Axions, neutrinos, and Higgs-like resonances, a fuzzy combination.

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Outline

The $U(1)_A$ problem

- In the limit whre the SM fermions are massless the QCD lagrangian has the symmetry $U(N)_V \otimes U(N)_A$, for the first family $N = 2$. $U(2)_V = SU(2)_V \otimes U(1)_V = SU(2)_I$ (isospin symmetry) $\otimes U(1)_B$ (Baryon number conservation).

The $U(1)_A$ problem

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- The global $U(2)_A = SU(2)_A \otimes U(1)_A$ symmetry is broken spontaneously by the quark condensate, thus, one expects 4 Nambu-Goldstone bosons $(\pi^0, \pi^-, \pi^+, \eta(?))$, but η is too heavy. Although pions are light, there is no clue of another light state in the hadronic spectrum. Weinberg dubbed this the $U(1)_A$ problem, suggesting that, somehow, there was no $U(1)_A$ symmetry in QCD.

The Strong CP problem

1. 'T Hooft realized that the current associated with the $U(1)_A$ symmetry is anomalous, i.e., $\partial_\mu J_5^\mu = \frac{g^2 N}{32\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}$ where N is the number of massless quarks. From this it is possible to add to the lagrangian the CP violating term:

$$\mathcal{L} = \theta \frac{g^2 N}{32\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu} \propto \theta \frac{g^2 N}{32\pi^2} \vec{E}\cdot\vec{B} \ , \tag{1}$$

Under parity $\vec{E} \to -\vec{E}$ and $\vec{B} \to \vec{B}$ (polar vector). Under charge conjugation $\vec{E}, \vec{B} \to -\vec{E}, -\vec{B}$, so that this term is not CP invariant. This interaction produces an electric dipole moment for the neutron $d_n = e(m_q/m_n)\theta \approx 10^{-16}\theta e$-cm. The current bound from PSI collaboration set $d_n < 2.9 \times 10^{-26} e$-cm. Why θ is so small?, this is the strong CP problem.

Paccei-Quinn Solution

- Peccei and Quinn suggested that the SM has an additional $U(1)_{PC}$ chiral (global) symmetry which drives $\theta \to 0$. This global $U(1)_{PQ}$ symmetry was named after Roberto Peccei and Helen Quinn.

What is our work about?

Our aim is to use the $U(1)_{PQ}$ symmetry to generate quark textures motivated from the data. In particular, we are interested in the hermitian textures

$$\begin{aligned} M^U &= \begin{pmatrix} 0 & 0 & |C_u|e^{i\phi_{C_u}} \\ 0 & A_u & |B_u|e^{i\phi_{B_u}} \\ |C_u|e^{-i\phi_{C_u}} & |B_u|e^{-i\phi_{B_u}} & D_u \end{pmatrix}, \\ M^D &= \begin{pmatrix} 0 & |C_d| & 0 \\ |C_d| & 0 & |B_d| \\ 0 & |B_d| & A_d \end{pmatrix}, \end{aligned} \tag{2}$$

The PQ charges of the SM fermions are:

Particles	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$Q_{\mathsf{PQ}}(i=1)$	$Q_{\mathsf{PQ}}(i=2)$	$Q_{\mathsf{PQ}}(i=3)$	$U(1)_{\mathsf{PQ}}$
q_{Li}	1/2	3	2	1/6	$-2s_1+2s_2+\alpha$	$-s_1+s_2+\alpha$	α	x_{q_i}
u_{Ri}	1/2	3	1	2/3	$s_1+\alpha$	$s_2+\alpha$	$-s_1+2s_2+\alpha$	x_{u_i}
d_{Ri}	1/2	3	1	-1/3	$2s_1-3s_2+\alpha$	$s_1-2s_2+\alpha$	$-s_2+\alpha$	x_{d_i}
ℓ_{Li}	1/2	1	2	$-1/2$	$\frac{x_{Q_R}-x_{Q_L}}{2}+s_1-2s_2$	$\frac{x_{Q_R}-x_{Q_L}}{2}+s_1-2s_2$	$\frac{x_{Q_R}-x_{Q_L}}{2}+s_1-2s_2$	x_{ℓ_i}
e_{Ri}	1/2	1	1	-1	$\frac{x_{Q_R}-x_{Q_L}}{2}+2s_1-4s_2$	$\frac{x_{Q_R}-x_{Q_L}}{2}+2s_1-4s_2$	$\frac{x_{Q_R}-x_{Q_L}}{2}+2s_1-4s_2$	x_{e_i}
ν_{Ri}	1/2	1	1	0	$\frac{x_{Q_R}-x_{Q_L}}{2}$	$\frac{x_{Q_R}-x_{Q_L}}{2}$	$\frac{x_{Q_R}-x_{Q_L}}{2}$	x_{ν_i}

Table: The columns 6-8 are the PQ (Q_{PQ}) charges for the SM quarks in each family. The subindex $i=1,2,3$ stands for the family number in the interaction basis. The parameters s_1, s_2 and α are reals with $s_1 \neq s_2$.

In order to generate these matrices at least 4 higgs doublets are needed.

Particles	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Q_{PQ}	$U(1)_{\text{PQ}}$
Φ_1	0	1	2	1/2	s_1	x_{ϕ_1}
Φ_2	0	1	2	1/2	s_2	x_{ϕ_2}
Φ_3	0	1	2	1/2	$-s_1+2s_2$	x_{ϕ_3}
Φ_4	0	1	2	1/2	$-3s_1+4s_2$	x_{ϕ_4}
Q_L	1/2	3	1	0	x_{Q_L}	x_{Q_L}
Q_R	1/2	3	1	0	x_{Q_R}	x_{Q_R}
S_1	0	1	1	0	s_1-s_2	x_{s_1}
S_2	0	1	1	0	$x_{Q_R}-x_{Q_L}$	x_{s_2}

Table: Beyond SM scalar and fermion fields and their respective PQ charges. The parameters s_1, s_2 are reals, with $s_1 \neq s_2$.

conveniently choosing the VEVvenient way it is possible to reproduce the quark masses and the CKM mixing matrix and Yukawa couplings of order 1.

$$Y_{ij}^{U,D} \sim 1. \tag{3}$$

So, by setting various Yukawa (for quarks) couplings close to 1 (except y_{23}^{U2}, y_{23}^{D3} and y_{13}^{U1}) we obtain:

$$\hat{v}_1 = 1.71\,\text{GeV}, \quad \hat{v}_2 = 2.91\,\text{GeV}, \quad \hat{v}_3 = 174.085\,\text{GeV}, \quad \hat{v}_4 = 13.3\,\text{MeV}. \tag{4}$$

High energy Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathsf{LO}} \supset\ & (D_\mu \Phi^\alpha)^\dagger D^\mu \Phi^\alpha + \sum_\psi i\bar{\psi}\gamma^\mu D_\mu \psi + \sum_{i=1}^{2} (D_\mu S_i)^\dagger D^\mu S_i \\
& - \Big(\bar{q}_{Li} y_{ij}^{D\alpha} \Phi^\alpha d_{Rj} + \bar{q}_{Li} y_{ij}^{U\alpha} \tilde{\Phi}^\alpha u_{Rj} \\
& + \bar{\ell}_{Li} y_{ij}^{E} \Phi_3 e_{Rj} + \bar{\ell}_{Li} y_{ij}^{\nu} \tilde{\Phi}_3 \nu_{Rj} + \frac{1}{2} Y_{ij}^{N} \overline{\tilde{\nu}_{Ri}^{c}} \tilde{\nu}_{Rj} S_2 + \mathsf{h.c} \Big) \\
& + (\lambda_Q \bar{Q}_R Q_L S_2 + \mathsf{h.c}) - V(\Phi, S_1, S_2)\,.
\end{aligned} \tag{5}
$$

In our model, the Yukawa interaction term generates the neutrino mass matrices

$$\mathcal{L} \supset y_{ij}^{\nu} \frac{v_{\Phi_3}}{\sqrt{2}} \bar{\nu}_{Li} \nu_{Rj} + \frac{1}{2} Y_{ij}^{N} \overline{\nu_{Ri}^{c}} \nu_{Rj} \frac{v_{S2}}{\sqrt{2}} + \text{h.c.} \ , \tag{6}$$

in such a way that the left-handed neutrinos mass matrix is

$$m_{ij}^{\nu} = -\left(m_D m_N^{-1} m_D\right)_{ij} = -\frac{v_{\text{SM}}^2}{\sqrt{2} v_{s_2}} \left(Y^{\nu} Y_N^{-1} Y^{\nu}\right)_{ij} \tag{7}$$

where $m_{ij}^{N} = \frac{1}{\sqrt{2}} Y_{ij}^{N} v_{S_2}$, the Dirac neutrino mass matrix is $m_{ij}^{D} = Y_{ij}^{\nu} v_{\text{SM}}/\sqrt{2}$, with $v_{\Phi_3} \sim v_{\text{SM}} \sim 246$GeV.

At the same time, as discussed in the previous section, the axion decay constant is given by $f_a \simeq v_{S_2} x_{S_2}$. From $\boxed{f_a^2 = \sum_{i=1}^{n} x_i^2 v_i^2}$ the axion mass $m_a \propto f_a^{-1} \simeq (x_{S_2} v_{S_2})^{-1}$, in such a way that the neutrino mass matrix is proportional to the axion mass, i.e.,

$$m^{\nu}_{ij} = -x_{S_2} \left(Y^{\nu} Y_N^{-1} Y^{\nu} \right)_{ij} \times 7.5 \times 10^6 m_a \ .$$

in the standard model $(Y^{\nu})^2$ ranges from $\sim 10^{-11}$ for the electron to 1 for the top quark, which implies a wide range for the axion masses. The neutrino mass matrix must be less than the cosmological bound (i.e., $\sum_i m_{\nu_i} \leq 0.12\text{eV}$), which implies

$$|x_{S_2} \text{Tr} \left(Y^{\nu} Y_N^{-1} Y^{\nu} \right) \times 10^6 m_a| \leq 0.12\text{eV} \qquad (8)$$

On the other hand, the trace of the neutrino mass determinant is the product of the three neutrino masses, i.e., $m_1^\nu m_2^\nu m_3^\nu = -\frac{(\det Y^\nu)^2}{\det Y^N}\left(x_{S_2} 7.5 \times 10^6 m_a\right)^3$. Combining the neutrino oscillation data and the cosmological upper bound on the neutrino masses it is posible to bound the neutrino mass determinant $\det(m^\nu) = m_1^\nu m_2^\nu m_3^\nu$,

$$0 \leq |m_1^\nu m_2^\nu m_3^\nu| \leq \begin{cases} (0.038\text{eV})^3, & \text{N.O.} \\ (0.034\text{eV})^3, & \text{I.O.} \end{cases}, \tag{9}$$

which implies

$$\left| x_{S_2} \frac{(\det Y^\nu)^{2/3}}{(\det Y^N)^{1/3}} \right| m_a \leq \begin{cases} 5.1 \times 10^{-9}\text{eV}, & \text{N.O.} \\ 4.5 \times 10^{-9}\text{eV}, & \text{I.O.} \end{cases} \tag{10}$$

Scalar Potential

The scalar potential compatible with the symmetries includes quadratic, quartic, and specific trilinear interactions:

$$
\begin{aligned}
V(\Phi_i, S_j) \quad = \quad & \sum_{i=1}^{4} \mu_i^2 \Phi_i^\dagger \Phi_i + \sum_{k=1}^{2} \mu_{s_k}^2 S_k^* S_k + \sum_{i=1}^{4} \lambda_i \left(\Phi_i^\dagger \Phi_i\right)^2 \\
+ \quad & \sum_{k=1}^{2} \lambda_{s_k} \left(S_k^* S_k\right)^2 + \sum_{i=1}^{4} \sum_{k=1}^{2} \lambda_{i s_k} \left(\Phi_i^\dagger \Phi_i\right) \left(S_k^* S_k\right) \\
+ \quad & \underbrace{\sum_{i,\, j\, =\, 1}^{4}}_{i<j} \left(\lambda_{ij} \left(\Phi_i^\dagger \Phi_i\right) \left(\Phi_j^\dagger \Phi_j\right) + J_{ij} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_j^\dagger \Phi_i\right)\right) \\
+ \quad & \lambda_{s_1 s_2} \left(S_1^* S_1\right) \left(S_2^* S_2\right) + K_1 \left(\left(\Phi_1^\dagger \Phi_2\right) \left(\Phi_3^\dagger \Phi_2\right) + h.c.\right) \\
+ \quad & K_2 \left(\left(\Phi_3^\dagger \Phi_4\right) \left(\Phi_3^\dagger \Phi_1\right) + h.c.\right) \\
+ \quad & K_3 \left(\left(\Phi_3^\dagger \Phi_4\right) S_1^2 + h.c.\right) + K_4 \left(\left(\Phi_1^\dagger \Phi_3\right) S_1^2 + h.c.\right) \\
+ \quad & F_1 \left(\left(\Phi_2^\dagger \Phi_3\right) S_1 + h.c.\right) + F_2 \left(\left(\Phi_1^\dagger \Phi_2\right) S_1 + h.c.\right) \\
+ \quad & \frac{1}{2} \left(m_{\zeta_{S_2}}\right)^2_{\mathsf{SB}} \zeta_{S_2}^2 + \frac{1}{2} \left(m_{\xi_{S_2}}\right)^2_{\mathsf{SB}} \xi_{S_2}^2 \; .
\end{aligned}
\tag{11}
$$

To preserve naturalness, all dimensionless parameters in the potential (11) were chosen in the range:

$$\lambda_i, \lambda_{s_k}, \lambda_{is_k}, \lambda_{ij}, J_{ij}, \lambda_{s_1 s_2}, K_i \in [-1.5, 1.5] \ . \tag{12}$$

Despite being dimensionless, K_3 and K_4 are subject to stronger constraints, leading to narrower intervals. Similar considerations apply to F_1 and F_2:

$$\begin{aligned} F_1 &\in [-10, 10] \text{ GeV} \ , \\ F_2 &\in [-160, 160] \text{ GeV} \ , \\ K_3 &\in [-1.5, 0.5] \ , \\ K_4 &\in [-1.5, 0.1] \ . \end{aligned} \tag{13}$$

distribution of scalar mass spectrum

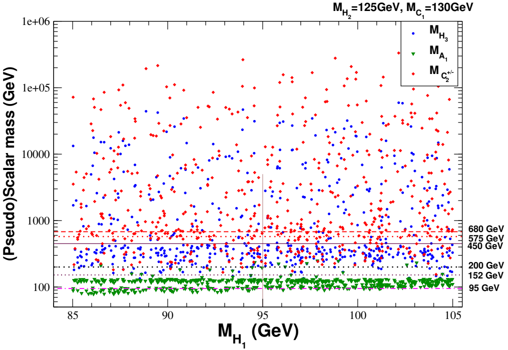

Figure: To connect with current phenomenology, we set $M_{H_1} = 95$ GeV, $M_{H_2} = 125$ GeV, $M_{H_1^\pm} = 130$ GeV. Heavier scalars span the ranges $M_{H_3} \in [200\text{ GeV}, 50\text{ TeV}]$, $M_{C_2^\pm} \in [200\text{ GeV}, 300\text{ TeV}]$, $M_{A_1} \in [95\text{ GeV}, 200\text{ GeV}]$. Higher states are taken above the current LHC reach.

It is possible to obtain dimension five effective lagrangians by means of the non-linear transformation

$$\begin{aligned}
S^i &\longrightarrow e^{i\frac{x_{S^i}}{\Lambda}a}S^i,\\
\Phi^\alpha &\longrightarrow e^{i\frac{x_{\Phi^\alpha}}{\Lambda}a}\Phi^\alpha,\\
\psi_L &\longrightarrow e^{i\frac{x_{\psi_L}}{\Lambda}a}\psi_L,\\
\psi_R &\longrightarrow e^{i\frac{x_{\psi_R}}{\Lambda}a}\psi_R,
\end{aligned} \tag{14}$$

Because the axial symmetry is anomalous, the path-integral measure is not invariant:

$$\mathcal{D}\bar{\psi}\mathcal{D}\psi \longrightarrow \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\frac{i}{16\pi^2}\int d^4x\frac{a(x)}{f_a}\left(N_f g_s^2 G^a_{\mu\nu}\tilde{G}^{a\mu\nu}+\cdots\right)$$

Under parity $G_{0i}\to -G_{0i}$ and $\tilde{G}_{ij}\to\tilde{G}_{ij}$, such that $G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$ is proportional to the contraction of the "electric" like fields with the magnetic ones, i.e., $B\cdot E$ both invariant under charge conjugation, hence $G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$ is CP violiting.

Effective quark-axion interaction vertex.

from $\mathcal{L}_{K^\Psi}$ we obtain the flavour-violating derivative couplings:

$$\Delta\mathcal{L}_{K^D} = -\,\partial_\mu a \bar{d}_i \gamma^\mu \left(g^V_{af_if_j} + \gamma^5 g^A_{af_if_j} \right) d_j, \tag{15}$$

where;

$$g^{V,A}_{ad_id_j} = \frac{1}{2 f_a c_3^{\text{eff}}} \Delta^{Dij}_{V,A}, \tag{16}$$

In this expression we made the substitution $\Lambda = f_a c_3^{\text{eff}}$. The axial and vector couplings are:

$$\Delta^{Dij}_{V,A} = \Delta^{Dij}_{RR}(d) \pm \Delta^{Dij}_{LL}(q), \tag{17}$$

with $\Delta^{Fij}_{LL}(q) = \left(U^D_L \; x_q \; U^{D\dagger}_L \right)^{ij}$ and $\Delta^{Fij}_{RR}(d) = \left(U^D_R \; x_d \; U^{D\dagger}_R \right)^{ij}_{.}$

Constraints from Semileptonic decays

it is shown that the decay widths of pseudoscalar $K^{\pm}$(B) mesons into an axion and a charged pion (vector K^*) are given by

$$\Gamma(K^{\pm}\to\pi^{\pm}a) = \frac{m_K^3}{16\pi}\left(1-\frac{m_\pi^2}{m_K^2}\right)^2\lambda_{K\pi a}^{1/2}f_0^2(m_a^2)|g_{ads}^V|^2,$$
$$\Gamma(B\to K^*a) = \frac{m_B^3}{16\pi}\lambda_{BK^*a}^{3/2}A_0^2(m_a^2)|g_{asb}^A|^2, \quad (18)$$

where $\lambda_{Mma} = \left(1-\frac{(m_a+m)^2}{M^2}\right)\left(1-\frac{(m_a-m)^2}{M^2}\right)$

Constraints from Semileptonic decays

Collaboration	upper bound
N62 Collaboration[1]	$\mathcal{B}\left(K^{+} \to \pi^{+} a\right) < (13.0^{+3.3}_{-3.0}) \times 10^{-11}$
CLEO [2]	$\mathcal{B}\left(B^{\pm} \to \pi^{\pm} a\right) < 4.9 \times 10^{-5}$
CLEO [2]	$\mathcal{B}\left(B^{\pm} \to K^{\pm} a\right) < 4.9 \times 10^{-5}$
BELLE [3]	$\mathcal{B}\left(B^{\pm} \to \rho^{\pm} a\right) < 21.3 \times 10^{-5}$
BELLE [3]	$\mathcal{B}\left(B^{\pm} \to K^{*\pm} a\right) < 4.0 \times 10^{-5}$

(19)

Table: These inequalities come from the window for new physics in the branching ratio uncertainty of the meson decay in a pair $\bar{\nu}\nu$.

Frame Title

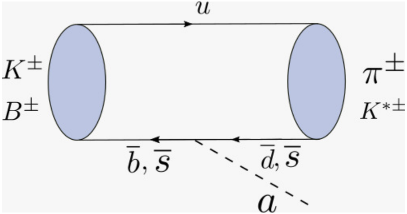

Figure: Tree level diagram contribution to the FCNC processes $K^{\pm} \to \pi^{\pm} a$ and $B^{\pm} \to K^{*\pm} a$.

Frame Title

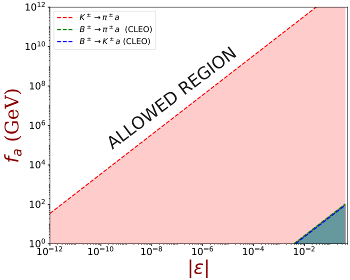

Figure: Allowed regions by lepton decays. For the down-type quarks and charged leptons the non-universal part of the PQ charges just depend on the diference $s_2 - s_1 = N\epsilon/9$, hence the flavor-changing neutral-current couplings (the off diagonal elements) just depend on ϵ.

Frame Title

$$\mathcal{L} \supset -c_1^{\text{eff}} \frac{\alpha_1}{8\pi} \frac{a}{\Lambda_{\text{PQ}}} B_{\mu\nu} \tilde{B}^{\mu\nu} - c_2^{\text{eff}} \frac{\alpha_2}{8\pi} \frac{a}{\Lambda_{\text{PQ}}} W_{\mu\nu}^3 \tilde{W}^{3\mu\nu} - c_3^{\text{eff}} \frac{\alpha_3}{8\pi} \frac{a}{f_a} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \ .$$

The decay width of an axion decaying in two photons is

$$\Gamma(a \to \gamma\gamma) = \frac{4\pi\alpha^2 m_a^3}{\Lambda_{\text{PQ}}^2} |C_{\gamma\gamma}^{\text{eff}}|^2 \ , \tag{20}$$

where

$$C_{\gamma\gamma}^{\text{eff}} = -\frac{c_3^{\text{eff}}}{32\pi^2} \left(\frac{c_1^{\text{eff}} + c_2^{\text{eff}}}{c_3^{\text{eff}}} - 2.03 \right)$$

Frame Title

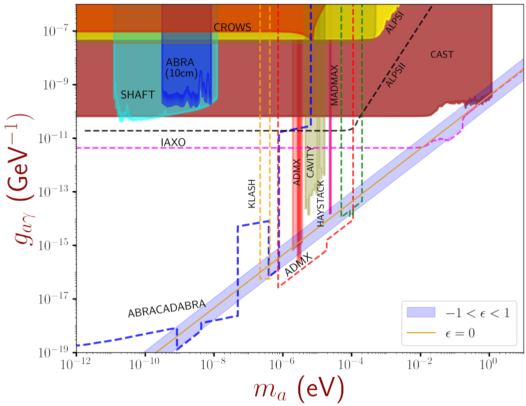

Figure: The excluded parameter space by various experiments corresponds to the colored regions, the dashed-lines correspond to the projected bounds of coming experiments looking for axion signals, the blue region corresponds to the parameter space scanned by our model in the interval $-1 \leq \epsilon \leq 1$.

S_1 and S_2 potential.

$$
\begin{aligned}
V(\Phi, S_i) &= \sum_{k=1}^{2} \mu_{s_k}^2 S_k^* S_k \\
&+ \sum_{k=1}^{2} \lambda_{s_k} \left(S_k^* S_k\right)^2 + \sum_{i=1}^{4} \sum_{k=1}^{2} \lambda_{i s_k} \left(\Phi_i^\dagger \Phi_i\right) \left(S_k^* S_k\right) \\
&+ \lambda_{s_1 s_2} \left(S_1^* S_1\right) \left(S_2^* S_2\right) \\
&+ F_1 \left(\left(\Phi_2^\dagger \Phi_3\right) S_1 + h.c. \right) \\
&+ F_2 \left(\left(\Phi_1^\dagger \Phi_2\right) S_1 + h.c. \right) \\
&+ \frac{1}{2} \left(m_{\zeta_{S_2}}\right)_{\text{SB}}^2 \zeta_{S_2}^2 + \frac{1}{2} \left(m_{\xi_{S_2}}\right)_{\text{SB}}^2 \xi_{S_2}^2 .
\end{aligned} \tag{21}
$$

In these expressions $S_i = \frac{v_{S_i} + \xi_{S_i} + i\zeta_{S_i}}{\sqrt{2}}$; $i = 1, 2$.

dark matter connexion

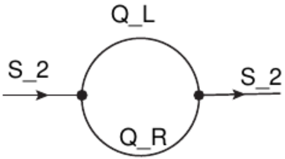

Figure: The scalar potential $V(\phi_\alpha, S_1, S_2)$ is invariant under the symmetry $S_2 \longrightarrow S_2^\dagger$ (which is equivalent to a Z_2 symmetry), but this symmetry is broken by the interaction term $\lambda_Q \bar{Q}_R Q_L S_2 +$ h.c.. In fact, from this interaction, it is also possible to generate, at one loop, a mass term for the CP-odd field $\frac{1}{2}\left(m_{\zeta_{S_2}}\right)^2_{\mathsf{SB}} \zeta^2_{S_2}$ in the effective Weinberg-Coleman potential.

Conclusions

- In this work we have proposed a PQ symmetry that gives rise to quark mass matrices with five texture-zeros. This texture can adjust in a non-trivial way the six masses of the quarks and the three CKM mixing angles and the CP violating phase.
- Since in our model the PQ charges are non-universal there are FCNC at the tree level. We calculated the tree level FCNC couplings from the effective interaction Lagrangian between the kinetic term of the quarks and the axion, these couplings are well known in the literature.
- In our model, the elements of the neutrino mass matrix can be related to the axion mass, thereby linking the smallness of both mass scales within a common framework.

Scalar Potential

1. "Appears and dissapears." Burchell et al (1998)
2. Appears and dissapears.

Scalar Potential

3. Reappears
    - Appears;
    - Appears
    - Appears
    - Appears

Descriptive statistics

Add an image or table.

Name	Turn	Height
Juan	1	1.9
Jose	2	1.7
Michael	3	1.95

Model

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_{2i} + \beta_3 Other_i + \gamma S_i + u_i \quad (22)$$

Y: Externality

- Var1.
- Var2.
- Var3.
- Var4.

Var7

Var8: Socioeconomic characteristics

- Var8.1
- Var8.2.
- Var8.3.
- Median income.
- Median home value.

S: State dummies

Outline

Frame title

Name	Turn	Height
Juan	1	1.9
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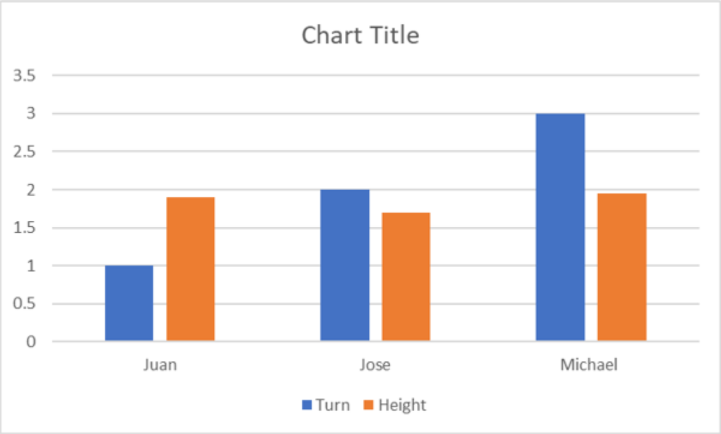
ReSULT



Result

Name	Turn	Height
Juan	1	1.9
Jose	2	1.7
Michael	3	1.95

Discussion

1. Comment1
2. Comment2.
3. Comment3.
4. Comment4.

Frame Title

E. Cortina Gil *et al.*, "Observation of the $K^+ \to \pi^+ \nu\bar{\nu}$ decay and measurement of its branching ratio," *JHEP*, vol. 02, p. 191, 2025.

R. Ammar *et al.*, "Search for the familon via B+- —¿ pi+- X0, B+- —¿ K+- X0, and B0 —¿ K0(S)X0 decays," *Phys. Rev. Lett.*, vol. 87, p. 271801, 2001.

O. Lutz *et al.*, "Search for $B \to h^{(*)}\nu\bar{\nu}$ with the full Belle $\Upsilon(4S)$ data sample," *Phys. Rev. D*, vol. 87, no. 11, p. 111103, 2013.