$SQED_4$ and QED_4 on the null-plane

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We studied the scalar electrodynamics ($SQED_4$) and the spinor electrodynamics (QED_4) in the null-plane formalism. We followed the Dirac's technique for constrained systems to perform a detailed analysis of the constraint structure in both theories. We imposed the appropriated boundary conditions on the fields to fix the hidden subset first class constraints which generate improper gauge transformations and obtain an unique inverse of the second class constraint matrix. Finally, choosing the null-plane gauge condition, we determined the generalized Dirac brackets of the independent dynamical variables which via the correspondence principle give the (anti)-commutators for posterior quantization.

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I. INTRODUCTION

Half the of last century Dirac [1] proposed three different forms of relativistic dynamics depending on the types of surfaces where the independent degree of freedom was initiated. The first one, named the *instant form*, is when a space-like surface is chosen to establish the fundamental Poisson brackets or commutations relations. It has been used most frequently so far and is usually called equal-time quantization. The second form, the *point form*, is to take a branch of hyperbolic surface $x^\mu x_\mu = \kappa^2$ with $x^0 > 0$. And, the third form, *front form or light front*, is when we choose the surface of a single light wave to study the field dynamics; it is commonly referred as the *null-plane formalism* and it took almost 30 years for Dirac's idea was applied in physical phenomena. An important advantage pointed out by Dirac is that seven of the ten Poincaré generators are kinematical on the null-plane while in the conventional theory constructed on the instant form only six have this property. Other notable feature of a relativistic theory on the null-plane is that it gives origin to singular Lagrangians, e.g. constrained dynamical systems, thus, Dirac's procedure [2] can be employed to analyze the constraint structure of a given theory. In general, it leads to a reduction in the number of independent field operators in the respective phase space.

At equal-time, any two different points are space-like separated and therefore the fields defined at these points are naturally independent quantities. In a null-plane surface the situation is different because the micro-causality principle leads to locality requirement in which only the transversal components are and the longitudinal component becomes non-local in the theory, although, such situation would not be unexpected [3]. It is possible to verify that the transformation from the usual coordinates to the null-plane coordinates is not a Lorentz transformation and the structure of the phase space is different when we compare with the conventional one. As such, the description of a physical system in the null-plane formalism could give additional information from those provided by the conventional formalism [3]. For example, the momentum four-vector is (k^+, k^-, k^T) where k^+ is the null-plane energy while k^T and k^- indicates the transverse and the longitudinal components of the momentum. Therefore, a massive particle on the mass shell, $k^- = \frac{m^2+(k^T)^2}{2k^+}$, has positive definite values for $k\pm$ in contrast to $-\infty \leq k^{1,2,3} \leq$ for the usual components. An immediate consequence is that the vacuum on the null-plane quantized theory may become simpler than the one

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Appendix C: Grassmann Algebras

A Grassmann algebra contains bosonic (self-commuting) and fermionic (self-anticommuting) variables [15]:

$$FB = (-1)^{n_A n_B} BF \quad , \tag{C1}$$

where $n = 0$ for a bosonic, and $n = 1$ for a fermionic variable. Note that the product of two fermionic variables is bosonic, and the product of a fermionic and a bosonic variables is fermionic.

The left derivative of a ψ_α fermionic variable is defined as

$$\frac{\partial}{\partial\psi_\alpha}\left\{ \psi_{\alpha_1}\psi_{\alpha_2} \ \cdots\psi_{\alpha_n}\right\} = -\delta_{\alpha\alpha_1}\psi_{\alpha_2} \ \cdots\psi_{\alpha_n} + \delta_{\alpha\alpha_2}\psi_{\alpha_1}\psi_{\alpha_3} \ \cdots\psi_{\alpha_n} + \ \cdots \ + (-1)^n\delta_{\alpha\alpha_n}\psi_{\alpha_1}\psi_{\alpha_2} \ \cdots\psi_{\alpha_{n-1}} \ . \tag{C2}$$

The Poisson Brackets can be defined similar to ordinary mechanics [16]. The phase space is spanned by q_i, p^i which are bosons and ψ_α and π^α, fermions. Denote by B(F) a bosonic (Fermionic) element of the Grassmann algebra, then

$$\begin{aligned}
\{B_1, B_2\} = -\{B_2, B_1\} &= \left\{\frac{\partial B_1}{\partial q_i}\frac{\partial B_2}{\partial p^i} - \frac{\partial B_2}{\partial q_i}\frac{\partial B_1}{\partial p^i}\right\} + \left\{\frac{\partial B_1}{\partial \phi_\alpha}\frac{\partial B_2}{\partial \pi^\alpha} - \frac{\partial B_2}{\partial \phi_\alpha}\frac{\partial B_1}{\partial \pi^\alpha}\right\} \\
\{F, B\} = -\{B, F\} &= \left\{\frac{\partial F}{\partial q_i}\frac{\partial B}{\partial p^i} - \frac{\partial B}{\partial q_i}\frac{\partial F}{\partial p^i}\right\} - \left\{\frac{\partial F}{\partial \phi_\alpha}\frac{\partial B}{\partial \pi^\alpha} + \frac{\partial B}{\partial \phi_\alpha}\frac{\partial F}{\partial \pi^\alpha}\right\} \\
\{F_1, F_2\} = \{F_2, F_1\} &= \left\{\frac{\partial F_1}{\partial q_i}\frac{\partial F_2}{\partial p^i} + \frac{\partial F_2}{\partial q_i}\frac{\partial F_1}{\partial p^i}\right\} - \left\{\frac{\partial F_1}{\partial \phi_\alpha}\frac{\partial F_2}{\partial \pi^\alpha} + \frac{\partial F_2}{\partial \phi_\alpha}\frac{\partial F_1}{\partial \pi^\alpha}\right\} \quad .
\end{aligned} \tag{C3}$$

It follow from its definition that the Poisson brackets has the properties

$$\begin{aligned}
\{A, B\} &= -(-1)^{n_A n_B}\{B, A\} \\
\{A, B + C\} &= \{A, B\} + \{A, C\} \\
\{A, BC\} &= (-1)^{n_A n_B} B\{A, C\} + \{A, B\}C \\
\{AB, C\} &= (-1)^{n_B n_C}\{A, C\}B + A\{B, C\} \\
(-1)^{n_A n_C}\{A, \{B, C\}\} &+ (-1)^{n_B n_A}\{B, \{C, A\}\} + (-1)^{n_C n_B}\{C, \{A, B\}\} = 0 \quad .
\end{aligned} \tag{C4}$$

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[18] See the definitions in the Appendix **A**.