ELECTROMAGNETIC MODES IN AN ELECTRONIC GAS WITH ONE-DIMENSIONAL HARMONIC MODULATION OF THE CARRIER CONCENTRATION

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ABSTRACT

With the aid of a Green function formalism, it is discussed the spectra of collective excitations in a system with a dielectric function which varies spatially due to the harmonic one-dimensional modulation of the carrier concentration. The band features are considered in the long wavelength region for arbitrary amplitudes of the modulated dielectric permittivity.

1. Problem: The new phenomena involved with the reduction of the dimensionality of semiconductor systems and the potential applications of such phenomena in optoelectronics have increased the interest on the study of optical properties of low dimensional systems. In this communication we consider the propagation along the x axis of TE electromagnetic modes with frequency ω and wave vector k_x in a system with one-dimensional harmonic modulation of period ? (along the superlattice axis x) of the dielectric permittivity. In a previous work [1] it was shown that the electric field amplitude y satisfies the differential equation:

$$L_x y(x) = -da^2(1+\cos(x))y(x)\,, \qquad (1)$$

where $L_x = \dfrac{d^2}{dx^2} - \alpha^2$, $\alpha^2 = b^2 + a^2(d-1)$, $a = \dfrac{\omega\sqrt{\mu\varepsilon}}{K}$, $b = \dfrac{k_x}{K}$, $K = \dfrac{2\pi}{\Lambda}$.

Here ε y μ are, respectively, the unmodulated dielectric permittivity and the magnetic permeability of the medium and d is the modulation amplitude. In [1] the approximate solutions, corresponding to small values of the parameter d were obtained with the aid of the Floquet's theorem. In the present work we obtain the corresponding solutions of (1) for arbitrary values of d by means of a formalism which uses the Green function method applied to superlattice systems [2]. In order to achieve this goal, we choose a function $G(x,x')$ such that:

$$L_{x'}G(x,x') = \delta(x-x') \text{ with } G(x,x') = 0 \text{ , when } |x| = \infty\,,$$

$$\lim_{h\to 0}\left(G(x,x')\big|_{x=x'+h} - G(x,x')\big|_{x=x'-h}\right) = 0\,,$$

$$\lim_{h\to 0}\left(\left.\frac{\partial G(x,x')}{\partial x}\right|_{x=x'+h} - \left.\frac{\partial G(x,x')}{\partial x}\right|_{x=x'-h}\right) = 1\,, \qquad (2)$$

where $x, x' \in (-\infty, \infty)$. Let us remark that the function $G(x,x')$ is the Green function of the equation $L_x y(x) = f(x)$. The solution of (2) and $\alpha^2 \geq 0$ can be easily found and it is given by $G(x,x') = -\frac{1}{2|\alpha|}e^{-|\alpha||x-x'|}$. If we multiply (1) by $G(x,x')$ and integrate by part, then we obtain:

$$y(x) = \frac{da^2}{2|\alpha|}\int_{-\infty}^{\infty} e^{-|\alpha||x-x'|}(1+\cos(x'))y(x')dx'. \tag{3}$$

We see that equation (1) has been transformed into an integral equation with the kernel $-da^2 G(x,x')(1+\cos(x'))$. In order to transform the integration interval into a finite interval, let us write:

$$1+\cos(x') = \sum_{n=-\infty}^{+\infty} F_n(x'), \tag{4}$$

where $F_n(x') = \begin{cases} 1+\cos(|x'-2n\pi|), & if|x'-2n\pi| \le \pi \\ 0 & if|x'-2n\pi| > \pi \end{cases}$.

After replacing (4) into (3) and performing the variable change $u = x' - 2n\pi$ we obtain:

$$y(x) = \frac{da^2}{2|\alpha|}\int_{-\pi}^{+\pi}\sum_{n=-\infty}^{+\infty} e^{-|\alpha||x-u-2n\pi|}(1+\cos(u))y(u+2n\pi)du. \tag{5}$$

According to Floquet's theorem, the solutions $y_\beta(u)$ of Mathieu's equation (1) are such, that:

$$y_\beta(u+2n\pi) = e^{i2\pi n\beta} y_\beta(u), \tag{6}$$

where β is a parameter, which can be real or complex. When we take into account this property, (5) is transformed into:

$$y_\beta(x) = \frac{da^2}{2|\alpha|}\int_{-\pi}^{+\pi}\sum_{n=-\infty}^{+\infty} e^{-|\alpha||x-u-2n\pi|}(1+\cos(u))e^{i2\pi n\beta} y_\beta(u)du. \tag{7}$$

This is the final form of the integral equation with respect to $y_\beta(x)$, which is equivalent to Mathieu's equation (1), with solutions satisfying Floquet's theorem (6). In the following we will apply the approximate methods for solving this equation.

2. Solution of the integral equation. The approximate solution of the integral equation (7) can be constructed in the following form [3]: the integral interval is divided in k equal parts such that $\Delta u = \Delta x = \frac{2\pi}{k}$ and the functions depending on x y u which enter (7) are evaluated at the points $x = -\pi + \frac{\pi}{k}(2p-1)$ and $u = -\pi + \frac{\pi}{k}(2q-1)$, where $p, q = 1,2,...,k$. By this means (7) can be discretized and we obtain:

$$\sum_{q=1}^{k}\left[\delta_{pq}-\frac{\pi d a^{2}}{k|\alpha|}L_{pq}\right]y_{\beta q}=0,\; y_{\beta_q}=y_{\beta}\left(-\pi+\frac{\pi}{k}(2q-1)\right), \tag{8}$$

where $$L_{pq}=\sum_{n=-\infty}^{\infty} e^{i2\pi n\beta}\, e^{\frac{-2\pi|\alpha|-p+q+n\pi||}{k}}\left(1+\cos\left(-1+\frac{2q-1}{k}\right)\right).$$

The system (8) of k algebraic equations with respect to unknowns $y_{\beta q}$, $q=1,2,\ldots,k$ approximates the solution of the integral equation (7), this solution being exact when $k\to\infty$. In order to have non trivial solutions it is required that the system determinant be equal to zero:

$$\left[\delta_{pq}-\frac{\pi d a^{2}}{k|\alpha|}L_{pq}\right]=0. \tag{9}$$

This is a determinant of infinite order. In our numerical analysis we have truncated it to an order of $k=16$.

3. Results. Equation (9) establishes a relationship between parameters a, d and b of the Mathieu's equation (1) and the parameter β of the Floquet's theorem (6). We will limit our analysis to solution $y_{\beta}(x)$ with real β. In this case each solution of (9) is a periodic function of β with unit period. For this reason it is enough to consider the interval $-0.5\leq\beta\leq 5.0$. If we fix d and b in the expression (9), we can obtain a as a function of β. It is of interest to consider the real part of this function, which is an even function of β. In Table 1 the obtained data are shown for $d=1$ and $b=0$. These data are distributed in bands, which are illustrated graphically in Fig 1. The bands are separated by stop bands (gaps), which are intervals of forbidden values of a. For these values the solutions of (1), in the form established by the Floquet`s theorem, correspond to complex values of β. The gap of a is generated by the threshold value $\beta=0.5$. This behaviour of a vs b is illustrated in Table *2* and in Figs. *2* y *3* for $d=1$, $\beta=0$ and $d=1, \beta=0.5$. The splitting of the dispersion curve shown in Fig.1 is represented in Fig. *3*. For a given value of b, the separation between curves gives the width of the forbidden region of a.

TABLE 1

β	a band 1	a band 2	β	a band 1	a band2	β	a band 1	a band2
0	0.0013	0.9485	0.125	0.1255	0.867	0.325	0.3185	0.709
0.005	0.0017	0.9485	0.15	0.1505	0.839	0.35	0.3385	0.701
0.01	0.0043	0.9475	0.175	0.1765	0.811	0.375	0.3555	0.695
0.015	0.011	0.9465	0.2	0.2015	0.785	0.4	0.3705	0.691
0.02	0.015	0.9455	0.225	0.2265	0.763	0.425	0.3825	0.688
0.05	0.0514	0.9335	0.25	0.2515	0.745	0.45	0.3905	0.686

0.075	0.0755	0.9165	0.275	0.2745	0.730	0.475	0.3965	0.684
0.1	0.1005	0.8935	0.3	0.2975	0.718	0.5	0.3975	0.684

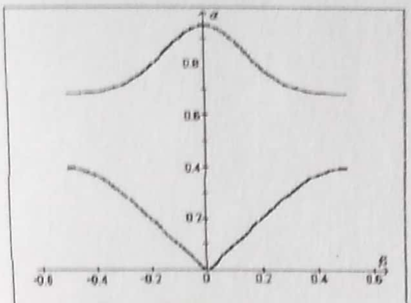
Figura 1. Band structure a vs β

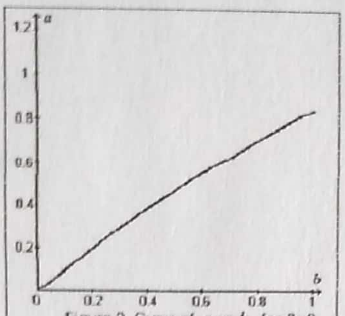
Figura 2. Curve of a vs b for β=0

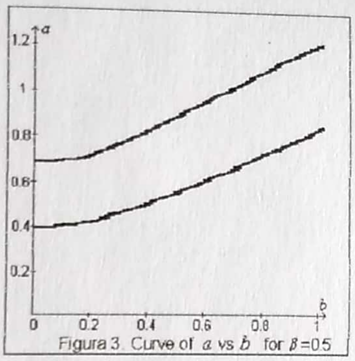
Figura 3. Curve of a vs b for β=0.5

TABLE 2

$d=1, \beta=0$											
b	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
a	0.01	0.1	0.2	0.3	0.39	0.47	0.56	0.63	0.71	0.78	0.85
$d=1, \beta=0.5$											
b	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
a	0.4	0.41	0.43	0.47	0.51	0.56	0.62	0.66	0.74	0.8	0.87
a	0.69	0.7	0.73	0.78	0.84	0.91	0.98	1.05	1.12	1.19	1.25

4. Conclusion. We have discussed the features of the spectra of collective excitations in a system with a harmonic one-dimensional modulation of the dielectric permittivity. For arbitrary amplitude of modulation there appear allowed and stop bands which strongly change the spectra of the free electromagnetic wave. This problem can be extended to more complex systems with broken translational symmetry along the superlattice axis and with temporal dispersion of the dielectric permittivity.

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