Families and Irreducible Anomaly-Free Sets for 3–3–3–1 Gauge Theories

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Based on the paper "Non-universal flipped trinification models with arbitrary β", *Eur. Phys. J. C* 85, 897 (2025)

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Diciembre 2025

Universidad de **Nariño**

5th Colombian Meeting on Flavor Physics

9–11 de diciembre de 2025
Universidad del Tolima, ibague

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article -Theoretical Physics

Non-universal flipped trinification models with arbitrary β

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Abstract We explore the recently proposed gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_X$, which naturally embeds both the Left-Right symmetric model and the 3-3-1 model as subgroups. Within this unified framework, we propose four families of leptons and quarks. A detailed analysis of their contributions to gauge anomaly cancellation is carried out for a general value of the parameter β. From this analysis, eight non-universal anomaly-free three-family models and four non-universal two-family anomaly free sets were identified. The three-family models offer realistic extensions of the Standard Model, retaining several appealing features of the 3-3-1 models, while the two-family sets provide flexibility for constructing models with even numbers of families. We also report LHC bounds on the Z' mass for the particular case $\beta = 1/\sqrt{3}$, considering all possible combi-

group; this class of models can explain the number of families of the Standard Model as 3-3-1 models do [10–14], and the origin of parity violation as it happens in the Left-Right models (LR) [3,15]. This class of models is also suitable for explaining dark-matter stability.

The left-right symmetric models are among the simplest extensions of the SM. This class of models represents an ideal theoretical framework for understanding the origin of parity violation in gauge theories, neutrino masses, and dark matter. Similarly, 3-3-1 models have been used for many years to explain neutrino masses and dark matter. Additionally, they can provide a relation between the number of families and the number of colors [12]. FT manages to integrate in a single model the advantages of the LR and the 3-3-1 models, cre-

Overview

Abstract

- We develop a model-independent framework to construct quark–lepton families in 3–3–3–1 gauge theories (flipped trinification),

$$G_{\text{FT}} = SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_X,$$

  for arbitrary electric-charge embedding.

- We introduce **Irreducible Anomaly-Free Sets (IAFS)**: minimal sets of fermion multiplets that cancel on their own all gauge and mixed anomalies

$$[SU(3)_{L,R}]^3, \ [SU(3)_{L,R}]^2 U(1)_X, \ [SU(3)_C]^2 U(1)_X, \ U(1)_X^3, \ \text{grav}^2 U(1)_X.$$

- Standard-Model (SM) families appear as unions of only a few IAFS, yielding general constraints that relate family replication to color and reproducing 3-3-1–like non-universality as a limiting case.

- For each IAFS we provide consistent charge assignments, a minimal scalar sector for sequential breaking, and renormalizable Yukawa interactions that generate realistic quark and lepton masses (including right-handed neutrinos).

- The classification has direct phenomenological meaning: it predicts patterns for extra neutral currents (Z', Z''), exotic fermion charges, possible residual discrete symmetries, and characteristic flavor textures.

Motivation and context

- The SM does not explain:
  - Why electric charge is quantized.
  - Why there are exactly three fermion families.

- Anomaly cancellation strongly constrains hypercharges but still allows a large model-building freedom.

- Flipped trinification,

$$G_{\text{FT}} = SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_X,$$

combines features of left–right and 3-3-1 models:
  - Parity can be broken dynamically.
  - Family replication is linked to the number of colors.
  - Extra neutral currents and exotic fermions are generic.

- **Goal of this work:** reorganize the fermion spectrum in terms of quark–lepton families and their *irreducible* anomaly-free combinations (IAFS).

Electric charge operator

- The electric charge is parametrized as

$$Q = T_{3L} + T_{3R} + \beta\left(T_{8L} + T_{8R}\right) + X,$$

  where β fixes the embedding of $U(1)_Q$ into $SU(3)_L \times SU(3)_R \times U(1)_X$.
- For a given β (or, equivalently, for a given charge q of the third leptonic component), all X charges of the multiplets are fixed.
- The SM hypercharge is recovered as

$$Y = T_{3R} + \beta\left(T_{8L} + T_{8R}\right) + X,$$

  which reproduces $Y = T_{3R} + \frac{1}{2}(B - L)$ at low energies.
- We work with a **single** value of β for the whole spectrum, so that the gauge adjoint remains unique.

Quark–lepton families: basic idea

- We define **families** that group quarks and leptons:
  - SM-like $SU(2)_L$ doublets for quarks and leptons.
  - Right-handed partners embedded in $SU(3)_R$.
  - Exotic states with non-standard electric charges required to complete $SU(3)$ multiplets.
- We construct:
  - Four lepton families S_{L1}–S_{L4}.
  - Four quark families S_{Q1}–S_{Q4}.
- All families are consistent with the gauge structure and depend on the choice of β.

Examples of family structures

- Schematic structure:

$$S_{L1} : (3,1) \oplus (1,3), \qquad S_{Q1} : (3,1) \oplus (1,3),$$

  with suitable color and $U(1)_X$ charges.
- Variants with $i = 2, 3, 4$ exchange $3 \leftrightarrow \bar{3}$ in some entries and add extra singlets.
- Each family contains:
  - One SM generation (quarks or leptons).
  - A set of exotic fermions (e.g. quarks with $Q = 5/3, -4/3$, multi-charged leptons, etc.).
- These families are the **building blocks** used to assemble anomaly-free spectra.

Irreducible anomaly-free sets

- For each family S_{Li} and S_{Qi} we compute contributions to:

$$[SU(3)_{L,R}]^3, \quad [SU(3)_{L,R}]^2 U(1)_X, \quad [SU(3)_C]^2 U(1)_X, \quad U(1)_X^3, \quad \text{grav}^2 U(1)_X.$$
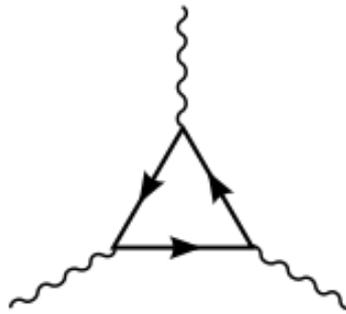
- An **IAFS** is a *minimal* combination of families that cancels all these anomalies on its own.

- A systematic search over all combinations yields:
  - Eight irreducible sets with three families.
  - Additional irreducible sets with two families, useful as building blocks for models with an even number of generations.

The eight three-family IAFS

- The three-family irreducible sets can be written as

$$
\begin{aligned}
M_1 &= 3S_{L4} + S_{Q1} + S_{Q2} + S_{Q3}, \\
M_2 &= 3S_{L4} + 2S_{Q1} + S_{Q4}, \\
M_3 &= 3S_{L3} + 2S_{Q2} + S_{Q3}, \\
M_4 &= 3S_{L3} + S_{Q1} + S_{Q2} + S_{Q4}, \\
M_5 &= 3S_{L2} + S_{Q2} + 2S_{Q3}, \\
M_6 &= 3S_{L2} + S_{Q1} + S_{Q3} + S_{Q4}, \\
M_7 &= 3S_{L1} + S_{Q2} + S_{Q3} + S_{Q4}, \\
M_8 &= 3S_{L1} + S_{Q1} + 2S_{Q4}.
\end{aligned}
$$

- Each M_i is anomaly-free and contains exactly three lepton and three quark families (including exotics).

- SM families then appear as unions of a *small* number of IAFS, which is crucial for model building.

Family replication and non-universality

- In all three-family IAFS:
  - The **leptonic sector is universal**: the three generations have the same gauge charges.
  - The **quark sector is non-universal**: at most two generations share the same $SU(3)_L$ embedding.

- This structure naturally reproduces the 3-3-1 relation between the number of families and colors and explains why extra neutral currents are non-universal.

- Non-universality generically induces tree-level flavor-changing neutral currents (FCNC) in the quark sector.

- Experimental data from K, D, and B mesons impose stringent constraints on allowed embeddings and mixing patterns.

Scalar sector and spontaneous breaking

- We introduce a minimal set of scalars transforming under $SU(3)_L \times SU(3)_R \times U(1)_X$ such that

$$G_{\text{FT}} \xrightarrow{\langle \Phi_{L,R} \rangle} SU(2)_L \times U(1)_Y \xrightarrow{\langle H \rangle} U(1)_Q.$$

- Typical ingredients:
  - Bi-triplets $(3, \bar{3})$ connecting left and right sectors.
  - $SU(3)_L$ and $SU(3)_R$ triplets that reduce the rank and give large masses to exotics.

- Vacuum-alignment conditions:
  - Leave a light SM-like Higgs doublet.
  - Generate multi-TeV masses for exotic fermions and for the extra neutral bosons Z' and Z''.
  - Can preserve residual discrete symmetries, stabilizing potential dark-matter candidates.

Yukawa sector and fermion masses

- For each IAFS we write the renormalizable Yukawa Lagrangian compatible with the chosen scalar content:

$$\mathcal{L}_Y = -\,\overline{Q_L}\,Y_q\,\Phi\,Q_R - \overline{L_L}\,Y_\ell\,\Phi\,L_R + \text{h.c.} \quad + \text{ terms with triplets and singlets.}$$

- After spontaneous symmetry breaking:
  - SM quarks and charged leptons acquire realistic hierarchical mass matrices.
  - Right-handed neutrinos allow for Dirac/Majorana terms and seesaw mechanisms.
  - Exotic fermions obtain large masses and small mixings with SM states, keeping them under control phenomenologically.

- The modular language in terms of IAFS lets us modify the family content without spoiling anomaly cancellation or Yukawa textures.

Extra neutral bosons and collider bounds

- The extended gauge group predicts two extra neutral gauge bosons, Z' and Z''.

- Their couplings to SM fermions depend on:
    - The embedding parameter β.
    - The choice of IAFS (how families sit in $SU(3)_L$ and $SU(3)_R$).
    - A mixing angle θ that diagonalizes the neutral gauge-boson mass matrix.

- We analyze LHC bounds on $M_{Z'}$ in a reference case (e.g. $\beta = -1/\sqrt{3}$), scanning over all possible quark–lepton family assignments.

- Bounds are highly sensitive to θ, since it controls the couplings to light quarks and leptons.

Flavor constraints and textures

- Non-universal neutral currents induce:
  - Neutral-meson mixing: K^0, D^0, B_d, B_s.
  - Rare decays and other FCNC processes.
- The IAFS classification helps identify which embeddings are compatible with these stringent flavor constraints.
- Typical predictions:
  - Specific patterns of flavor-violating Z' couplings.
  - Texture zeros or hierarchical structures in Yukawa matrices.
  - Possible links between flavor and dark-matter sectors via residual discrete symmetries.

Conclusions

- We presented a **modular** framework to construct flipped-trinification models from:
  - Basic quark and lepton families S_{Li}, S_{Qi}.
  - Their *irreducible* anomaly-free combinations (IAFS).

- The classification clarifies:
  - How the three SM families emerge from a small number of IAFS.
  - Why leptons tend to be universal while quarks are typically non-universal.
  - Which embeddings remain compatible with present collider and flavor data.

- The approach provides a flexible toolkit to:
  - Design models with well-defined dark-matter candidates.
  - Explore flavor textures and sources of CP violation.
  - Connect flipped trinification to grand-unified scenarios.

Outlook

- Extend the IAFS classification to:
  - Different choices of β.
  - Variants with additional discrete symmetries.

- Detailed collider studies of Z', Z'' and exotic fermion production, including future hadron and lepton colliders.

- Systematic exploration of dark-matter candidates from residual symmetries in the scalar and fermion sectors.

- Embedding flipped trinification and its IAFS structure into explicit GUT constructions.