{1f} is diagonal, from the relation $z^{1f} = \cos \beta V_L^{f\dagger} y^{1f} + \sin \beta V_L^{f\dagger} y^{2f}$ and from the fact that if $(V_L^f y^{1f})_{ij} = 0$ then $(V_L^f y^{2f})_{ij} \neq 0$ and the opposite, it must be true that each of the contributions must be diagonal. From the expressions (73) we have

$$\begin{aligned} z_{ij}^{1f} &= z_i^{1f} \delta_{ij} = \cos \beta (V_L^f y^{1f})_{ij}, \text{ for any } i \text{ and } j = 1, 2. \\ z_{ij}^{1f} &= z_i^{1f} \delta_{ij} = \sin \beta (V_L^f y^{2f})_{ij}, \text{ for any } i \text{ and } j = 3. \end{aligned} \quad (74)$$

That is to say, $(V_L^f y^{1f})_{ij}$ and $(V_L^f y^{2f})_{ij}$ are diagonal matrix with $(V_L^f y^{1f})_{33} = 0$ and $(V_L^f y^{2f})_{11} = (V_L^f y^{2f})_{22} = 0$. From these results, the Yukawa couplings of \mathcal{H}_2 with the physical fermions turn out to be diagonal:

$$z^{2f} = -\sin \beta (V_L^f y^{1f})_{ij} + \cos \beta (V_L^f y^{2f})_{ij} = -\tan \beta z_i^{1f} \delta_{ij|i,j \in (1,2)} + \cot \beta z_i^{1f} \delta_{i3}.$$

In the last step we obtained the expressions for $(V_L^f y^{1f})_{ij}$ and $(V_L^f y^{2f})_{ij}$ from equation (74). In matrix form this result can be written as

$$z^{2f} = \begin{pmatrix} -\tan \beta z_1^{1f} & 0 & 0 \\ 0 & -\tan \beta z_2^{1f} & 0 \\ 0 & 0 & \cot \beta z_3^{1f} \end{pmatrix}. \quad (75)$$

This result is important because it shows that the coupling of the neutral scalars in the mass eigenstates of the SM fermions is diagonal. Therefore, our model does not present FCNC in the scalar sector. In most cases, the exact values of the matrix y_{ij}^{af} are entirely unknown, and what we know are the diagonal couplings, $z_i = \frac{\sqrt{2}}{v} m_i^f$, where m_i^f is a diagonal matrix whose elements correspond to the fermion masses in the SM so that the Yukawa couplings will be given by

$$y_{ij}^{1f} = V_{Lik}^{f\dagger} \frac{\sqrt{2}}{v \cos \beta} m_k^{1f} \delta_{kj}, \quad 0 = m_3^{1f} \text{ where .}$$

Here m^{1f} is a diagonal matrix with the first two eigenvalues equal to the masses of the particles in the SM and a third element equal to zero. We have a similar expression for the second term in equation (68)

$$y_{ij}^{2f} = V_{Lik}^{f\dagger} \frac{\sqrt{2}}{v \sin \beta} m_k^{2f} \delta_{kj}, \quad 0 = m_2^{2f} = m_1^{2f} \text{ where .}$$

Here m^{2f} is a diagonal matrix with the first two eigenvalues equal to zero and a third element equal to the corresponding mass in the SM. On the other hand, the Yukawa couplings inducing flavor-changing charged currents can be written as shown below:

$$\begin{aligned} z^{2ud} &= -\sin \beta [V_L^{u\dagger} V_L^d (V_L^{d\dagger} y^{1d} V_R^d)] + \cos \beta [V_L^{u\dagger} V_L^d (V_L^{d\dagger} y^{2d} V_R^d)], \\ z^{2du} &= -\sin \beta [V_L^{d\dagger} V_L^u (V_L^{u\dagger} y^{1u} V_R^u)] + \cos \beta [V_L^{d\dagger} V_L^u (V_L^{u\dagger} y^{2u} V_R^u)], \\ z^{2ve} &= -\sin \beta [V_L^{\nu\dagger} V_L^e (V_L^{e\dagger} y^{1e} V_R^e)] + \cos \beta [V_L^{\nu\dagger} V_L^e (V_L^{e\dagger} y^{2e} V_R^e)], \\ z^{2e\nu} &= -\sin \beta [V_L^{e\dagger} V_L^\nu (V_L^{\nu\dagger} y^{1\nu} V_R^\nu)] + \cos \beta [V_L^{e\dagger} V_L^\nu (V_L^{\nu\dagger} y^{1\nu} V_R^\nu)]. \end{aligned}$$

Remembering that $V_{\text{CKM}} = V_L^{u\dagger} V_L^d \equiv V$ and $V_{\text{PMNS}} = V_L^{e\dagger} V_L^\nu \equiv U$, it is easy to see from equation (69) that

$$z^{2ud} = Vz^{2d}, \quad z^{2du} = V^\dagger z^{2u}, \quad z^{2ve} = U^\dagger z^{2e}, \quad z^{2e\nu} = Uz^{2\nu}. \quad (76)$$

Appendix D. $Z-Z'$ mixing

The $U(1)'$ charge assignments of the Higgs fields Q_{Φ}' in a specific model, generate the $\theta_{Z-Z'}$ mixing [61, 75]

$$\theta_{Z-Z'} = C \frac{g'}{g_1} \left(\frac{m_Z}{m_{Z'}} \right)^2 = - \frac{\sum_i t_{3i} Q_{\Phi_i}' v_{\Phi_i}^2 g'}{\sum_i t_{3i}^2 v_i^2} \frac{g'}{g_1} \left(\frac{m_Z}{m_{Z'}} \right)^2, \quad (77)$$

where t_{3i} is the third component of weak isospin of Φ_i , g' and g_1 (~ 0.743) are the Z' and Z coupling strength constants, respectively and $m_{Z'}$ and m_Z its corresponding masses.

$$\begin{aligned} \theta_{Z-Z'} &= -2 \left(x - z + \frac{v_1^2 (y - x)}{v_{SM}^2} \right) \frac{1}{g_1} \left(\frac{m_Z}{m_{Z'}} \right)^2 \\ &= -2 \left(y - z + \frac{v_2^2 (x - y)}{v_{SM}^2} \right) \frac{1}{g_1} \left(\frac{m_Z}{m_{Z'}} \right)^2 \sim -\frac{2x}{g_1} \left(\frac{m_Z}{m_{Z'}} \right)^2, \end{aligned} \quad (78)$$

In the last step, we use the approximation $v_{SM} \gtrsim v_2^2 \gg v_1^2$ and $x \gg z$. By imposing the condition $|\theta_{Z-Z'}| < 10^{-3}$ which, roughly speaking, is the upper limit of the $\theta_{Z-Z'}$ mixing for the leptophobic model in reference [61]. We impose this bound for our model since the couplings to the leptons are proportional to the coupling z , which, by collider constraints is less than 10^{-2} for Z' masses below 2 TeV where the $Z-Z'$ mixing constraints are strong. Under these considerations we obtain $\frac{2|x|}{g_1} \left(\frac{m_Z}{m_{Z'}} \right)^2 < 10^{-3}$, which is an almost model-independent result [61]. This is equivalent to

$$|x| < \left(\frac{m_{Z'}}{4.68 \text{ TeV}} \right)^2. \quad (79)$$

These constraints are very restrictive for Z' masses below 2 TeV as shown in figure 1.

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References

- [1] Donoghue J F, Golowich E and Holstein B R 2014 Dynamics of the Standard Model *Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology* 2 edn (Cambridge University Press) (<https://doi.org/10.1017/CBO9780511524370>)
- [2] Nisati A and Sharma V 2017 *Discovery of the Higgs Boson* (World Scientific) (<https://doi.org/10.1142/8595>)
- [3] Workman R L *et al* 2022 Review of particle physics *PTEP* **2022** 083C01
- [4] Aad G *et al* 2016 Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV *J. High Energy Phys.* **JHEP08(2016)045**
- [5] Ma E 1990 Hierarchical radiative quark and lepton mass matrices *Phys. Rev. Lett.* **64** 2866–9
- [6] Langacker P 1981 Grand unified theories and proton decay *Phys. Rept.* **72** 185
- [7] Dine M, Fischler W and Srednicki M 1981 A simple solution to the strong cp problem with a harmless axion *Phys. Lett. B* **104** 199–202

- [8] Benavides R H, Forero D V, Muñoz L, Muñoz J M, Rico A and Tapia A 2023 Five texture zeros in the lepton sector and neutrino oscillations at DUNE *Phys. Rev. D* **107** 036008
- [9] Alonso R, Gavela M B, Isidori G and Maiani L 2013 Neutrino mixing and masses from a minimum principle *J. High Energy Phys.* **JHEP11(2013)187**
- [10] Mohapatra R N and Smirnov A Y 2006 Neutrino mass and new physics *Ann. Rev. Nucl. Part. Sci.* **56** 569–628
- [11] Perlmutter S 2000 Supernovae, dark energy, and the accelerating universe: The Status of the cosmological parameters *Int. J. Mod. Phys. A* **15S1** 715–39
- [12] Joyce A, Jain B, Khoury J and Trodden M 2015 Beyond the cosmological standard model *Phys. Rept.* **568** 1–98
- [13] Golowich E and Pal P B 1990 Charge quantization from anomalies *Phys. Rev. D* **41** 3537–40
- [14] Decamp D *et al* 1989 Determination of the number of light neutrino species *Phys. Lett. B* **231** 519–29
- [15] Ellis J 2012 Outstanding questions: physics beyond the standard model *Phil. Trans. R. Soc. Lond. A* **370** 818–30
- [16] Capozziello S and De Laurentis M 2011 Extended theories of gravity *Phys. Rept.* **509** 167–321
- [17] Abdalla E *et al* 2022 Cosmology intertwined: a review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies *JHEAp* **34** 49–211
- [18] Davidson A, Koca M and Wali K C 1979 $U(1)$ as the minimal horizontal gauge symmetry *Phys. Rev. Lett.* **43** 92
- [19] Marshak R E and Mohapatra R N 1980 Quark-Lepton symmetry and B–L as the $U(1)$ generator of the electroweak symmetry group *Phys. Lett. B* **91** 222–4
- [20] Ma E 2002 New $U(1)$ gauge symmetry of quarks and leptons *Mod. Phys. Lett. A* **17** 535–41
- [21] Barr S M and Dorsner I 2005 The origin of a peculiar extra U *Phys. Rev. D* **72** 015011
- [22] Adhikari R, Erler J and Ma E 2009 Seesaw neutrino mass and new $U(1)$ gauge symmetry *Phys. Lett. B* **672** 136–40
- [23] Okada N and Orikasa Y 2012 Dark matter in the classically conformal B–L model *Phys. Rev. D* **85** 115006
- [24] Langacker P and Plumacher M 2000 Flavor changing effects in theories with a heavy Z' boson with family nonuniversal couplings *Phys. Rev. D* **62** 013006
- [25] Langacker P 2009 The Physics of Heavy Z' Gauge Bosons *Rev. Mod. Phys.* **81** 1199–228
- [26] Crispim Romao M, King S F and Leontaris G K 2018 Non-universal Z' from fluxed GUTs *Phys. Lett. B* **782** 353–61
- [27] Antusch S, Hohl C, King S F and Susic V 2018 Non-universal Z' from $SO(10)$ GUTs with vector-like family and the origin of neutrino masses *Nucl. Phys. B* **934** 578–605
- [28] Pisano F and Pleitez V 1992 An $SU(3) \times U(1)$ model for electroweak interactions *Phys. Rev. D* **46** 410–7
- [29] Frampton P H 1992 Chiral dilepton model and the flavor question *Phys. Rev. Lett.* **69** 2889–91
- [30] Pleitez V 2021 Challenges for the 3-3-1 models *5th Colombian Meeting on High Energy Physics* p 12 hep-ph 2112.10888.
- [31] Benavides R H, Giraldo Y, Muñoz L, Ponce W A and Rojas E 2022 Systematic study of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ local gauge symmetry *J. Phys. G* **49** 105007
- [32] Suarez E, Benavides R H, Giraldo Y, Ponce W A and Rojas E 2024 Alternative 3-3-1 models with exotic electric charges *J. Phys. G* **51** 035004
- [33] Benavides R, Muñoz L A, Ponce W A, Rodríguez O and Rojas E 2017 Minimal nonuniversal electroweak extensions of the standard model: A chiral multiparameter solution *Phys. Rev. D* **95** 115018
- [34] Tang Y and Wu Y-L 2018 Flavor non-universal gauge interactions and anomalies in B-meson decays *Chin. Phys. C* **42** 033104
- [35] Maji P, Nayek P and Sahoo S 2019 Implication of family non-universal Z' model to rare exclusive $b \rightarrow s(\bar{l}l, \nu\bar{\nu})$ transitions *PTEP* **2019** 033B06
- [36] García-Duque C H, Muñoz J H, Quintero N and Rojas E 2021 Extra gauge bosons and lepton flavor universality violation in Υ and B meson decays *Phys. Rev. D* **103** 073003
- [37] Athron P, Martinez R and Sierra C 2024 B meson anomalies and large $B^+ \rightarrow K^+ \nu \bar{\nu}$ in non-universal $U(1)'$ models. *J. High Energy Phys.* **JHEP02(2024)121**
- [38] Kumar Alok A, Dighe A, Gangal S and Kumar J 2021 The role of non-universal Z couplings in explaining the Vus anomaly *Nucl. Phys. B* **971** 115538

- [39] Alvarado J S, Mantilla S F, Martinez R, Ochoa F and Sierra C 2023 Gauged nonuniversal U(1)_X model to study muon $g-2$ and B meson anomalies *Phys. Rev. D* **108** 095040
- [40] Frank M, Hiçyılmaz Y, Mondal S, Özdal Özer and Ün C S 2021 Electron and muon magnetic moments and implications for dark matter and model characterisation in non-universal U(1)' supersymmetric models *J. High Energy Phys.* **JHEP10(2021)063**
- [41] Greljo A, Stangl P, Eller Thomsen A and Zupan J 2022 On $(g-2)_\mu$ from gauged U(1)_X *J. High Energy Phys.* **JHEP07(2022)098**
- [42] Alok A K, Chundawat N R S and Kumar D 2022 Impact of $b \rightarrow s\ell\ell$ anomalies on rare charm decays in non-universal Z' models *Eur. Phys. J. C* **82** 30
- [43] Aram Hayrapetyan et al. 2024 Search for a neutral gauge boson with nonuniversal fermion couplings in vector boson fusion processes in proton-proton collisions at $\sqrt{s} = 13$ TeV. 12 2024.
- [44] Salvioni E, Strumia A, Villadoro G and Zwirner F 2010 Non-universal minimal Z' models: present bounds and early LHC reach *J. High Energy Phys.* **JHEP03(2010)010**
- [45] Branco G C, Emmanuel-Costa D and Gonzalez Felipe R 2000 Texture zeros and weak basis transformations *Phys. Lett. B* **477** 147–55
- [46] Gupta M and Ahuja G 2012 Flavor mixings and textures of the fermion mass matrices *Int. J. Mod. Phys. A* **27** 1230033
- [47] Verma R 2013 Minimal weak basis textures and quark mixing data *J. Phys. G* **40** 125003
- [48] Sharma S, Fakay P, Ahuja G and Gupta M 2015 Finding a unique texture for quark mass matrices *Phys. Rev. D* **91** 053004
- [49] Emmanuel-Costa D and González Felipe R 2017 More about unphysical zeroes in quark mass matrices *Phys. Lett. B* **764** 150–6
- [50] Benavides R H, Giraldo Y, Muñoz L, Ponce W A and Rojas E 2020 Five texture zeros for dirac neutrino mass matrices *J. Phys. G* **47** 115002
- [51] Navas S *et al* 2024 Review of particle physics *Phys. Rev. D* **110** 030001
- [52] Atwood D, Reina L and Soni A 1997 Phenomenology of two Higgs doublet models with flavor changing neutral currents *Phys. Rev. D* **55** 3156–76
- [53] Aoki M, Kanemura S, Tsumura K and Yagyu K 2009 Models of Yukawa interaction in the two Higgs doublet model, and their collider phenomenology *Phys. Rev. D* **80** 015017
- [54] Glashow S L and Weinberg S 1977 Natural conservation laws for neutral currents *Phys. Rev. D* **15** 1958
- [55] Erler J, Langacker P, Munir S and Rojas E 2011 Z' Bosons at Colliders: a Bayesian Viewpoint *J. High Energy Phys.* **JHEP11(2011)076**
- [56] Salazar C, Benavides R H, Ponce W A and Rojas E 2015 LHC Constraints on 3-3-1 Models *J. High Energy Phys.* **JHEP07(2015)096**
- [57] Rojas E and Erler J 2015 Alternative Z' bosons in E_6 *J. High Energy Phys.* **JHEP10(2015)063**
- [58] Benavides R H, Muñoz L, Ponce W A, Rodríguez O and Rojas E 2018 Electroweak couplings and LHC constraints on alternative Z' models in E_6 *Int. J. Mod. Phys. A* **33** 1850206
- [59] Aad G *et al* 2019 Search for high-mass dilepton resonances using 139 fb⁻¹ of pp collision data collected at $\sqrt{s}=13$ TeV with the ATLAS detector *Phys. Lett. B* **796** 68–87
- [60] Aaboud M *et al* 2018 Search for additional heavy neutral Higgs and gauge bosons in the ditau final state produced in 36 fb⁻¹ of pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector *J. High Energy Phys.* **JHEP01(2018)055**
- [61] Erler J, Langacker P, Munir S and Rojas E 2009 Improved constraints on Z-prime Bosons from electroweak precision data *J. High Energy Phys.* **JHEP08(2009)017**
- [62] Leike A 1994 Model independent Z' constraints at future e^+e^- colliders *Z. Phys. C* **62** 265–70
- [63] Erler J 1999 Global fits to electroweak data using GAPP *Physics at Run II: QCD and Weak Boson Physics Workshop: 2nd General Meeting* p 6
- [64] Sirunyan A M *et al* 2019 Search for a standard model-like Higgs boson in the mass range between 70 and 110 GeV in the diphoton final state in proton–proton collisions at $\sqrt{s}= 8$ and 13 TeV *Phys. Lett. B* **793** 320–47
- [65] Aad G *et al* 2023 Search for a light charged Higgs boson in $t \rightarrow H^\pm b$ decays, with $H^\pm \rightarrow cb$, in the lepton+jets final state in proton–proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector *J. High Energy Phys.* **JHEP09(2023)004**
- [66] Arhrib A, Krab M and Semlali S 2024 Accommodating the LHC charged Higgs boson excess at 130 GeV in the general two-Higgs doublet model *J. Phys. G* **51** 115003

- [67] Crivellin A, Fang Y, Fischer O, Bhattacharya S, Kumar M, Malwa E, Mellado B, Rapheeha N, Ruan X and Sha Q 2023 Accumulating evidence for the associated production of a new Higgs boson at the LHC *Phys. Rev. D* **108** 115031
- [68] Banik S, Crivellin A, Iguro S and Kitahara T 2023 Asymmetric di-Higgs signals of the next-to-minimal 2HDM with a $U(1)$ symmetry *Phys. Rev. D* **108** 075011
- [69] Sirunyan A M *et al* 2017 Measurements of properties of the Higgs boson decaying into the four-lepton final state in pp collisions at $\sqrt{s} = 13$ TeV *J. High Energy Phys.* JHEP11(2017)047
- [70] Armen Tumasyan *et al.* 2024 Search for a new resonance decaying into two spin-0 bosons in a final state with two photons and two bottom quarks in proton-proton collisions at $\sqrt{s} = 13$ TeV. *J. High Energy Phys.* JHEP05(2024)316
- [71] Crivellin A and Mellado B 2024 Anomalies in particle physics and their implications for physics beyond the standard model *Nat. Rev. Phys.* **6** 294–309
- [72] Aad G *et al* 2024 Search for a resonance decaying into a scalar particle and a Higgs boson in the final state with two bottom quarks and two photons in proton–proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector *J. High Energy Phys.* JHEP11(2024)047
- [73] Ponce W A, Giraldo Y and Sanchez L A 2003 Minimal scalar sector of 3-3-1 models without exotic electric charges *Phys. Rev. D* **67** 075001
- [74] Georgi H and Nanopoulos D V 1979 Suppression of flavor changing effects from neutral spinless meson exchange in gauge theories *Phys. Lett. B* **82** 95–6
- [75] Langacker P and Luo M 1992 Constraints on additional Z bosons *Phys. Rev. D* **45** 278–92

Alternative 3-3-1 models: a comprehensive analysis

In collaboration with W. A. Ponce, E. Rojas, R. H. Benavides and L. Muñoz

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Systematic study of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ local gauge symmetry

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Abstract

We review in a systematic way how anomaly free $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ models without exotic electric charges can be constructed, using as basis closed sets of fermions which includes each one the particles and antiparticles of all the electrically charged fields. Our analysis reproduces not only the known models in the literature, but also shows the existence of several more independent

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Abstract

- We review in a systematic way how anomaly free $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ models without exotic electric charges can be constructed, using as basis closed sets of fermions which includes each one the particles and antiparticles of all the electrically charged fields.
- Our analysis reproduces not only the known models in the literature, but also shows the existence of several more independent models for one and three families not considered so far.
- A phenomenological analysis of the new models is done, where the lowest limits at a 95 % CL on the gauge boson masses are presented.

Introduction

- The impressive success of the Standard Model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, has not been able enough to provide explanation for several fundamental issues.
- Minimal extensions of the SM arise either by adding new fields, or by enlarging the local gauge group (adding a right handed neutrino field constitute its simples extension). The electroweak gauge group is $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (3-3-1 for short) in which the electroweak sector of the standard model $SU(2)_L \otimes U(1)_Y$ is extended to $SU(3)_L \otimes U(1)_X$.
- Our analysis is to obtain the alternative embeddings for some of the well-known 3-3-1 models in the literature.

3-3-1 Models

Two classes of models will show up: universal one family models where the anomalies cancel in each family as in the SM, and family models where the anomalies cancel by an interplay between the several families.

For the 3-3-1 models, the most general electric charge operator in the extended electroweak sector is

$$Q = a\lambda_3 + \frac{1}{\sqrt{3}}b\lambda_8 + XI_3, \quad (1)$$

where λ_α , $\alpha = 1, 2, \dots, 8$ are the Gell-Mann matrices for $SU(3)_L$ normalized as $\text{Tr}(\lambda_\alpha\lambda_\beta) = 2\delta_{\alpha\beta}$ and $I_3 = Dg(1, 1, 1)$ is the diagonal 3×3 unit matrix. $a = 1/2$, the isospin $SU(2)_L$ of the SM is entirely embedded in $SU(3)_L$ and if one wishes to avoid exotic electric charges in the fermion and boson sectors as the ones present in the minimal (3-3-1) model, one must choose $b = 1/2$.

Models Without Exotic Electric Charges

- $S_1 = [(\nu_e^0, e^-, E_1^-) \oplus e^+ \oplus E_1^+]_L$ with quantum numbers $(1, 3, -2/3)$; $(1, 1, 1)$ and $(1, 1, 1)$ respectively.
- $S_2 = [(e^-, \nu_e^0, N_1^0) \oplus e^+]_L$ with quantum numbers $(1, 3^*, -1/3)$ and $(1, 1, 1)$ respectively.
- $S_3 = [(d, u, U) \oplus u^c \oplus d^c \oplus U^c]_L$ with quantum numbers $(3, 3^*, 1/3)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ and $(3^*, 1, -2/3)$ respectively.
- $S_4 = [(u, d, D) \oplus u^c \oplus d^c \oplus D^c]_L$ with quantum numbers $(3, 3, 0)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ and $(3^*, 1, 1/3)$ respectively.
- $S_5 = [(N_2^0, E_2^+, e^+) \oplus E_2^- \oplus e^-]_L$ with quantum numbers $(1, 3^*, 2/3)$, $(1, 1, -1)$, and $(1, 1, -1)$ respectively.
- $S_6 = [(E_3^+, N_3^0, N_4^0) \oplus E_3^-]_L$ with quantum numbers $(1, 3, 1/3)$ and $(1, 1, -1)$ respectively.
- $S_7 = [(e^-, \nu_e^0, N_1^0) \oplus (N_2^0, E^+, e^+) \oplus E^-]_L$ with quantum numbers $(1, 3^*, -1/3)$; $(1, 3^*, 2/3)$ and $(1, 1, -1)$ respectively.
- $S_8 = [(\nu_e^0, e^-, E^-) \oplus (E^+, N_1^0, N_2^0) \oplus e^+]_L$ with quantum numbers $(1, 3, -2/3)$, $(1, 3, 1/3)$ and $(1, 1, 1)$ respectively.
- $S_9 = [(e^-, \nu_e, N_1^0) \oplus (E^-, N_2^0, N_3^0) \oplus (N_4^0, E^+, e^+)]_L$ with quantum numbers $(1, 3^*, -1/3)$; $(1, 3^*, -1/3)$ and $(1, 3^*, 2/3)$ respectively.
- $S_{10} = [(\nu_e, e^-, E_1^-) \oplus (E_2^+, N_1^0, N_2^0) \oplus e^+ \oplus (N_3^0, E_2^-, E_3^-) \oplus E_1^+ \oplus E_3^+]_L$ with quantum numbers $(1, 3, -2/3)$; $(1, 3, 1/3)$; $(1, 1, 1)$; $(1, 3, -2/3)$; $(1, 1, 1)$, and $(1, 1, 1)$ respectively.
- $S_{11} = [(e^-, \nu_e, N_1^0) \oplus (N_2^0, E_1^+, e^+) \oplus (N_3^0, E_2^+, E_3^+) \oplus E_1^- \oplus E_2^- \oplus E_3^-]_L$ with quantum numbers $(1, 3^*, -1/3)$; $(1, 3^*, 2/3)$; $(1, 3^*, 2/3)$; $(1, 1, -1)$; $(1, 1, -1)$, and $(1, 1, -1)$ respectively.
- $S_{12} = [(\nu_e^0, e^-, E_1^-) \oplus (E_1^+, N_1^0, N_2^0) \oplus (E_2^+, N_3^0, N_4^0) \oplus e^+ \oplus E_2^-]_L$ with quantum numbers $(1, 3, -2/3)$; $(1, 3, 1/3)$; $(1, 3, 1/3)$; $(1, 1, 1)$, and $(1, 1, -1)$ respectively.

Irreducible anomaly free sets

i	Vector-like lepton sets (L_i)	One quark set (Q_i^I)	Two quark sets (Q_i^{II})	Three quark sets (Q_i^{III})
1	$S_1 + S_5$	$S_4 + S_9$	$S_1 + S_2 + S_3 + S_4$	$3S_2 + S_3 + 2S_4$
2	$S_2 + S_6$	$S_3 + S_{10}$	$2S_1 + S_3 + S_4 + S_7$	$3S_1 + 2S_3 + S_4$
3	$S_7 + S_8$	$S_2 + S_4 + S_7$	$2S_2 + S_3 + S_4 + S_8$	
4	$S_{10} + S_{11}$	$S_1 + S_3 + S_8$	$3S_2 + S_3 + S_4 + S_{12}$	
5	$S_9 + S_{12}$	$2S_1 + S_3 + S_6$	$3S_1 + 2S_3 + S_{12}$	
6	$S_1 + S_6 + S_7$	$2S_2 + S_4 + S_5$	$3S_2 + 2S_4 + S_{11}$	
7	$S_6 + S_8 + S_9$	$S_1 + S_4 + 2S_7$	$3S_1 + S_3 + S_4 + S_{11}$	
8	$S_2 + S_5 + S_8$	$S_2 + S_3 + 2S_8$		
9	$S_5 + S_7 + S_{10}$	$S_1 + S_2 + S_3 + S_{12}$		
10	$S_2 + S_7 + S_{12}$	$S_1 + S_2 + S_4 + S_{11}$		
11	$S_1 + S_8 + S_{11}$	$S_4 + 3S_7 + S_{10}$		
12	$S_1 + 2S_6 + S_9$	$S_3 + 3S_8 + S_9$		
13	$S_6 + 2S_7 + S_{10}$	$2S_1 + S_3 + S_7 + S_{12}$		
14	$S_5 + 2S_8 + S_9$	$2S_1 + S_4 + S_7 + S_{11}$		
15	$S_5 + S_6 + S_9 + S_{10}$	$2S_2 + S_3 + S_8 + S_{12}$		
16	$S_2 + 2S_5 + S_{10}$	$2S_2 + S_4 + S_8 + S_{11}$		
17	$S_1 + 2S_7 + S_{12}$	$3S_2 + S_3 + 2S_{12}$		
18	$S_1 + S_2 + S_{11} + S_{12}$	$3S_2 + S_4 + S_{11} + S_{12}$		
19	$S_2 + 2S_8 + S_{11}$	$3S_1 + S_3 + S_{11} + S_{12}$		
20	$2S_1 + S_6 + S_{11}$	$3S_1 + S_4 + 2S_{11}$		
21	$2S_2 + S_5 + S_{12}$			

IAFSs. Any general Anomaly Free-Set (AFS) containing quarks, must be a combination of IAFSs (i.e., L_i , Q_i^I , Q_i^{II} and Q_i^{III}) even for more than three families. For leptons, the second column (L) is not exhaustive and it was not possible to account for all the possibilities.

Collider Constraints

Model	j	SM Lepton Embeddings	Universal	2+1	Lepton Configuration	LHC-Lower limit (TeV)
A	-	$3S_2^{\ell+e^+}$	✓	×	$3C_2$	4.87
B	-	$3S_1^{\ell+e^+}$	✓	×	$3C_1$	5.53
C^j	1	$S_1^{\ell+e^+} + S_2^{\ell+e^+} + S_9^{\ell+e'^+}$	×	×	$C_1 + C_2 + C_3$	
	2	$(S_1^{\ell} + S_9^{e'^+}) + S_2^{\ell+e^+} + (S_9^{\bar{\ell}} + S_1^{e^+})$	×	✓	$2C_2 + C_4$	4.87
D^j	1	$S_1^{\ell+e^+} + S_2^{\ell+e^+} + S_{10}^{\ell+e^+}$	×	✓	$2C_1 + C_2$	5.53
	2	$S_1^{\ell+e^+} + S_{10}^{2\ell+2e^+}$	✓	×	$3C_1$	5.53

Alternative embeddings for the classical AFSs. The superscripts correspond to the particle content of the SM, where ℓ ($\bar{\ell}$) stands for a left-handed lepton doublet embedded in a $SU(3)_L$ triplet (anti-triplet), and e'^+ (e^+) is the right-handed charged lepton embedded in a $SU(3)_L$ triplet (singlet). The check mark ✓ means that at least two families (2+1) or three families (universal) have the same charges under the gauge symmetry, the cross × stands for the opposite. LHC constraints are obtained for embeddings for which we can choose the same Z' charges for the first two families, otherwise we leave the space blank.

Conclusions

The main conclusions of this work are:

- 1 Restricting ourselves to models without exotic electric charges, we have built 12 sets of particles S_i from triplets, antitriplets and singlets of $SU(3)_L \otimes U(1)_X$. These sets are constructed in such a way that they contain the charged particles and their respective antiparticles.
- 2 With these sets, we built the IAFSs L_i , Q_i^I , Q_i^{II} and Q_i^{III} depending on their quark content. From the IAFSs it is possible to systematically build 3-3-1 models. It is important to realize that if we restrict the AFSs to a minimum content of vector-like structures (i.e, L_i), having a lepton and quark sector consistent with the SM, our analysis is reduced to the AFSs that contain the classical 3-3-1 models.
- 3 If we allow alternative embeddings for SM particles within S_i , we get new phenomenological distinguishable model.
- 4 We found 1682 models which could be of phenomenological interest.
- 5 We can see that, independent of the model, the mass value of the new neutral gauge boson for all the 3-3-1 models without exotic electric charges is above 4.87 TeV.

THANK YOU!



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Carlos Yaguna
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On behalf of the Organizing Committee



Classification for Alternative 3-3-1 models with exotic electric charges

In collaboration with E. Suarez, W. A. Ponce, E. Rojas and R. H. Benavides

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Alternative 3-3-1 models with exotic electric charges

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(Dated: November 30, 2023)

We report the most general classification of 3-3-1 models with $\beta = \sqrt{3}$. We found several solutions where anomaly cancellation occurs among fermions of different families. These solutions are particularly interesting as they generate non-universal heavy neutral vector bosons. Non-universality in the SM fermion charges under an additional gauge group generates Charged Lepton Flavor Violation (CLFV) and Flavor Changing Neutral Currents (FCNC); we discuss under what conditions the new models can evade constraints coming from these processes. In Addition, we also report LHC constraints.

I. INTRODUCTION

Models with exotic fermions based on the gauge group symmetry $SU(3) \otimes SU(3) \otimes U(1)$ (hereafter 3-3-1 models for short) have been proposed since the early 1970s [1–11]; however, many of these models lacked important properties of what is known nowadays as 3-3-1 models. For a model to be interesting from a modern perspective [12], it must be chiral, the triangle anomalies must be canceled out only with a number of generations multiple of 3, and most importantly, it must contain the Standard Model (SM).

In the 1990s, non-universal models without exotic lep-

β does not exist in the literature, and therefore a work in this line is **necessary**. It is important to notice that there are solutions for arbitrary β [49]; however, this solution does not account for all the possible models for a given β . As we will see, the parameter β cannot be arbitrarily large, from the matching conditions $|\beta| \lesssim \cot \theta_W \sim 1.8$. This condition constitutes a very important restriction regarding the possible realizations of the 3-3-1 symmetry at low energies as it limits the number of possible non-trivial cases to a countable set.

In section II, we review the basics of the 3-3-1 models. In section III, we propose sets of fermions corresponding to families of quarks and leptons with the left-handed

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Abstract

- We present a comprehensive classification of 3-3-1 models with $\beta = \sqrt{3}$.
- We have identified multiple solutions where anomaly cancellation takes place across fermions from distinct families.
- These solutions are particularly intriguing as they give rise to non-universal heavy neutral vector bosons.
- Non-universality in the charges of Standard Model fermions under an additional gauge group leads to Charged Lepton Flavor Violation (CLFV) and Flavor Changing Neutral Currents (FCNC); we explore the conditions under which the new models can circumvent constraints arising from these processes.
- We provide an overview of constraints from the LHC.

Introduction

- Models with exotic fermions based on the gauge group symmetry $SU(3) \otimes SU(3) \otimes U(1)$ (hereafter 3-3-1 models for short) have been proposed since the early 1970s. These models must be chiral, the triangle anomalies must be canceled out only with a number of generations multiple of 3, and most importantly, it must contain the Standard Model (SM).
- In the 1990s, non-universal models without exotic leptons gained popularity as they were very convenient in addressing flavor problems
- Pleitez and Frampton proposed the non-universal 3-3-1 models as examples of electroweak extensions with lepton number violation, where the number of families is determined by anomaly cancellation. It has exotic electric charges in the quark sector and corresponds to what is known in the literature as $\beta = \sqrt{3}$. The parameter β cannot be arbitrarily large, from the matching conditions $|\beta| \lesssim \cot \theta_W \sim 1.8$.

3-3-1 Models

The most complete electric charge operator for this electroweak sector is

$$Q = \alpha T_{L3} + \beta T_{L8} + X\mathbf{1}, \quad (1)$$

assuming $\alpha = 1$, the $SU(2)_L$ isospin group of the SM is fully covered in $SU(3)_L$.

The parameter $\beta = \frac{2b}{\sqrt{3}}$ is a free parameter that defines the model (β is proportional to b present in the electric charge of the exotic vector boson K_μ). The X values are determined through anomaly cancellation. The 8 gauge fields A_μ^a of $SU(3)_L$ can be expressed as

$$\sum_a \lambda_a A_\mu^a = \sqrt{2} \begin{pmatrix} D_{1\mu}^0 & W_\mu^+ & K_\mu^{(b+1/2)} \\ W_\mu^- & D_{2\mu}^0 & K_\mu^{(b-1/2)} \\ K_\mu^{-(b+1/2)} & K_\mu^{-(b-1/2)} & D_{3\mu}^0 \end{pmatrix}, \quad (2)$$

$b = 3/2$ (i.e., $\beta = \sqrt{3}$).

3-3-1 Models

- Lepton generation $S_{L1} = [(\nu_e^0, e^-, E_2^{--}) \oplus e^+ \oplus E_2^{++}]_L$ with quantum numbers $(1, 3, -1)$; $(1, 1, 1)$ and $(1, 1, 2)$ respectively.
- Set $S_{L2} = [(e^-, \nu_e^0, E_1^+) \oplus e^+ \oplus E_1^-]_L$ with quantum numbers $(1, 3^*, 0)$; $(1, 1, 1)$ and $(1, 1, -1)$, respectively.
- Set $S_{L3} = [(e^-, \nu_e^0, e^+)]_L$ with quantum numbers $(1, 3^*, 0)$.
- Set $S_{Q1} = [(d, u, Q_1^{5/3}) \oplus u^c \oplus d^c \oplus Q_1^c]_L$ with quantum numbers $(3, 3^*, 2/3)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ and $(3^*, 1, -5/3)$, respectively.
- Set $S_{Q2} = [(u, d, Q_2^{-4/3}) \oplus u^c \oplus d^c \oplus Q_2^c]_L$ with quantum numbers $(3, 3, -1/3)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ and $(3^*, 1, 4/3)$, respectively.
- Triplets and anti-triplets of exotic leptons; for example,
 $S_{E1} = [(N_1^0, E_4^+, E_3^{++}) \oplus E_4^- \oplus E_3^{--}]_L$ with quantum numbers $(1, 3^*, 1)$; $(1, 1, -1)$ and $(1, 1, -2)$, respectively.
- $S_{E2} = [(E_5^+, N_2^0, E_6^-) \oplus E_5^- \oplus E_6^+]_L$ with quantum numbers $(1, 3, 0)$; $(1, 1, -1)$ and $(1, 1, 1)$, respectively.

Irreducible anomaly free sets

Contribution to the anomalies for each family of quarks S_{Q_i} , leptons S_{L_i} and exotics S_{E_i} , for 3-3-1 models with $\beta = \sqrt{3}$.

Anomalías	S_{L1}	S_{L2}	S_{L3}	S_{Q1}	S_{Q2}	S_{E1}	S_{E2}
$[SU(3)_C]^2 U(1)_X$	0	0	0	0	0	0	0
$[SU(3)_L]^2 U(1)_X$	-1	0	0	2	-1	1	0
$[\text{Grav}]^2 U(1)_X$	0	0	0	0	0	0	0
$[U(1)_X]^3$	6	0	0	-12	6	-6	0
$[SU(3)_L]^3$	1	-1	-1	-3	3	-1	1

AFSs for $\beta = \sqrt{3}$. We have classified the AFS according to the content of quark families, i.e., Q_i^I , Q_i^{II} , and Q_i^{III} . Combinations of these sets with three SM quark and three SM lepton families can be considered as 3-3-1 models.

i	Vector-like lepton set (L_i)	One quark set (Q_i^I)	Two quarks set (Q_i^{II})	Three quarks set (Q_i^{III})
1	$S_{E2} + S_{L2}$	$S_{E2} + 2S_{L1} + S_{Q1}$	$S_{L1} + S_{L2} + S_{Q1} + S_{Q2}$	$3S_{L1} + 2S_{Q1} + S_{Q2}$
2	$S_{E1} + S_{L1}$	$S_{E1} + 2S_{L2} + S_{Q2}$	$S_{L1} + S_{L3} + S_{Q1} + S_{Q2}$	$3S_{L2} + S_{Q1} + 2S_{Q2}$
3	$S_{E2} + S_{L3}$	$S_{E1} + S_{L2} + S_{L3} + S_{Q2}$		$3S_{L3} + S_{Q1} + 2S_{Q2}$
4		$S_{E1} + 2S_{L3} + S_{Q2}$		$2S_{L2} + S_{L3} + S_{Q1} + 2S_{Q2}$
5				$S_{L2} + 2S_{L3} + S_{Q1} + 2S_{Q2}$

Collider Constraints

Model	j	SM Lepton Embeddings	Universal	2 + 1	Quark Configuration	LHC-Lower limit
$M3 = Q_3^{III}$ (Minimal)	-	$[3S_{L3}^{\bar{\ell}+e'+}]$	✓	×	$2S_{Q2} + S_{Q1}$	6.4 TeV
$M4 = Q_4^{III}$	-	$[2S_{L2}^{\bar{\ell}+e'+} + S_{L3}^{\bar{\ell}+e'+}]$	×	✓	$2S_{Q2} + S_{Q1}$	6.4 TeV
$M6 = (Q_1^I + Q_1^{II})^j$	1	$[3S_{L1}^{\ell'+e'+}] + S_{L2} + S_{E2}$	✓	×	$2S_{Q1} + S_{Q2}$	SC
	2	$[2S_{L1}^{\ell'+e'+} + S_{L2}^{\ell'+e'+}] + S_{L1} + S_{E2}$	×	✓	$2S_{Q1} + S_{Q2}$	SC
$M17 = (Q_2^I + Q_3^I + Q_4^I)^j$	1	$[3S_{L2}^{\bar{\ell}+e'+}] + 3S_{L3} + 3S_{E1}$	✓	×	$3S_{Q2}$	SC
	2	$[3S_{L3}^{\bar{\ell}+e'+}] + 3S_{L2} + 3S_{E1}$	✓	×	$3S_{Q2}$	6.4 TeV
	3	$[2S_{L2}^{\bar{\ell}+e'+} + S_{L3}^{\bar{\ell}+e'+}] + S_{L2} + 2S_{L3} + 3S_{E1}$	×	✓	$3S_{Q2}$	6.4 TeV
	4	$[S_{L2}^{\bar{\ell}+e'+} + 2S_{L3}^{\bar{\ell}+e'+}] + 2S_{L2} + S_{L3} + 3S_{E1}$	×	✓	$3S_{Q2}$	6.4 TeV
$M10 = (Q_1^I + Q_2^{II})^j$	1	$[3S_{L1}^{\ell'+e'+}] + S_{L3} + S_{E2}$	✓	×	$2S_{Q1} + S_{Q2}$	SC
	2	$[2S_{L1}^{\ell'+e'+} + S_{L3}^{\ell'+e'+}] + S_{L1} + S_{E2}$	×	✓	$2S_{Q1} + S_{Q2}$	7.3 TeV

TABLE X: Alternative embeddings of the SM fields for some of the models in Table VIII. The lepton sets in square brackets (blue) contain the standard model fields. The superscripts correspond to the particle content of the SM, where ℓ ($\bar{\ell}$) stands for a left-handed lepton doublet embedded in a $SU(3)_L$ triplet (anti-triplet), and e'^+ (e^+) is the right-handed charged lepton embedded in a $SU(3)_L$ triplet (singlet). The check mark ✓ means that at least two (2+1) or three (universal) families have the same charges under the gauge symmetry. The cross × stands for the opposite. LHC constraints are obtained from Table IX for embeddings in which we can choose the same Z' charges for the first two families, otherwise, we leave the space blank. *To avoid a strongly coupled model in the Lepton sector, it is necessary to embed the first Lepton family (electron and electron neutrino) in S_{L3} . This feature will be helpful to distinguish between the different embeddings. The embedding also defines the content of exotic particles in each case.*

Conclusions

- ① Since that for 3-3-1 models, the absolute value of the parameter β must be less than $\beta \lesssim \cot \theta_W = 1.8$ (for $\sin^2 \theta_W = 0.231$ in the $\overline{\text{MS}}$ renormalization scheme at the Z -pole energy scale), and the values of β are further limited by the requirement that the vector boson charges be integers, the possible values of this parameter are reduced to a few cases.
- ② We have constructed three sets of lepton families, S_{Li} , two quark families, S_{Qi} , and two exotic lepton families S_{Ei} , and we calculated their contribution to anomalies. In our analysis, we obtained 14 irreducible AFSs, from which we built 33 non-trivial 3-3-1 models (without considering the different embeddings) with at least three quark and three lepton families for each case. Each of these embeddings constitutes a phenomenologically distinguishable model; however, we limited our analysis of the possible embeddings to a few cases.
- ③ In the same way, from our analysis of the 3-3-1 models with $\beta = \sqrt{3}$ we report the couplings of the SM fields to the Z' boson for all the possible quark and lepton families and the corresponding lower limits on the Z' mass.
- ④ We also discuss the conditions under which the reported models avoid FCNC and CLFV. We also observed that strongly coupled models appear naturally and require a high value for the Z' mass. They can be helpful in specific phenomenological approaches based on models with strong dynamics.

THANK YOU!



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Participated as a speaker with the talk “Classification for Alternative 3-3-1 models with exotic electric charges ”

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Carlos Eduardo Vera
Universidad del Tolima Colombia
On behalf of the Organizing Committee



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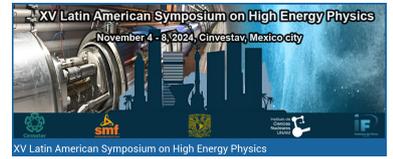
Exploring Higgs-like Resonant Signals within a 2HDM Framework

Yithsbey Giraldo

University of Nariño

5 noviembre 2024

Abstract



Universidad de Nariño

In this work, we explore Higgs-like resonant signals within the framework of Two-Higgs-Doublet Models (2HDM). We address the recent anomalies observed at the LHC and propose extensions to the Standard Model that could explain them, focusing on the inclusion of a scalar singlet σ . We analyze the resulting particle spectrum and how it aligns with experimental observations without the need for fine-tuning the model parameters.

Beyond the Standard Model Motivations

- Existence of neutrino mass.
- Evidence for dark matter.
- Observed matter-antimatter asymmetry of the universe.

This requires extending the Standard Model (SM) either in the scalar, gauge, or fermion sector.

Search for Higgs Bosons Beyond the SM at the LHC

- **ATLAS and CMS** have intensively searched for new Higgs bosons beyond the SM.
- Analyzing various decay channels and production processes to detect new scalar particles.
- **Decays into two photons ($\gamma\gamma$):** Excesses around 95 GeV and 151 GeV.
- **Decays into $Z\gamma$:** Observed in combinations of associated production channels.
- **Decays into pairs of bottom quarks ($b\bar{b}$):** Indicative in the excess around 95 GeV.
- **Decays into pairs of W^+W^- and ZZ bosons:** Suggest heavy scalar bosons.
- **Production of Higgs pairs (di-Higgs):** Especially in asymmetric processes like $H \rightarrow Sh$ or $H \rightarrow SS$.

Mass Range of Scalar Resonances

Around 95 GeV:

- Indications of a possible new scalar boson.
- Observed in channels like $\gamma\gamma$, $\tau^+\tau^-$, and $b\bar{b}$.
- Local and global significances in ATLAS and CMS.

Around 151 GeV:

- Significant deviations in $\gamma\gamma$ and $Z\gamma$ channels.
- Suggest the existence of a new scalar in this mass range.

Two-Higgs-Doublet Models (2HDM) and Extensions

2HDM Type II and Type IV:

- Can accommodate an additional scalar boson around 95 GeV.
- Compatible with excesses in $\gamma\gamma$ and $\tau^+\tau^-$.

N2HDM (Next-to-Minimal Two-Higgs-Doublet Model):

- Includes an additional doublet or scalar singlet.
- Allows interactions between multiple scalars.

S2HDM (2HDM Extended with a Singlet):

- Explains simultaneously the excesses in $\gamma\gamma$ and $b\bar{b}$ around 95 GeV.

Δ 2HDMs Model:

- Extension with two Higgs doublets, a real singlet, and a real triplet.
- Explains observed excesses in multiple channels.

The Proposed Model:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_\alpha \otimes U(1)_\beta$$

Fermion and Scalar Content:

	Fields	\hat{Y}	\hat{Q}
Leptons	ℓ_i, ν_{iR}, e_{iR}	$-1, 0, -2$	$(0, -1), 0, -1$
Quarks	q_i, u_{iR}, d_{iR}	$\frac{1}{3}, \frac{4}{3}, -\frac{2}{3}$	$(\frac{2}{3}, -\frac{1}{3}), \frac{2}{3}, -\frac{1}{3}$
Scalars	Φ_α, σ	$1, 0$	$(1, 0), 0$

Spontaneous Symmetry Breaking:

$$SU(2)_L \otimes U(1)_\alpha \otimes U(1)_\beta \xrightarrow{\langle \sigma \rangle} SU(2)_L \otimes U(1)_Y$$

$$\xrightarrow{\langle \Phi_1, \Phi_2 \rangle} U(1)_Q$$

Scalar Potential

- Scalar content:

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ v_i + h_i + im_i \end{pmatrix}, \quad \sigma = \frac{v_\sigma + \xi + i\zeta}{\sqrt{2}}$$

- VEV hierarchy: $v_\sigma > v_2 \gg v_1$
- General scalar potential:

$$V(\Phi_1, \Phi_2, \sigma) = \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 + \mu_\sigma^2 |\sigma|^2 + \lambda_1 |\Phi_1|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \lambda_{1\sigma} |\Phi_1|^2 |\sigma|^2 + \text{cubic or quartic terms in } \sigma$$

- Cubic term: $\mu[(\Phi_1^\dagger \Phi_2)\sigma + \text{h.c.}]$
- Quartic term: $\lambda[(\Phi_1^\dagger \Phi_2)\sigma^2 + \text{h.c.}]$

Mass Spectrum of the Neutral Scalar Sector

- After minimizing the potential, the mass matrix for neutral scalars is obtained.
- Two cases are considered: potential with cubic term and with quartic term.
- Mass hierarchy: $M_{H_1} < M_{H_2} < M_{H_3}$

Numerical Results (cubic term)

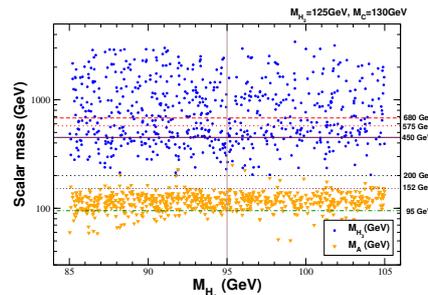


Figure: Distribution of the scalar masses M_{H_3} (blue dots) and the pseudoscalar M_A (orange triangles) for the potential with cubic term.

Mass Spectrum of the Neutral Pseudoscalar Sector

- Mass matrix for neutral pseudoscalars in both cases (cubic and quartic).
- The mass of the pseudoscalar A is:

$$M_A^2 = \begin{cases} \frac{\mu(v_1^2 v_2^2 + v^2 v_\sigma^2)}{\sqrt{2} v_1 v_2 v_\sigma} & (\text{cubic case}) \\ \frac{\lambda(4v_1^2 v_2^2 + v^2 v_\sigma^2)}{2v_1 v_2} & (\text{quartic case}) \end{cases}$$

Mass Spectrum of the Charged Scalar Sector

- Mass matrix for charged scalars in both cases.
- The mass of the charged Higgs C^\pm is:

$$M_{C^\pm}^2 = \begin{cases} \frac{v^2}{2} \left(\sqrt{2} \frac{\mu v_\sigma}{v_1 v_2} + \lambda_4 \right) & (\text{cubic case}) \\ \frac{v^2}{2} \left(\frac{\lambda v_\sigma^2}{v_1 v_2} + \lambda_4 \right) & (\text{quartic case}) \end{cases}$$

Conclusions

- Recent anomalies observed at the LHC can be explained through extensions of the SM, specifically 2HDMs.
- The inclusion of a scalar singlet σ that provides mass to a new boson Z' enhances the model's ability to explain these anomalies.
- The model predicts a particle spectrum consistent with experimental observations, including:
 - Three CP-even scalar bosons: H_1, H_2, H_3 .
 - One CP-odd scalar boson: A .
 - One charged scalar boson: C^\pm .
- The masses and parameters of the model naturally align with the observed anomalies without the need for fine-tuning.

Numerical Results (quartic term)

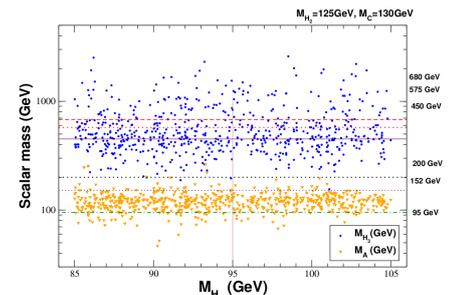


Figure: Distribution of the scalar masses M_{H_3} (blue dots) and the pseudoscalar M_A (orange triangles) for the potential with quartic term.

Work in progress, in collaboration with E. Rojas, W. Ponce, R. Benavides, and O. Rodríguez, based in part on arXiv:2408.16177 [hep-ph]

"The Standard Model of Particle Physics as an effective theory from two non-universal $U(1)$'s"

To whom it may concern (Universidad de Nariño),

It is my pleasure to acknowledge the participation at the XV Latin-American Symposium on High-Energy Physics (SILAFEA), which was held at the Cinvestav premises in November, 4-8, 2024, in Mexico City, Mexico, of the professors (delivering the presentation, specifying type and title, indicated):

- Germán Ramos Zambrano (poster, ‘Symplectic formulation for $Q.E.D._2$ on the null plane’)
- Juan Carlos Salazar Montenegro (poster, ‘A minimal axion model for mass matrices with five texture-zeros’)
- Yithsbey Giraldo Üsuga (poster, ‘Exploring Higgs-like Resonant Signals within a 2HDM Framework’)
- Edwin Delgado Insuasty (poster, ‘Possibility to observe Earth matter effects via non-standard neutrino properties and a robust analysis of supernova neutrino spectra’)
- Eduardo Rojas Peña (talk, ‘Alternative 3-3-1 models and Collider constraints’)

We are particularly grateful for your participation and involvement in the whole event, particularly in the planning of the future of our community. More information on the event can be checked at its indico <https://indico.nucleares.unam.mx/event/2125/>.

If any further information is needed, please do not hesitate to contact me through my emails pablo.roig@cinvestav.mx or paroig@gmail.com.



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Researcher (staff) at Phys. Dept. Cinvestav, Mexico City, Mexico
Chair of XV SILAFEA

12th AstroTwinCoLO School
Sterrewacht Leiden - Astronomía UdeA



Statement of participation for

Maderli Selena Toro

who attended the 12th AstroTwinCoLO School, a 20 h school on
Weak Gravitational Lensing Techniques

The school was held on 25-29 Nov 2024 at the Universidad de Antioquia campus (Medellín).

Germán Chaparro
Universidad de Antioquia (Colombia)
Coordinador - Pregrado de Astronomía
Instituto de Física



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