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Collisional parton energy loss in a finite size QCD medium reexamined: Off-mass-shell effects

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We study the collisional energy loss mechanism for particles produced off mass shell in a finite size QCD medium. The off-mass-shell effects introduced consider particles produced in wave packets instead of plane waves and a length scale associated with an in-medium particle lifetime. We show that these effects reduce the energy loss as compared to the case when the particles are described as freely propagating from the source. The reduction in energy loss is stronger as the scale becomes of the order of or smaller than the medium size. We discuss possible consequences of the result on the description of the energy loss process in the parton recombination scenario.

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I. INTRODUCTION

The problem of collisional parton energy loss in a QCD medium has been revived by recent RHIC data on nonphotonic single electrons [1] that are not well described within radiative energy loss calculations. Collisional energy loss has been a subject of research for a long time [2–4]. During the pioneering years, an important question was to understand how to handle the infrared singularities in perturbative calculations at finite temperature. The advent of resummation techniques clarified this point and allowed the reliable computation of the energy loss of a heavy parton traversing an infinite medium to lowest order in perturbation theory [5,6]. Shortly thereafter, it was estimated that radiative energy loss in a finite size medium was a more important mechanism to account for energy losses of energetic partons [7–9]. Nonetheless, even more recent studies [10,11] suggested that for a range of parameters relevant to RHIC energies, radiative and collisional energy losses for heavy quarks are of the same order of magnitude. These last calculations where done for infinite QCD media. The outstanding question was whether collisional energy loss for finite size media was also significant.

In this context there were two results seemingly in contradiction [12,13]. In Ref. [12] a semiclassical approach based on linear response theory was used to compute the collisional energy loss by means of the work done by the response chromoelectric field on the color-charged heavy parton traversing the medium. The infinite medium limit of this description agrees with the collisional energy loss result at high temperature—up to color factors—obtained from a perturbative approach using Hard Thermal Loop (HTL) effective propagators [5]. The original claim that the finite size medium induced energy loss is strongly suppressed compared to the infinite medium case was later revised by properly subtracting the kinetic energy associated with producing the particle within the medium [14].

However, in Ref. [13] a lowest order perturbative calculation using HTL propagators finds that finite size effects on the collisional energy loss are not significantly suppressed as compared to the infinite medium case. The formulation of the problem is based on the assumption that the scattered particle originates within the medium but otherwise is produced on mass shell.

However, when particles are emitted by sources lasting a finite amount of time they are not necessarily produced on their mass shell since the source emits over a (wide) range of energies. A physical consequence is the possibility that the particle *loses* its identity within the medium. In the midst of a high-energy heavy-ion collision, such a possibility can be realized in the recombination of a jet parton with the partons from the surrounding medium. Recall that during the propagation inside a deconfined QCD plasma, a fast parton can have not only induced gluon radiation but also *induced absorption* from thermal gluons. This process can be fairly well considered as parton recombination, which is one of the accepted mechanisms used to describe the distinct features of meson and baryon spectra that include a baryon to meson ratio larger than one for $p_t \gtrsim 2$ GeV in central Au+Au collisions at RHIC [15] and their different azimuthal anisotropies [16]. This point is addressed in the context of the modification of parton fragmentation functions induced by medium effects in Ref. [17]. When a parton recombines it certainly losses its original identity and the energy loss can no longer be described in terms of parton degrees of freedom. Parton recombination from a jet with thermal partons to form intermediate p_t hadrons is a viable scenario in the case of light flavors, given the features of the proton to pion ratio, and even for s quarks, given the features of the Λ to kaon ratio [18]. Other off-mass-shell effects can be of relevance as well when studying if and how the virtuality of the propagating parton affects the in-medium splitting functions [19].

In this work we study one such off-mass-shell effect, namely a possible finite *lifetime* of the scattered partons—originating within and traversing the finite size medium—in the description of the collisional energy loss mechanism. We introduce the possibility that the scattered parton can be described in terms of a propagator containing a parameter associated to the parton's lifetime. We argue that, for recombining partons, this picture could be used to consider collisional energy losses only up to times when these recombine with thermal partons from the medium from where the energy loss process should be described in terms of hadronic degrees of freedom.

The work is organized as follows: In Sec. II we rewrite the expression for the collisional energy loss in a finite size QCD medium based on the formalism used in Ref. [13], allowing for particles being emitted off mass shell and having a finite lifetime. In Sec. III we give the numerical estimates for the collisional energy losses of heavy and light quarks using parameters relevant for RHIC energies. We use the cases studied in Ref. [13] as a base to compare our results. We finally conclude and give an outlook of the consequences of the result in Sec. IV.

II. ENERGY LOSS

We start by describing the elemental interaction of a fermion with momentum $P^\mu = (p_0, \mathbf{p})$ (not necessarily on its mass shell), magnitude of its velocity $v \equiv |\mathbf{v}| = p/E$, mass M, and spin s with a massless fermion in the nonexpanding medium, with momentum $K^\mu = (k, \mathbf{k})$ and spin λ, through the exchange of a gauge boson with momentum $q^\mu = (\omega, \mathbf{q})$ and by means of a coupling constant g. For elastic collisions, these particles retain their identities and after the scattering they have momenta $P'^\mu = (E', \mathbf{p}')$ and $K'^\mu = (k', \mathbf{k}')$ and spins s' and λ', respectively, and $E' = \sqrt{p'^2 + M^2}$. The scattering diagram associated with the process is depicted in Fig. 1. When the incoming massive fermion is on its mass shell, (i.e., $p_0 = E = \sqrt{p^2 + M^2}$), the expression for the matrix element describing this process is given by (see Eq. (6) in Ref. [13])

$$
\begin{aligned}
i\mathcal{M} = -g^2 \int d^4x\, j(t,\mathbf{x}) e^{iP\cdot x} \int d^4x_1 \int d^4x_2 \\
\times \int \frac{d^3p}{(2\pi)^3 2E} \int \frac{d^4q}{(2\pi)^4} D_{\alpha\beta}(q) e^{iq\cdot(x_1-x_2)} \\
\times \bar{u}(p',s') e^{iP'\cdot x_1} \gamma^\alpha u(p,s) e^{-iP\cdot x_1} \\
\times \bar{u}(k',\lambda') e^{iK'\cdot x_2} \gamma^\beta u(k,\lambda) e^{-iK\cdot x_2} \\
\times \theta(t_1 - t)\theta[L/v - (t_1 - t)],
\end{aligned} \tag{1}
$$

where $D_{\alpha\beta}$ will become the effective HTL gluon propagator, $j(t,\mathbf{x})e^{iP\cdot x}$ is the amplitude associated with the source to produce an incoming particle with momentum P, and

$$
\begin{aligned}
x^\mu &= (t, \mathbf{x}), \\
x_i^\mu &= (t_i, \mathbf{x}_i), \quad i = 1, 2.
\end{aligned} \tag{2}
$$

The step functions in Eq. (1) represent the conditions that the produced fermion interacts within the plasma at t_1 after being produced at t and before it leaves the plasma at $L/v + t$, under the approximation that its velocity remains constant.

When the source produces particles off their mass shell, the matrix element describing the process can be written as

$$
\begin{aligned}
i\mathcal{M} = -\frac{g^2}{2M} \int d^4x\, j(t,\mathbf{x}) \int d^4x_1 \int d^4x_2 \\
\times \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} D_{\alpha\beta}(q) e^{iq\cdot(x_1-x_2)} \\
\times \bar{u}(p',s') e^{iP'\cdot x_1} \gamma^\alpha S(P) e^{iP\cdot(x-x_1)} u(p,s) \\
\times \bar{u}(k',\lambda') e^{iK'\cdot x_2} \gamma^\beta u(k,\lambda) e^{-iK\cdot x_2} \\
\times \theta(t_1 - t)\theta[L/v - (t_1 - t)],
\end{aligned} \tag{3}
$$

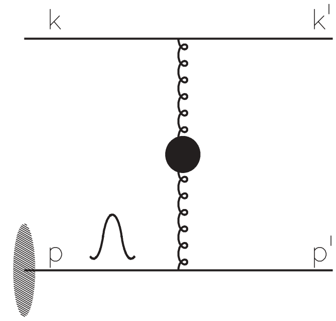

FIG. 1. Scattering diagram describing the lowest order contribution to the energy loss from the collision of fermions with incoming and outgoing momenta P and P' with medium fermions with initial and final momenta K and K', respectively. The blob in the intermediate gluon line represents the effective HTL propagator. Incoming fermions are produced by the source in wave packets and not necessarily on their mass shell.

where $S(P)e^{iP\cdot(x-x_1)}u(p,s)$ represents the amplitude to propagate a fermion mode with momentum P from x to x_1. Notice that Eq. (3) reduces to Eq. (1) when $S(P) \to S_0(P) = (2\pi)(2M)\Lambda_+(P)\delta(P^2 - M^2)\theta(p_0)$, where

$$
\begin{aligned}
\Lambda_+(P) &= \sum_s u(p,s)\bar{u}(p,s) \\
&= \frac{\not{P} + M}{2M}
\end{aligned} \tag{4}
$$

is the projector for positive-energy solutions.

Let us however consider the situation where $S(P)$ does not describe free, on-mass-shell propagation, but instead the propagation of a wave packet with a finite width η. To explore a simple scenario, let us recall that

$$
\lim_{\eta\to 0^+} \frac{\eta}{(p_0 - E_p)^2 + \eta^2} = (2E_p\pi)\delta(P^2 - M^2)\theta(p_0), \tag{5}
$$

where $E_p = +\sqrt{p^2 + M^2}$. To consider a finite width, we take η finite and write

$$
S(P) = \left(\frac{2M}{E_p}\right) \frac{\eta}{(p_0 - E_p)^2 + \eta^2} \Lambda_+(P). \tag{6}
$$

Upon the change of variable $y = x_1 - x$ and after integration over d^4x_2, d^4y, d^3p, d^4q, and d^4x in Eq. (3) we get

$$
\begin{aligned}
i\mathcal{M} = 2i\left(\frac{\eta}{E_p}\right) \int_{-\infty}^{\infty} \frac{dp_0}{(2\pi)} e^{-i(p_0 - E' - \omega)L/2v} \\
\times \frac{\sin[(p_0 - E' - \omega)L/2v]}{(p_0 - E' - \omega)[(p_0 - E_p)^2 + \eta^2]} \\
\times i\mathcal{M}_0(p_0)\tilde{j}(P),
\end{aligned} \tag{7}
$$

where $\tilde{j}$ is the Fourier transform of $j(t, \mathbf{x})$ and

$$i\mathcal{M}_0(p_0) = ig^2 D_{\alpha\beta}(K' - K) \times \bar{u}(p', s')\gamma^{\alpha} u(p, s)\bar{u}(k', \lambda')\gamma^{\beta} u(k, \lambda) \quad (8)$$

and where we used $\Lambda_+(P)u(p, s) = u(p, s)$. $\mathcal{M}_0(p_0)$ is the matrix element describing the scattering process in an infinite volume and for the following discussion, we have emphasized its dependence on p_0.

To perform the integral in Eq. (7), we write

$$\frac{1}{(p_0 - E_p)^2 + \eta^2} = \frac{1}{[(p_0 - E_p) + i\eta][(p_0 - E_p) - i\eta]}, \quad (9)$$

and for convergence, we close the contour of integration on the lower p_0 complex half-plane, which selects the pole at $p_0 = E_p - i\eta$. This results in the following expression for the matrix element:

$$i\mathcal{M} = i\left(\frac{1}{E_p}\right) e^{-\eta L/2v} e^{-i(E_p - E' - \omega)L/2v} \times \frac{\sin[(E_p - E' - \omega - i\eta)L/2v]}{(E_p - E' - \omega - i\eta)} \times i\mathcal{M}_0(E_p - i\eta)\tilde{j}(P). \quad (10)$$

Notice that for a consistent description we require the condition $\eta \ll E_p$, meaning that the central energy of the wave packet is much larger than its width. Therefore in Eq. (10) we can approximate

$$\mathcal{M}_0(E_p - i\eta) \simeq \mathcal{M}_0(E_p). \quad (11)$$

This approximation cannot be made for the rest of the factors in Eq. (10) since the term $E_p - E' - \omega$ is of order of the transferred momentum, which in turn is of order of the medium's temperature, which may not be much larger than the wave packet's width.

The square of the matrix element given in Eq. (10), averaged over the initial spin s and summed over all other spins, is therefore

$$\frac{1}{2}\sum_{s,s',\lambda,\lambda'} |\mathcal{M}|^2 = e^{-\eta L/v}\left(\frac{1}{E_p^2}\right)|\tilde{j}(P)|^2 \times \left|\frac{\sin[(\omega - \mathbf{v}\cdot\mathbf{q} + i\eta)L/2v]}{\omega - \mathbf{v}\cdot\mathbf{q} + i\eta}\right|^2 \times \frac{1}{2}\sum_{s,s',\lambda,\lambda'} |\mathcal{M}_0(E_p)|^2, \quad (12)$$

where we have used $E_p - E' \simeq \mathbf{v}\cdot\mathbf{q}$ for an energetic incoming fermion. It is worth mentioning that, if instead of using the propagator in Eq. (6), one uses the free Feynman propagator, the result for the square of the matrix element, averaged over the initial spin and summed over all other spins, yields the result found in Ref. [13].

Hereafter, we specialize to the description of the scattering of the fermion (quark) in the QCD plasma. The differential energy loss dE is related to the differential collisional interaction rate $d\Gamma$ by $dE = \omega d\Gamma$ [5]. $d\Gamma$ is given by

$$d^3N d\Gamma = \frac{1}{2}\sum_{s,s',\lambda,\lambda'} |\mathcal{M}|^2 \frac{d^3p'}{(2\pi)^3 2E'}\frac{d^3k}{(2\pi)^3 2k}\frac{d^3k'}{(2\pi)^3 2k'} \times \sum_{\xi=q,\bar{q},g} n_{\text{eq}}^{\xi}(k)[1 \pm n_{\text{eq}}^{\xi}(k')] \simeq \frac{1}{2}\sum_{s,s',\lambda,\lambda'} |\mathcal{M}|^2 \frac{d^3p'}{(2\pi)^3 2E'}\frac{d^3k}{(2\pi)^3 2k}\frac{d^3k'}{(2\pi)^3 2k'} \times n_{\text{eq}}(k), \quad (13)$$

where

$$d^3N = d_R|j(P')|^2[u(p', s')\bar{u}(p', s')]^2\frac{d^3p'}{(2\pi)^3 2E'} = d_R|j(P')|^2\frac{d^3p'}{(2\pi)^3 2E'} \quad (14)$$

represents the number of (nonscattered) particles into the phase space volume in the interval $\mathbf{p}'$ and $\mathbf{p}' + d^3p'$ with $d_R = 3$ for the SU(3) fundamental representation. Also, in Eq. (13), when describing the collisional energy loss, we have used $n_{\text{eq}}^{\xi}(k)[1 \pm n_{\text{eq}}^{\xi}(k')] = n_{\text{eq}}^{\xi}(k)$ since the term proportional to $n_{\text{eq}}^{\xi}(k)n_{\text{eq}}^{\xi}(k')$ is odd under the exchange of k and k' and integrates to zero [5], and we have defined

$$n_{\text{eq}}(k) = \sum_{\xi=q,\bar{q},g} n_{\text{eq}}^{\xi}(k). \quad (15)$$

For a source producing energetic particles with a large spread in momentum, we can take the approximation $|j(P')|^2 \simeq |j(P)|^2$. Therefore, by considering the finite width of the scattered wave packet, the collisional energy loss can be written as

$$\Delta E \simeq C_R\frac{e^{-\eta L/v}}{E_p^2}\int\frac{d^3k}{(2\pi)^3 2k}n_{\text{eq}}(k)\int\frac{d^3k'}{(2\pi)^3 2k'}\omega \times \left|\frac{\sin[(\omega - \mathbf{v}\cdot\mathbf{q} + i\eta)L/2v]}{\omega - \mathbf{v}\cdot\mathbf{q} + i\eta}\right|^2 \times \frac{1}{2}\sum_{s,s',\lambda,\lambda'} |\mathcal{M}_0(E_p)|^2. \quad (16)$$

Notice that Eq. (16) is modified with respect to the corresponding expression in Ref. [13] by the η-dependent exponential factor and the η dependence in the arguments of the sine function and in the energy denominator. When $\eta \to 0$, the corresponding expression for the energy loss in Ref. [13] is recovered. To find the explicit expression for Eq. (16), recall that the effective gluon propagator can be written as

$$D^{\mu\nu} = -P^{\mu\nu}\Delta_T - QP^{\mu\nu}\Delta_L, \quad (17)$$

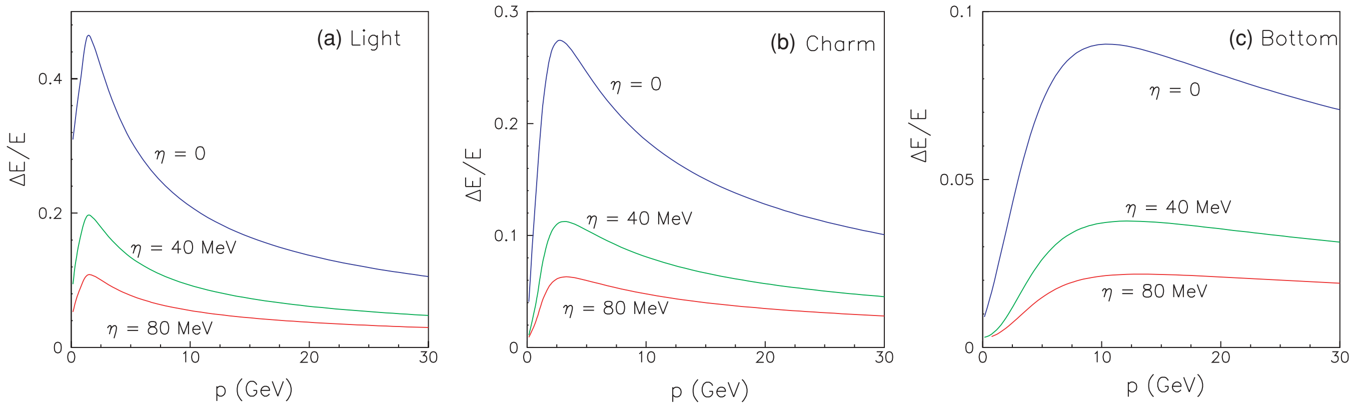

FIG. 2. (Color online) Fractional energy loss for (a) light, (b) charm, and (c) bottom quarks as a function of their momenta for a fixed medium's length $L = 5$ fm. The uppermost curve in each case corresponds to the description without off-mass-shell effects. This is compared to the case with off-mass-shell effects for two values of η : 40 and 80 MeV. In each case, the fractional energy loss decreases as the value of η increases.

where, in the HTL approximation, the effective transverse and longitudinal gluon propagators are given by [20]

$$\begin{aligned}\Delta_T^{-1} &= \omega^2 - q^2 - \frac{m_D^2}{2} - \frac{(\omega^2-q^2)m_D^2}{2q^2} \\ &\quad\times\left(1+\frac{\omega}{2q}\ln\left|\frac{\omega-q}{\omega+q}\right|\right), \\ \Delta_L^{-1} &= q^2 + m_D^2\left(1+\frac{\omega}{2q}\ln\left|\frac{\omega-q}{\omega+q}\right|\right),\end{aligned} \tag{18}$$

where $m_D^2 = g^2T^2(1+N_f/6)$ is the square of the Debye mass and, if we work in Coulomb gauge, the only nonvanishing components of the transverse and longitudinal projectors are

$$\begin{aligned}P^{ij} &= \delta^{ij} - \frac{q^iq^j}{q^2}, \\ Q^{00} &= 1.\end{aligned} \tag{19}$$

By using Eqs. (17)–(19), the matrix element squared describing the underlying scattering process in vacuum, averaged over the initial spin and summed over all other spins, is given by

$$\begin{aligned}\frac{1}{2}\sum_{s,s',\lambda,\lambda'}|\mathcal{M}_0|^2 = 16g^4E_p^2\Bigg\{&|\Delta_L(q)|^2\left(kk'+\mathbf{k}\cdot\mathbf{k}'\right) \\ &+2\mathrm{Re}[\Delta(q)_L\Delta(q)_T^*] \\ &\times\left[k\left(\mathbf{v}\cdot\mathbf{k}'-\frac{\mathbf{v}\cdot\mathbf{q}\mathbf{k}'\cdot\mathbf{q}}{q^2}\right)\right. \\ &\left.+k'\left(\mathbf{v}\cdot\mathbf{k}-\frac{\mathbf{v}\cdot\mathbf{q}\mathbf{k}\cdot\mathbf{q}}{q^2}\right)\right] \\ &+|\Delta_T(q)|^2\left[2\left(\mathbf{v}\cdot\mathbf{k}-\frac{\mathbf{v}\cdot\mathbf{q}\mathbf{k}\cdot\mathbf{q}}{q^2}\right)\right. \\ &\times\left(\mathbf{v}\cdot\mathbf{k}'-\frac{\mathbf{v}\cdot\mathbf{q}\mathbf{k}'\cdot\mathbf{q}}{q^2}\right) \\ &\left.+\left(kk'-\mathbf{k}\cdot\mathbf{k}'\right)\left(v^2-\frac{\mathbf{v}\cdot\mathbf{q}\mathbf{v}\cdot\mathbf{q}}{q^2}\right)\right]\Bigg\}.\end{aligned} \tag{20}$$

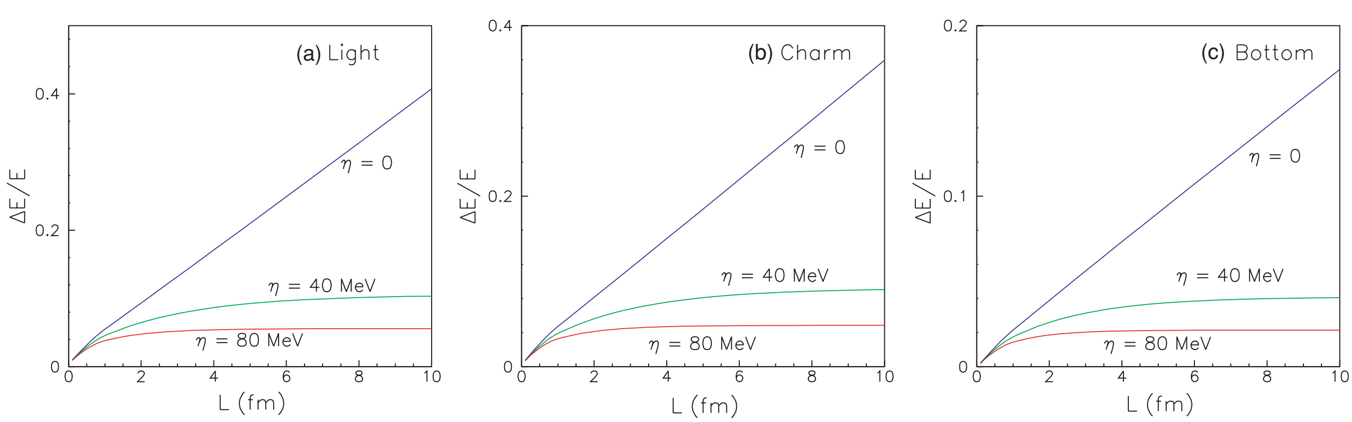

FIG. 3. (Color online) Fractional energy loss for (a) light, (b) charm, and (c) bottom quarks as a function of the medium's length for a fixed quark momentum $p = 10$ GeV. The uppermost curve in each case corresponds to the description without off-mass-shell effects. This is compared to the case with off-mass-shell effects for two values of η : 40 and 80 MeV. In each case, the fractional energy loss decreases as the value of η increases.

For a nonexpanding medium, the energy loss does not depend on the direction of $\mathbf{v}$, so we can simplify Eq. (16) by averaging over the direction of $\mathbf{v}$ [5]. This is most conveniently performed by introducing the auxiliary functions

$$\mathcal{J}_i = \int \frac{d\Omega}{4\pi} \left| \frac{\sin[(\omega - \mathbf{v}\cdot\mathbf{q} + i\eta)L/2v]}{\omega - \mathbf{v}\cdot\mathbf{q} + i\eta} \right|^2 (\omega - \mathbf{v}\cdot\mathbf{q})^i, \quad i = 1, 2, 3, \tag{21}$$

in terms of which the average over the different powers of $\mathbf{v}$ appearing in Eq. (20) can be expressed. The functions in Eq. (21) are explicitly given in the Appendix. After averaging over the directions of $\mathbf{v}$, the expression for the energy loss can be written as

$$\begin{aligned}\Delta E = {} & \frac{C_R g^4}{2\pi^4} e^{-\eta L/v} \int_0^\infty n_{\rm eq}(k) dk \\ & \times \left(\int_0^k q dq \int_{-q}^{q} \omega d\omega + \int_k^{q_{\rm max}} q dq \int_{q-2k}^{q} \omega d\omega \right) \\ & \times \Bigg(|\Delta_L(q)|^2 \frac{(2k+\omega)^2 - q^2}{2} \mathcal{J}_0 \\ & + |\Delta_T|^2 \frac{[q^2 - \omega^2][(2k+\omega)^2 + q^2]}{4q^4} \\ & \times [(v^2 q^2 - \omega^2)\mathcal{J}_0 + 2\omega\mathcal{J}_1 - \mathcal{J}_2] \Bigg),\end{aligned} \tag{22}$$

where the limits of integration over ω and q take into account that, for the considered scattering amplitude, the transferred four-momentum is spacelike and

$$q_{\rm max} = \frac{2k(1 + k/E_p)}{1 - v + 2k/E_p} \tag{23}$$

is obtained from the approximation that the maximum energy transferred occurs for backward scattering [21].

III. NUMERICAL RESULTS

To present the quantitative behavior for the energy loss, we take standard values for the parameters involved. We give examples of the effect for light as well as for heavy flavors, both to study the mass effect and to directly compare to the findings of Ref. [13]. The plasma temperature is taken as $T = 0.225$ GeV, the effective number of flavors as $N_f = 2.5$, the strength of the coupling constant as $\alpha = g^2/4\pi = 0.3$, and the Debye mass as $m_D = 0.5$ GeV. The bottom quark mass is taken as 4.5 GeV whereas the charm quark mass is taken as 1.2 GeV. We take the mass of the light quarks as 0.2 GeV.

Figure 2 shows the fractional energy loss $\Delta E/E$ for light, charm, and bottom quarks in a finite size medium with $L = 5$ fm, for the cases with and without off-mass-shell effects. For the curves describing the off-mass-shell effects, we consider two values, $\eta = 40$ MeV and $\eta = 80$ MeV. Notice that in all cases a finite value of η produces less of an energy loss than the case $\eta = 0$. The decrease is more important for larger values of η.

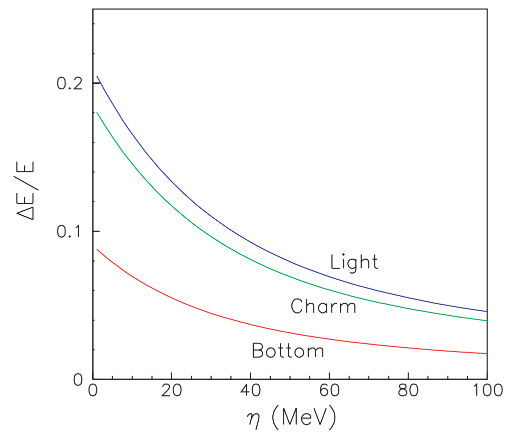

FIG. 4. (Color online) Fractional collisional energy loss for light, charm, and bottom quarks as a function of η for a fixed quark momentum $p = 10$ GeV and a fixed medium's size $L = 5$ fm. The fractional energy loss decreases with increasing η and the decrease is similar in shape, independent of the quark mass.

Figure 3 shows the fractional energy loss for light, charm, and bottom quarks as a function of the medium's length L for a fixed quark momentum $p = 10$ GeV, comparing also the cases with and without off-mass-shell effects. For the curves describing the off-mass-shell effects, we consider once more the two values $\eta = 40$ MeV and $\eta = 80$ MeV. Notice that a finite value of η causes the fractional energy loss to asymptotically reach a maximum value as the medium's size increases. This is in sharp contrast with the case where no off-mass-shell effects are considered, where for large L the energy loss increases linearly.

Figure 4 shows the behavior of the fractional energy loss as a function of η also for light, charm, and bottom quarks for a fixed value of the quark momentum $p = 10$ GeV and a fixed value of the medium's size $L = 5$ fm. The fractional energy loss decreases with increasing η and the decrease is similar in shape, regardless of the quark mass.

IV. DISCUSSION AND CONCLUSIONS

In this work we have studied the off-mass-shell effects on the collisional energy loss of particles, produced and scattered within a finite size QCD medium, associated with the introduction of a finite width wave packet and therefore a finite particle lifetime. We have shown that this effect decreases the energy loss as compared to the case when these particles are produced on mass shell and therefore these particles live longer than the medium, fragmenting outside it. We have argued that this picture should be applied in particular to energetic partons that recombine with thermal partons and thus hadronize within the medium.

Recall that the length scales playing a role for the energy loss mechanisms in a finite size, thermal, nonexpanding medium are the medium's size L, the average distance between collisions, $d \sim 1/T$, the Debye radius r_D, the mean free path $\delta \sim 1/g^2T$, and the particle's formation time $t_f \sim 1/E_p$. When considering the in-medium particle lifetime, one also introduces the length scale η^{-1}. For the description of the scattering process in terms of a perturbative picture, it is required that the hierarchy of scales

$$1/T \ll r_D \ll \delta \tag{24}$$

be satisfied. The requirement that the scattering particle can be described in terms of an oscillating mode means that this mode is not damped too fast, which in turn translates into the condition

$$1/E_p \ll 1/\eta, \tag{25}$$

which can be thought of as the condition that the time the medium takes to produce the particle is much shorter than the in-medium particle lifetime. For very energetic partons, it is safe to assume that $t_f \sim 1/E_p$ is the smallest of all length scales. Moreover, for media sizes of the order expected to be produced in relativistic heavy-ion collisions, it is also safe to assume that L is larger than the mean free path and therefore the hierarchy of scales

$$1/E_p \ll 1/T \ll r_D \ll \delta \lesssim L \tag{26}$$

follows. In fact, the results in Refs. [13,14] can be viewed as meaning that as long as the medium size is larger than the Debye radius, the collisional energy loss for on-mass-shell particles in a finite size medium is not suppressed as compared to the infinite medium case.

The situation changes when introducing the in-medium particle lifetime. The results of this work show that when $\eta^{-1} \lesssim L$ the effects are strong. They cease to matter for $\eta \to 0$.

It should be emphasized that, in the context of this work, the term *loss of identity* does not refer to a parton change of color or phase, which are accounted for already in the description of the underlying QCD scattering process, neither to a change of flavor, which would be mediated by the weak interaction and is thus irrelevant for the time scales involved during the QCD plasma phase. This term is rather related to the onset of a hadronization mechanism that happens during the interaction of a fast (hard) parton with soft ones from the medium in such a way as to produce a hadron by means of recombination. This process has been referred to as *shower and thermal parton recombination* in Ref. [22]. In this way, the loss of identity is related to the fact that when this kind of parton forms a hadron, the energy loss can no longer be described in terms of parton degrees of freedom and must be described in terms of hadron degrees of freedom, thereby effectively causing the parton to disappear from the description.

Typical time scales involved in hard-soft parton recombination are of order $\tau_{\rm recomb} \sim 1.5$ fm, despite the low momentum transfers involved, since, as argued for instance in Ref. [23], this recombination need not be local and it can be mediated by a QCD string. Such a time scale is well within the lifetime of the QCD medium for central collisions and the largest nuclei, where it is estimated to be of order 5 fm.

When the medium length shortens and falls below $\tau_{\rm recomb}$, hadronization is more likely to happen outside the medium. This means in particular that, for peripheral collisions or collisions of smaller systems, the energy loss description in terms of partonic degrees of freedom is appropriate. This is accounted for in our description when we take $\eta^{-1} > L$, for which the energy losses with and without the use of the parameter η coincide. Indeed, a change in the energy loss of intermediate momentum hadrons should exist as a function of centrality and system size. To quantify such a change it is necessary to quantitatively estimate the energy loss of a hadron within a QCD medium. This calculation is for the moment outside the scope of the present work.

In the context of recombination, the use of the parameter η should be introduced into a statistical scenario that also incorporates the evolution of the colliding system with energy density. In addition, a realistic geometry, including adequate probability profiles to produce jets [24] and the effects of an expanding medium, should be used. All this work is for the future.

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APPENDIX

The functions defined in Eq. (21) are explicitly given by

$$\begin{aligned}
\mathcal{J}_0 &\equiv \int \frac{d\Omega}{4\pi}\left|\frac{\sin[(\omega-\mathbf{v}\cdot\mathbf{q}+i\eta)L/2v]}{\omega-\mathbf{v}\cdot\mathbf{q}+i\eta}\right|^2\\
&= -\frac{1}{8qv\eta}\Bigg[i\cosh(\eta L/v)\\
&\quad\times\Bigg(\sum_{l,l'=\pm1}\mathrm{sgn}(l)\mathrm{sgn}(l')Ci[(\omega-lqv-l'i\eta)L/v]\\
&\quad+2i\left[\arctan\left(\frac{qv+\omega}{\eta}\right)+\arctan\left(\frac{qv-\omega}{\eta}\right)\right]\Bigg)\\
&\quad+\sinh(\eta L/v)\Bigg(\sum_{l,l'=\pm1}\mathrm{sgn}(l)Si[(\omega-lqv-l'i\eta)L/v]\Bigg)\Bigg],\\
\mathcal{J}_1 &\equiv \int \frac{d\Omega}{4\pi}\left|\frac{\sin[(\omega-\mathbf{v}\cdot\mathbf{q}+i\eta)L/2v]}{\omega-\mathbf{v}\cdot\mathbf{q}+i\eta}\right|^2(\omega-\mathbf{v}\cdot\mathbf{q})\\
&= \frac{1}{8qv}\Bigg[\cosh(\eta L/v)\Bigg(\sum_{l,l'=\pm1}\mathrm{sgn}(l)Ci[(\omega-lqv\\
&\quad-l'i\eta)L/v]+\ln\left[\frac{(\omega+qv)^2+\eta^2}{(\omega-qv)^2+\eta^2}\right]\Bigg)-i\sinh(\eta L/v)\\
&\quad\times\Bigg(\sum_{l,l'=\pm1}\mathrm{sgn}(l)\mathrm{sgn}(l')Si[(\omega-lqv-l'i\eta)L/v]\Bigg)\Bigg],
\end{aligned}$$

$$\mathcal{J}_2 \equiv \int \frac{d\Omega}{4\pi} \left| \frac{\sin[(\omega - \mathbf{v}\cdot\mathbf{q} + i\eta)L/2v]}{\omega - \mathbf{v}\cdot\mathbf{q} + i\eta} \right|^2 (\omega - \mathbf{v}\cdot\mathbf{q})^2$$
$$= \frac{1}{8qv}\Bigg[\eta\cosh(\eta L/v)\Bigg(i\sum_{l,l'=\pm1}\mathrm{sgn}(l)\mathrm{sgn}(l')Ci \times[(\omega - lqv - l'i\eta)L/v] - 2\Bigg[\arctan\left(\frac{qv+\omega}{\eta}\right) + \arctan\left(\frac{qv-\omega}{\eta}\right)\Bigg] + \frac{4qv}{\eta}\Bigg) + \eta\sinh(\eta L/v) \times\left(\sum_{l,l'=\pm1}\mathrm{sgn}(l)Si[(\omega - lqv - l'i\eta)L/v]\right) - \left(\frac{4v}{L}\right)\cos(L\omega/v)\sin(Lq)\Bigg], \tag{A1}$$

where sgn is the sign function and Ci and Si are the cosine and sine integrals, respectively. Despite their appearance, these functions are all real for real values of ω, q, v, L, and η.

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