Dynamical mass generation in QED with magnetic fields: arbitrary field strength and coupling constant

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We study the dynamical generation of masses for fundamental fermions in quenched quantum electrodynamics, in the presence of magnetic fields of *arbitrary strength*, by solving the Schwinger-Dyson equation (SDE) for the fermion self-energy in the rainbow approximation. We employ the Ritus eigenfunction formalism which provides a neat solution to the technical problem of summing over all Landau levels. It is well known that magnetic fields *catalyze* the generation of fermion mass m for arbitrarily small values of electromagnetic coupling α. For intense fields it is also well known that $m \propto \sqrt{eB}$. Our approach allows us to span all regimes of parameters α and eB. We find that $m \propto \sqrt{eB}$ provided α is small. However, when α increases beyond the critical value α_c which marks the onslaught of dynamical fermion masses in vacuum, we find $m \propto \Lambda$, the cut-off required to regularize the ultraviolet divergences. Our method permits us to verify the results available in literature for the limiting cases of eB and α. We also point out the relevance of our work for possible physical applications.

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It is well known that fermions can acquire mass dynamically, i.e., through self interactions by means of non perturbative effects without the need of a non zero bare mass. This phenomenon is called the dynamical mass generation (DMG) and can be studied in the continuum through the Schwinger-Dyson equations (SDEs). In quantum electrodynamics (QED), DMG takes place only if the electromagnetic coupling α exceeds a critical value α_c. However, the presence of magnetic fields brings about a drastic change and it is possible to generate fermion masses for any value of the coupling. This phenomenon, named *magnetic catalysis* [1, 2, 3, 4] has been extensively studied in the situation where the field is strong. In this case, only the lowest Landau level (LLL) is enough to describe it and the analysis simplifies considerably.

Physically, for strong fields, Landau levels are widely spaced making it energetically less favorable for virtual particles to populate levels other than the LLL. Moreover, for weak coupling, the dynamics describing the formation of a condensate is dominated by the LLL. The essence of the effect is a dimensional reduction brought about by the presence of the magnetic field that produces an effectively stronger interaction among virtual particles and antiparticles in the vacuum. This interaction is even stronger in the LLL since the only component of the momentum contributing to the energy is the longitudinal one and this makes it more easy for virtual pairs to meet each other and condense.

Nevertheless, one can envision a situation where the magnetic field is not so strong and therefore that low energy levels are close to the LLL in such a way that virtual particles in those levels also contribute to the dynamics of condensate formation. Furthermore one could study the phenomenon as a function of the coupling constant to interpolate between the small coupling domain where the dynamics is magnetic field driven to the strong coupling domain where the dominance should switch to the description of dynamical symmetry breaking in vacuum.

To the best of our knowledge, calculations aiming to describe magnetic catalysis have non been performed considering the contribution of all Landau levels, for arbitrary field strength and coupling constant, perhaps because such calculations involve the technical challenge of carrying out a seemingly prohibitive sum over these levels. In this work, we undertake the study of DMG in QED in the rainbow approximation in the presence of magnetic fields of arbitrary intensity. We present a solution to the above technical difficulty and use it to carry out a detailed quantitative analysis of the dynamically generated fermion mass for arbitrary values of the coupling constant and magnetic field intensities.

It has been shown [5] (see also Ref. [6]) that the mass operator in the presence of an electromagnetic field can be written as a combination of the structures $\gamma^\mu\Pi_\mu$, $\sigma^{\mu\nu}F_{\mu\nu}$, $(F_{\mu\nu}\Pi^\nu)^2$, $\gamma_5 F_{\mu\nu}\tilde{F}^{\mu\nu}$ which commute with the operator $(\gamma_\mu\Pi^\mu)^2$, where $\Pi_\mu = i\partial_\mu - eA_\mu^{\text{ext}}$, $F_{\mu\nu} = \partial_\mu A_\nu^{\text{ext}} - \partial_\nu A_\mu^{\text{ext}}$, $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\tau}F_{\lambda\tau}$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and A^{ext} is the external vector potential. We take $A^\mu_{\text{ext}} = B(0, -y/2, x/2, 0)$ in such a way that it gives rise to a constant magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. The two-point fermion Green's function $G(x, y)$ is found through the equation

$$\gamma \cdot \Pi(x)G(x,y) - \int d^4x' M(x,x')G(x',y) = \delta^4(x-y) \;, \quad (1)$$

where the mass operator $M(x, y)$ is described by its SDE in the rainbow approximation as

$$M(x,x') = -ie^2\gamma^\mu G(x,x')\gamma^\nu D^{(0)}_{\mu\nu}(x-x'). \quad (2)$$

In the presence of a constant external field, the fermion asymptotic states are no longer free particle states, but instead are described by eigenfunctions of the operator $(\gamma^\mu\Pi_\mu)^2$. Therefore, it is convenient to work in the representation spanned by these eigenfunctions, rendering the mass operator diagonal and allowing to write the equation for the mass function as [7],

$$\mathcal{M}(p_\parallel, n_p) = -ie^2 \sum_{\sigma_k,\sigma_p=\pm 1}\sum_{n_k,s_k} \frac{s_k!(n_p-\frac{(\sigma_p+1)}{2})!}{s_p!(n_k-\frac{(\sigma_k+1)}{2})!}\int \frac{d^4q}{(2\pi)^4}\frac{e^{-q_\perp^2/2\gamma}}{q^2+i\epsilon}\frac{\mathcal{M}((p-q)_\parallel,n_k)}{(p-q)_\parallel^2-2eBn_k-\mathcal{M}^2((p-q)_\parallel,n_k)}$$
$$\times\left(\frac{q_\perp^2}{4\gamma}\right)^{l_k-l_p-\frac{(\sigma_k-\sigma_p)}{2}}\left[2+\frac{(1-\xi)}{q^2}\left(q_\perp^2(1-\delta_{\sigma_p\sigma_k})-q_\parallel^2\delta_{\sigma_p\sigma_k}\right)\right]\left[L_{n_p-\frac{(\sigma_p+1)}{2}}^{n_k-n_p-\frac{(\sigma_k-\sigma_p)}{2}}\left(\frac{q_\perp^2}{4\gamma}\right)\right]^2\left[L_{s_k}^{s_p-s_k}\left(\frac{q_\perp^2}{4\gamma}\right)\right]^2, \quad (3)$$

where $q_\perp=(0,q_1,q_2,0)$, $q_\parallel=(q_0,0,0,q_3)$, and thus, in Minkowski space, $q^2=q_\parallel^2-q_\perp^2$. Also, $\gamma=eB/2$, $p^2=E_p^2-p_z^2-2eBn$, ξ is the covariant gauge parameter and L_n^m are Laguerre functions. We shall assume that the wave function renormalization equals one. After the structures upon which the mass operator M depends have been accounted for, the mass function should be a scalar matrix whose components can in principle be different in the transverse and longitudinal directions due to the presence of the field. Here, let us work with the ansatz that $\mathcal{M}$ is proportional to the unit matrix. The assumption is good when considering small momentum [2].

We expect that $\mathcal{M}((p-q)_\parallel,n_k)$ should be independent of s_k since the energy only depends on the principal quantum number n_k. We also assume that $\mathcal{M}((p-q)_\parallel,n_k)$ is a slowly varying function of n_k and thus make the approximation $\mathcal{M}((p-q)_\parallel,n_k)\sim\mathcal{M}((p-q)_\parallel,n_k=0)$. For consistency we consider the case $n_p=0$. Hereafter, we employ the more convenient notation $\mathcal{M}(k_\parallel,n_k=0)\equiv\mathcal{M}(k_\parallel)$ for generic arguments of the mass function. With these considerations the sum over s_k can be performed

$$\begin{aligned}\mathcal{M}(p_\parallel) =& -ie^2\sum_{\sigma_p,\sigma_k}\sum_{k=0}^{\infty}\int\frac{d^4q}{(2\pi)^4}\frac{e^{-\frac{q_\perp^2}{4\gamma}}}{q^2+i\varepsilon}\\ &\times\frac{\mathcal{M}((p-q)_\parallel)}{\left\{(q-p)_\parallel^2-2eB(k+(\sigma_k+1)/2)\right\}-\mathcal{M}^2\left((p-q)_\parallel\right)}\\ &\times\left\{2+(1-\xi)(1-\delta_{\sigma_p\sigma_k})\frac{q_\perp^2}{q^2}-(1-\xi)\delta_{\sigma_p\sigma_k}\frac{q_\parallel^2}{q^2}\right\}\\ &\times(-1)^{k-m}L_m^{k-m}\left(\frac{q_\perp^2}{4\gamma}\right)L_k^{-(k-m)}\left(\frac{q_\perp^2}{4\gamma}\right),\end{aligned} \quad (4)$$

where $k=n_k-\frac{\sigma_k+1}{2}$. It is worth mentioning that after summing over s_k, the resulting equation is the same as Eq. (50) in Ref. [2] when considering $n_k=0$. It corresponds to the strong field limit. Using the identity

$$\frac{1}{(p-q)_\parallel^2-2eB(k+\frac{\sigma_k+1}{2})-\mathcal{M}^2((p-q)_\parallel)}= -i\int_0^\infty ds e^{is[(p-q)_\parallel^2-2eB(k+\frac{\sigma_k+1}{2})-\mathcal{M}^2((p-q)_\parallel)+i\epsilon]}, \quad (5)$$

and carrying out the sums over k, σ_k and σ_p, we get

$$\begin{aligned}\mathcal{M}(p_\parallel) =& -e^2\int\frac{d^4Q}{(2\pi)^4}\frac{\mathcal{M}\left((\frac{p}{4\gamma}-Q)_\parallel\right)}{Q^2+i\varepsilon}\int_0^\infty d\tau\\ &\times\ e^{-Q_\perp^2[1-\exp(-i\tau)]}e^{i\left[(2\sqrt{\gamma}Q-p)_\parallel^2-\mathcal{M}^2\left((\frac{p}{4\gamma}-Q)_\parallel\right)+i\epsilon\right]\frac{\tau}{4\gamma}}\\ &\times\left[\left\{2-(1-\xi)\frac{Q_\parallel^2}{Q^2}\right\}+\left\{2+(1-\xi)\frac{Q_\perp^2}{Q^2}\right\}e^{-i\tau}\right],\end{aligned} \quad (6)$$

where $Q=\frac{q}{2\sqrt{\gamma}}$ and $\tau=4\gamma s$. This is the integral equation for the mass function in Minkowski space. To perform the integral in the variable τ we notice that the integrand dies out sufficiently rapidly for large imaginary values of τ. We can thus close the contour of integration on a path whose first leg is a horizontal line along the real τ-axis, continued along the quarter-circle at infinity in the right-lower quadrant and finally along the negative imaginary τ-axis. Using Cauchy's theorem, we can thus write Eq. (6) as

$$\begin{aligned}\mathcal{M}(p_\parallel) =& e^2\int\frac{d^4Q}{(2\pi)^4}\frac{\mathcal{M}((\frac{p}{4\gamma}-Q)_\parallel)}{Q^2}\int_0^\infty d\tau\\ &\times\ e^{-Q_\perp^2[1-\exp(-\tau)]}e^{-\left[(2\sqrt{\gamma}Q-p)_\parallel^2+\mathcal{M}^2\left((\frac{p}{4\gamma}-Q)_\parallel\right)\right]\frac{\tau}{4\gamma}}\\ &\times\left[\left\{2-(1-\xi)\frac{Q_\parallel^2}{Q^2}\right\}+\left\{2-(1-\xi)\frac{Q_\perp^2}{Q^2}\right\}e^{-\tau}\right].\end{aligned} \quad (7)$$

To guaranty convergence of the integral over τ, we need to consider momenta Q and p in Euclidian space and accordingly, a Wick rotation on Q has already been performed in Eq. (7). We now perform the change of variable

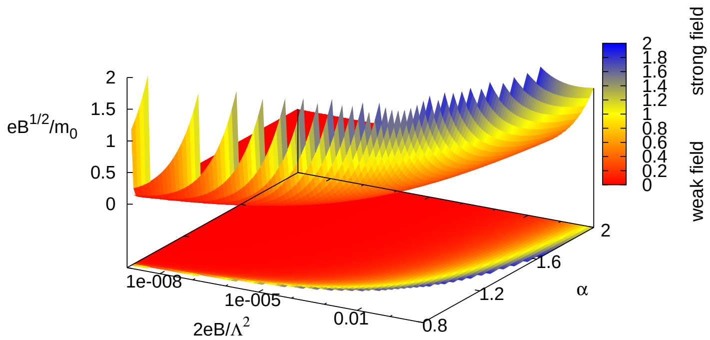

FIG. 1: (Color online) Relative strength of the magnetic field $\sqrt{eB}/m_0$, m_0 is the dynamical mass in the vacuum, as a function of the coupling α and the parameter $2eB/\Lambda^2$. Studying this dependence is possible only for $\alpha > \alpha_c = \pi/4$ where $m_0 \neq 0$. Towards yellow and blue, magnetic field becomes strong and towards red, it becomes weak. Note that for higher couplings, it is easier to access weak fields. On the other hand, lower couplings imply strong magnetic fields unless $eB << \Lambda^2$.

$x = e^{-\tau}$ to get

$$\mathcal{M}(p_\parallel) = e^2 \int \frac{d^4Q}{(2\pi)^4} \frac{\mathcal{M}((\frac{p}{4\gamma}-Q)_\parallel)}{Q^2} \int_0^1 dx e^{-Q_\perp^2(1-x)} \times x^{\left[(Q-p)_\parallel^2+\frac{\mathcal{M}^2}{2eB}-1\right]} \times \left[\left\{2-(1-\xi)\frac{Q_\parallel^2}{Q^2}\right\}+\left\{2-(1-\xi)\frac{Q_\perp^2}{Q^2}\right\}x\right], \quad (8)$$

where we have not shown the argument of $\mathcal{M}^2$ to avoid cumbersome notation.

To illustrate the calculation of the mass function, we work in the Feynman gauge, $\xi = 1$. It is well known that in the ladder approximation, the most preferable gauge is the Landau gauge $\xi = 0$ because it comes the closest to satisfying the Ward-Takahashi identity. However, within the ladder approximation, working with a different value of ξ gets reflected in the fact that–in the free field case–the coefficient of the coupling constant changes from 1 to $(1 + \xi/3)$. Therefore, in the ladder approximation, the physical picture of dynamical mass generation should be similar in all gauges close to the Landau gauge, including $\xi = 1$. For example, the critical coupling changes from approximately 1.04 for Landau gauge to 0.78 for the Feynman gauge (see fig. 4). The implicit assumption is that the same happens in the presence of the field. This can be entirely justified for the weak field limit since one always keeps close to vacuum. In the strong field case it is also justified, since our results reproduce this very well studied case (see for instance Ref. [2] where the analysis was also carried out in the Feynman gauge).

Upon the change of variables $Q \to Q + p_\parallel/\sqrt{4\gamma}$,

$$\mathcal{M}\left(p_\parallel/\sqrt{4\gamma}\right) = e^2 \int \frac{d^4Q}{(2\pi)^4} \frac{\mathcal{M}(Q_\parallel)}{(Q+p_\parallel)^2} \times \int_0^1 dx x^\lambda e^{-Q_\perp^2(1-x)}\ 2(1+x), \quad (9)$$

where $\lambda = Q_\parallel^2 + \frac{\mathcal{M}^2(Q_\parallel)}{4\gamma} - 1$. To perform the angular integration, we write $d^4Q = d^2Q_\perp d^2Q_\parallel = \frac{\pi}{2} dQ_\perp^2 dQ_\parallel^2 d\theta$, where θ is the angle between $Q_\parallel$ and $p_\parallel$. Hereafter, we assume that the mass function depends only on the magnitude of its argument. The angular integration is now easily carried out and the result can be expressed as

$$\mathcal{M}(p_\parallel/\sqrt{4\gamma}) = \frac{e^2}{2(2\pi)^2} \int_0^1 dx[1+x] \int dQ_\parallel^2 x^\lambda \mathcal{M}(Q_\parallel) \times \int_0^\infty \frac{dQ_\perp^2\ e^{-Q_\perp^2(1-x)}}{\sqrt{\left[Q_\parallel^2 - \frac{p_\parallel^2}{4\gamma}\right]^2 + 2Q_\perp^2\left[\frac{p_\parallel^2}{4\gamma}+Q_\parallel^2\right] + Q_\perp^4}}. \quad (10)$$

We now approximate the argument of the square root in the denominator by intervals. For $p_\parallel^2/4\gamma \geq Q_\parallel^2$ we take $\frac{p_\parallel^2}{4\gamma} + Q_\parallel^2 \sim \frac{p_\parallel^2}{4\gamma}$. Conversely, for $Q_\parallel^2 \geq p_\parallel^2/4\gamma$ we take $\frac{p_\parallel^2}{4\gamma} + Q_\parallel^2 \sim Q_\parallel^2$. With this approximation, the integral over $Q_\perp$ can be analytically performed and the result is

$$\mathcal{M}(p_\parallel/\sqrt{4\gamma}) = \frac{e^2}{2(2\pi)^2} \int_0^1 dx[1+x]x^\lambda \mathcal{M}(Q_\parallel) \times \left\{ \int_0^{\frac{p^2}{4\gamma}} dQ_\parallel^2 \exp\left[(1-x)\frac{p^2}{4\gamma}\right] \Gamma\left[0,(1-x)\frac{p^2}{4\gamma}\right] + \int_{p^2/4\gamma}^\infty dQ_\parallel^2 \exp\left[(1-x)Q_\parallel^2\right] \Gamma\left[0,(1-x)Q_\parallel^2\right] \right\}, \quad (11)$$

where $\Gamma(x, y)$ is the incomplete gamma function. Notice that the approximations leading to Eq. (11) make no reference to the strength of the magnetic field. Therefore, this equation is valid for arbitrary magnetic field intensities. In the following, we use the result in Eq. (11) to numerically explore the behavior of the mass function when varying either the magnetic field strength or the value of the coupling constant. The validity of our approximations gets a numerical affirmation when in certain limiting parametric regime of α and eB, we retrieve the results already known in the literature.

For $\alpha > \alpha_c (= \pi/4)$, we determine the strength of the magnetic field by comparing it with the dynamically generated mass m_0 in the vacuum. $\sqrt{eB}/m_0 >> 1$ is the strong field limit whereas $\sqrt{eB}/m_0 << 1$ corresponds to the weak field limit. In the Feynman gauge, this ratio has been depicted in fig. 1. For $\alpha < \alpha_c$, $m_0 = 0$. Therefore, the strength of the magnetic field can only be compared with the ultraviolet cut-off Λ.

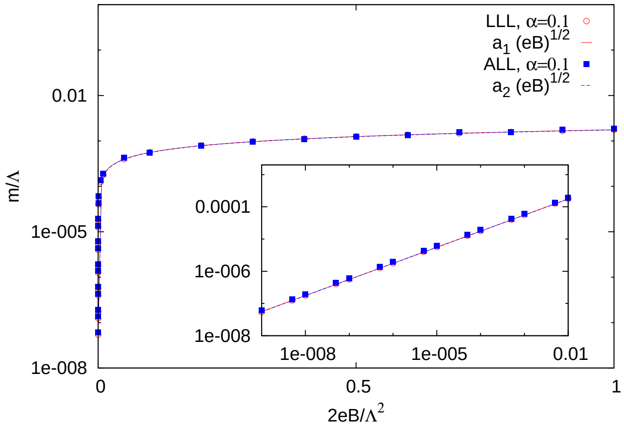

FIG. 2: (Color online) Dynamical mass as a function of the magnetic field in units of ultraviolet cut-off in the weak coupling regime. Blue dots correspond to summing over all Landau levels whereas red dots correspond to the LLL approximation. We have used $\alpha = 0.1$. Furthermore, $a_1 = 0.00176653$ and $a_2 = 0.00174613$. As $\alpha < \alpha_c$, we get $m \propto \sqrt{eB}$, no matter the strength of the magnetic field.

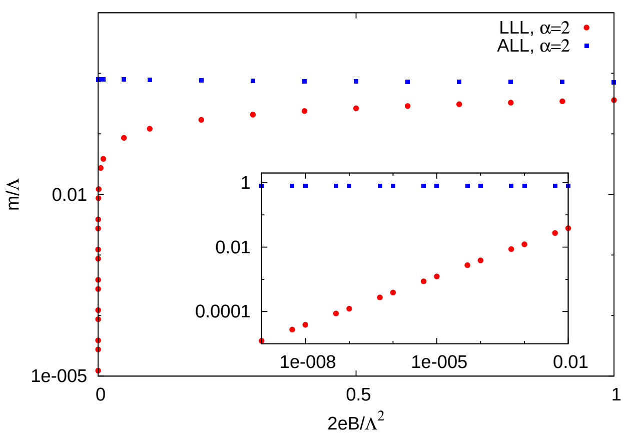

FIG. 3: (Color online) Dynamical mass as a function of the magnetic field in units of ultraviolet cut-off in the strong coupling regime. Blue dots correspond to summing over all Landau levels whereas red dots correspond to the LLL approximation. We have used $\alpha = 2$. As we practically get a flat line, we conclude that $m \propto \Lambda$.

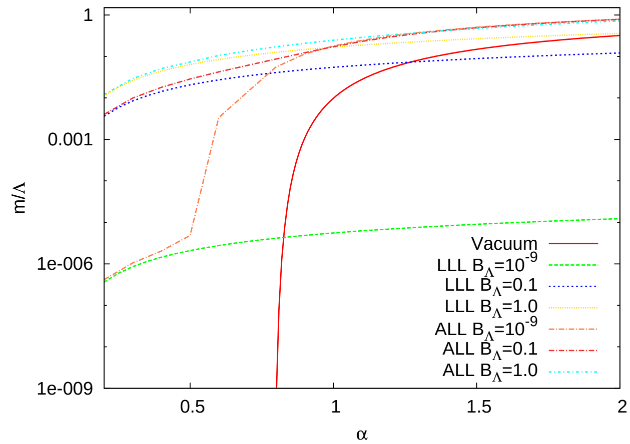

FIG. 4: (Color online) Dynamical mass as a function of the coupling constant for different values of the magnetic field. We have used the notation $\mathrm{B}_\Lambda = 2eB/\Lambda^2$. For comparison, we show the behavior of the dynamical mass in the absence of a magnetic field. Note that for large values of B_Λ the dynamical mass obtained by considering just the contribution of the LLL is practically the same as the one obtained by considering the contribution of all Landau levels. The situation changes for small values of B_Λ where it can be seen that other levels than just the LLL contribute to the dynamical mass.

Therefore, we first study the dependence of the mass function on the magnetic field strength for $\alpha < \alpha_c$. We concentrate on the dependence of the dynamically generated mass m, defined as the mass function evaluated at zero momentum. In fig. 2, we plot the solution to Eq. (11) containing the sum over all Landau levels for a very small value of the coupling, $\alpha = 0.1$, as a function of the magnetic field strength, ranging from eB/Λ^2 as large as 1 to as low as 10^{-9}. Our results are virtually the same as obtained from the LLL approximation [2] as expected.

We now turn our attention to the study of the mass function for $\alpha > \alpha_c$. Corresponding results are shown in fig. 3. Note that considering all Landau levels changes the results considerably in comparison with the predictions of the LLL. The dynamically generated mass $m \propto \Lambda$. Note how the proportionality of the mass function switches from $\sqrt{eB}$ to Λ when we move from $\alpha < \alpha_c$ to $\alpha > \alpha_c$.

Finally, we present the explicit dependence of m on the coupling constant in fig. 4 both for the LLL and all Landau levels contributing. For comparison, we show the corresponding dynamically generated mass in vacuum. Notice that, as is known, magnetic field of *arbitrary strengths, however small* ($eB << \Lambda^2$) catalyze the appearance of a dynamically generated mass for any value of α. For large values of $\mathcal{B}_\Lambda = 2eB/\Lambda^2$, the dynamical mass obtained by considering just the contribution of the LLL matches onto the one obtained by considering the contribution of all Landau levels. Going towards decreasing α, this matching gets triggered early on if $eB \approx \Lambda^2$. However, one has to go to very small values of α to achieve the same if $eB << \Lambda^2$ as we might expect on physical grounds. The situation changes for small values of $\mathcal{B}_\Lambda$ where it can be seen that other levels than just the LLL contribute to the dynamical mass. In this last case, the transition region is around the critical value of the coupling constant in vacuum, α_c. For $\alpha > \alpha_c$ the dynamics is dominated by the strength of the coupling constant, as the largest contribution to the mass comes from the intensity of self-interactions rather than from the magnetic field contribution.

Notice how the results depicted in fig. 4 are harmoniously consistent with those in figs. 2 and 3. Let us first focus on fig. 2. It has been drawn for $\alpha = 0.1$. This value of α corresponds to the far left of fig. 4 where our results including all the Landau levels are practically the same as the LLL results, i.e., the dynamically generated mass is proportional to $\sqrt{eB}$. Let us now compare fig. 4 with fig. 3. Figure 3 has been obtained by setting $\alpha = 2$, which corresponds to the far right of fig. 4. Note that at that end, our results including all Landau levels match onto each other for all values of the parameter $\mathcal{B}_\Lambda$ ranging from 10^{-9} to 1.0. This implies that for large values of α, m/Λ is in fact independent of $\mathcal{B}_\Lambda$. Therefore, we conclude that $m \propto \Lambda$, the result we earlier deduced from fig. 3. This shows how, fig. 4 is consistent both with figs. 2 and 3. In fact, it is complimentary. It clearly shows that for intermediate regions of α, we must take into account all Landau levels for a quantitatively correct description of magnetic catalysis.

In conclusion, we have studied the phenomenon of magnetic catalysis for arbitrary values of the coupling constant and magnetic field strength by taking into account the contribution from all the energy levels in the system. We have also shown that the phenomenon is *not always* restricted to the contribution of the LLL. This is especially evident for low and moderate values of the magnetic field. In the special case of the strong field limit and for small values of the coupling constant, studied extensively in the literature, our analysis confirms the well known behavior of the dynamically generated mass.

Our general results can have interesting cosmological consequences, as is exemplified for instance, in the study of the electroweak phase transition. Recall that this transition took place during the early universe for temperatures around $T = 100$ GeV. It has been recently shown that within the standard model, if a primordial magnetic field was present at this epoch, the phase transition becomes stronger first order [9]. The analysis is based on the study of the effective potential whose development with temperature is driven by the Higgs condensate. However a fermion mass generated by the magnetic field is tantamount of a fermion condensate whose contribution to the development of the phase transition, and in particular, to the true vacuum expectation value of the Higgs field could further modify the nature of the transition and influence the baryogenesis scenario.

Another interesting question concerns the study of higher order effects. It is well known that such effects change the behavior of the dynamically generated mass as a function of the coupling constant for strong fields and weak coupling (see the second to the last of Refs. [1]). In the context of the present work it is possible to study how this change evolves as the magnetic field and coupling constant take on arbitrary values. These issues deserve further investigation that we are currently pursuing and will be reported elsewhere.

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[1] V.P. Gusynin, V.A. Miransky and I.A. Shovkovy, Phys. Lett. B **349**, 477 (1995); Phys. Rev. D **52**, 4747 (1995); Nucl. Phys. B **462**, 249 (1996); Nucl. Phys. B **563**, 361 (1999); UKr. J. Phys. **45**, 603 (2000).

[2] D.-S. Lee, C.N. Leung and Y.J. Ng, Phys. Rev. D **55**, 6504 (1997).

[3] D.K. Hong, Phys. Rev. D **57**, 3759 (1998).

[4] E.J. Ferrer and V. de la Incera, Phys. Lett. B **481**, 287 (2000).

[5] V.I. Ritus, Ann. Phys. **69**, 555-582 (1972).

[6] E.J. Ferrer and V. de la Incera, Phys. Rev. D **58** 065008.

[7] A. Ayala, A. Bashir, A. Raya and E. Rojas, Phys. Rev. D **73**, 105009 (2006).

[8] G. Gangopadhyay, J. Phys. A **32**, L433 (1999).

[9] A. Sánchez, A. Ayala and G. Piccinelli, Phys. Rev. D **75** 043004 (2007).