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Renormalization of Generalized Quantum Electrodynamics

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Abstract

In the present work we shall study the renormalizability of Generalized Quantum Electrodynamics ($GQED_4$). The on-shell renormalization scheme is reviewed and applied to the theory and we calculate the explicit expressions for all the counter-terms of the $GQED_4$.

Keywords: Renormalization, Quantum field theory, Higher order derivative theory, Counter terms.

1. Introduction

The Generalized Electrodynamics [1] was originally conceived in order to get rid of some pathologies inherent in the Maxwell theory, however, as pointed out by Pimentel and Galvão [2], only the generalized Lorenz condition, $\Omega[A] = \left(1 + m_P^{-2}\partial^2\right)\partial^\mu A_\mu$, completely fixed the gauge freedom and also intrinsically related with determining the correct true degrees of freedom for the theory. A study of the finite-temperature free Podolskys theory has showed a correction to the Stefan-Boltzmann law and by using cosmic microwave background data it was possible to set a thermodynamical limit to the Podolskys parameter m_P [3].

A previous study of Generalized Electrodynamics showed that the free gauge field Green's function is given by [4]:

$$
\begin{aligned}
iD_{\mu\nu}(k) = {} & \frac{1}{k^2}\left[\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right] - \\
& \frac{1}{k^2 - m_P^2}\left[\eta_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - m_P^2}\right] + \\
& (1-2\xi)\frac{1}{\left(k^2 - m_P^2\right)k^2}k_\mu k_\nu + \frac{1}{\left(k^2 - m_P^2\right)^2}k_\mu k_\nu.
\end{aligned} \tag{1}
$$

At first glance, the above expression could leads to a naive interpretation of the photon propagator as a sum of two distinct sectors: a massless and a massive; indeed, this statement holds in the free theory. However, we can not read the mass-dependent terms in Eq.(1) as true degrees of freedom of the theory, though, if we will require that the photon propagator behaves as a truly Maxwell photon, this will suggest a reasonable and appropriated interpretation of the Podolsky term as a regulator term; which plays the role of a Pauli-Villars-Raisky term [5] (since the $m_P^2 \to \infty$ limit exists in the theory, and do the mapping to the Maxwell theory).

Although the idea of higher-derivative (HD) be successful in the case of the attempt to quantize gravity, many inherent issues are present in the classical analysis of HD theories. In particular, such theories have a Hamiltonian which is not bounded from below and the addition of HD terms leads to the existence of instabilities (ghosts states) jeopardizing the unitarity. Nevertheless, recently in Ref. [6] a procedure was suggested for including interactions in free HD systems without breaking their stability. Remarkably, they shown that the dynamics of the GQED is stable at both classical and quantum level.

This work is addressed to the issue of renormalizability of the Generalized Quantum Electrodynamics, and is organized as follows. We review and apply to the

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$GQED_4$ the on-shell renormalization program.

2. Renormalization Schedule

In this section, we recall the so-called on-shell renormalization scheme [7] and employ it in the $GQED_4$; which is the most suitable for calculation in field theories which have a natural scale. The first part of the current analysis is based on determine, formally, the constants Z_i under suitable renormalization conditions and the physical constants, e and m. Now, we define the renormalized Lagrangian with the generalized Lorenz condition gauge-fixing term $\Omega[A] = \left(1 + m_P^{-2}\partial^2\right)\partial^\mu A_\mu$ [2], and also introduce the counter-terms with the following prescription:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(i\hat{\partial} - m + e\hat{A}\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{1}{2m_P^2}\partial^\mu F_{\mu\beta}\partial_\alpha F^{\alpha\beta} - \frac{1}{2\xi}\left[\left(1 + m_P^{-2}\partial^2\right)\partial^\mu A_\mu\right]^2 \\ &+ \delta_{Z_2}\bar{\psi}i\hat{\partial}\psi - \delta_{Z_0}\bar{\psi}m\psi + \delta_{Z_1}e\bar{\psi}\hat{A}\psi - \delta_{Z_3}\frac{1}{4}F_{\mu\nu}F^{\mu\nu};\end{aligned} \tag{2}$$

where, we have introduced the following definition: $\delta_{Z_i} = Z_i - 1$. The relations between the bare and renormalized quantities are as follows:

$$\begin{aligned}A^{(0)} &= Z_3^{1/2}A^{(r)} \quad , \quad \psi^{(0)} = Z_2^{1/2}\psi^{(r)}, \\ \bar{\psi}^{(0)} &= Z_2^{1/2}\bar{\psi}^{(r)},\end{aligned} \tag{3}$$

and[1]

$$\begin{aligned}Z_2 m^{(0)} &= Z_0 m \quad , \quad Z_3^{1/2}e^{(0)} = Z_1 Z_2^{-1}e, \\ m_P^{(0)} &= Z_3^{1/2}m_P.\end{aligned} \tag{4}$$

Here, the Podolsky's parameter m_P has not a constant associated with its renormalization, in the same sense as the ξ parameter (gauge Ward-Fradkin-Takahashi identity (WFT) [4]); the above changing is only for matter of notation.

Before starting with a proper discussion, we need to pay attention to the following bare WFT identity [4]:

$$ik_\mu\tilde{\Gamma}^\mu\left(p, p'; q = p' - p\right) = \mathscr{S}^{-1}\left(p - p'\right) - \mathscr{S}^{-1}(p). \tag{5}$$

An interesting consequence of the renormalized theory fulfilling the WFT identity is that the ratio Z_1/Z_2 must be finite if the theory is renormalizable. Thus, the finiteness of the ratio Z_1/Z_2 implies that order-by-order in perturbation theory the equality $Z_1 = Z_2$ is identically satisfied. Such identity is also responsible by preserving the gauge invariance after the renormalization procedure has been applied. Thereby, the coupling constant e is determined only by Z_3: $e_0 = Z_3^{-1/2}e$.

From the Lagrangian (2), we obtain new Schwinger-Dyson-Fradkin equations for the theory; the renormalized self-energies (added the counter-terms δ_{Z_i}) will be denoted by the suffix $^{(R)}$. First, we will analyze the photon sector, which has now the renormalized self-energy function:

$$\Pi^{(R)}(k) = \Pi(k) + \delta_{Z_3}. \tag{6}$$

where $\Pi(k)$ is the polarization scalar written in terms of the renormalized quantities.

Now, we impose the first renormalization condition as follow: we require that the photon propagator (1), in the gauge $\xi = 1$ (without lost of generality), must behave itself as a truly on-shell Maxwell photon:

$$i\mathscr{D}_{\mu\nu}(k) = \frac{1}{k^2}\eta_{\mu\nu}, \quad for \quad k^2 \to 0; \tag{7}$$

which leads to the renormalization condition:

$$\Pi^{(R)}\left(k^2\right)\Big|_{k^2\to 0} = 0. \tag{8}$$

We obtain, then, the general expression for the counter-term δ_{Z_3}:

$$\delta_{Z_3} = Z_3 - 1 = -\,\Pi\left(k^2\right)\Big|_{k^2\to 0}. \tag{9}$$

Here, hence, from Eq.(7) we can state an appropriated interpretation of the true behavior of the Podolsky's terms, in Eq.(1), amounting to Pauli-Villar-Raisky regulator terms [5]. Going now to the fermionic sector, we have that the renormalized self-energy function is written as:

$$i\Sigma^{(R)}(p, m) = i\Sigma(p, m) - im\delta_{Z_0} + i\delta_{Z_2}\hat{p}; \tag{10}$$

where the function $\Sigma(p)$ is the radiative correction of the fermionic $1PI$ function: $\Gamma(p) = \hat{p} - m - \Sigma^{(R)}(p, m)$; where: $\Gamma(x, y) = -\frac{\delta^2\Gamma}{\delta\psi(y)\delta\bar{\psi}(x)}$. Due to the spinorial structure, we can even write down the electron self-energy function in the following general way: $\Sigma(p, m) = \Sigma_1\left(p^2\right)\hat{p} + \Sigma_2\left(p^2\right)I$.

In order to fix the fermionic counter-terms, we must impose two renormalization conditions. To the first one, we require that[2]:

$$\frac{\partial\Gamma(p)}{\partial\hat{p}}\Big|_{\hat{p}\to m_F} = 1, \tag{11}$$

[1]We could also introduce: $m_0 = Z_m m$, with $Z_m = \frac{Z_0}{Z_2}$; Z_m is the real mass renormalization constant, an ξ-independent quantity.

[2]m_F is defined as the zero of the electron $1PI$ function.

which results into:

$$\left.\frac{\partial\Sigma^{(R)}(p,m)}{\partial\hat{p}}\right|_{\hat{p}\to m_F} = 0. \tag{12}$$

From the condition (12) we obtain the following relation to the counter-term δ_{Z_2}:

$$\begin{aligned}\delta_{Z_2} &= Z_2 - 1 = -\left.\Sigma_1\left(p^2\right)\right|_{p^2\to m_F^2} \\ &- 2m_F^2 \left.\frac{\partial\Sigma_1\left(p^2\right)}{\partial p^2}\right|_{p^2\to m_F^2} - 2m_F \left.\frac{\partial\Sigma_2\left(p^2\right)}{\partial p^2}\right|_{p^2\to m_F^2}.\end{aligned} \tag{13}$$

Whereas, for the second fermionic renormalization condition, we require that:

$$\Gamma(p) = \hat{p} - m_F, \quad when \quad \hat{p} \to m_F; \tag{14}$$

which implies directly into:

$$\left.\Sigma^{(R)}(p,m)\right|_{\hat{p}\to m_F} = 0. \tag{15}$$

The counter-term δ_{Z_0} is thus written as:

$$\begin{aligned}m\delta_{Z_0} &= m(Z_0 - 1) = \left.\Sigma_2\left(p^2\right)\right|_{p^2\to m_F^2} \\ &- 2m_F^2\left[m_F \left.\frac{\partial\Sigma_1\left(p^2\right)}{\partial p^2}\right|_{p^2\to m_F^2} + \left.\frac{\partial\Sigma_2\left(p^2\right)}{\partial p^2}\right|_{p^2\to m_F^2}\right].\end{aligned} \tag{16}$$

We can state that the renormalization condition (11) determines the counter-term δ_{Z_2}, and that the condition (15), the counter-term δ_{Z_0}.

Now, let us come towards to the fourth renormalization condition to determine the counter-term δ_{Z_1}. Using the so-called Gordon decomposition, we can write the vertex part Λ in terms of the Dirac and Pauli form factors:

$$\begin{aligned}\Lambda^\rho(p,p') &= \gamma^\rho F_1\left(q^2\right) + \frac{i}{2m}\sigma^{\rho\nu}q_\nu F_2\left(q^2\right), \\ \sigma^{\rho\nu} &= \frac{i}{2}\left[\gamma^\rho,\gamma^\nu\right],\end{aligned} \tag{17}$$

where $q = p' - p$, is the transferred momentum. Therefore, the on-shell condition for the vertex part is given in a way that for on-shell external electron lines $p'^2 = p^2 = m^2$, and $q^2 \to 0$, we have:

$$\left.F_1\left(q^2\right)\right|_{q^2\to 0} = 0, \tag{18}$$

which results in determining the counter-term δ_{Z_1}.

3. Remarks and conclusions

Was studied here the process, and subsequent consequences, of the renormalization for the Generalized Quantum Electrodynamics. Structurally speaking, the $GQED_4$ has the same form of QED_4, then our formal discussion of the general on-shell renormalization scheme followed the guidelines of the well-known program. One of the most important features of this discussion was the choice of the renormalization conditions; more specific, the renormalization condition for the photon propagator; where we required that it, in the $k^2 \to 0$ limit, should behave as a truly Maxwell photon, i.e., a massless particle. Allowing us, thus, gain an importantly, and natural, physical meaning for the theory's behavior, the interpretation of the Podolsky term as a natural regulator term, such as a Pauli-Villars-Raisky term [5]. However, hitherto there is not any proof regarding the relation between higher-order derivative terms with Pauli-Villars-Raisky regularization procedure [8].

We showed, through the explicit expressions of the radiative functions [4], in a general gauge ξ, the calculation of all four counter-terms for the theory (up to α-order), although the fermionic and vertex counter-terms: δ_{Z_0}, δ_{Z_2}, δ_{Z_1}, were all ultraviolet finite, leading us thus to a naive conclusion that the theory is entirely finite (in the fermionic and vertex sector).

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