Revised Criteria for Stability in the General Two-Higgs Doublet Model

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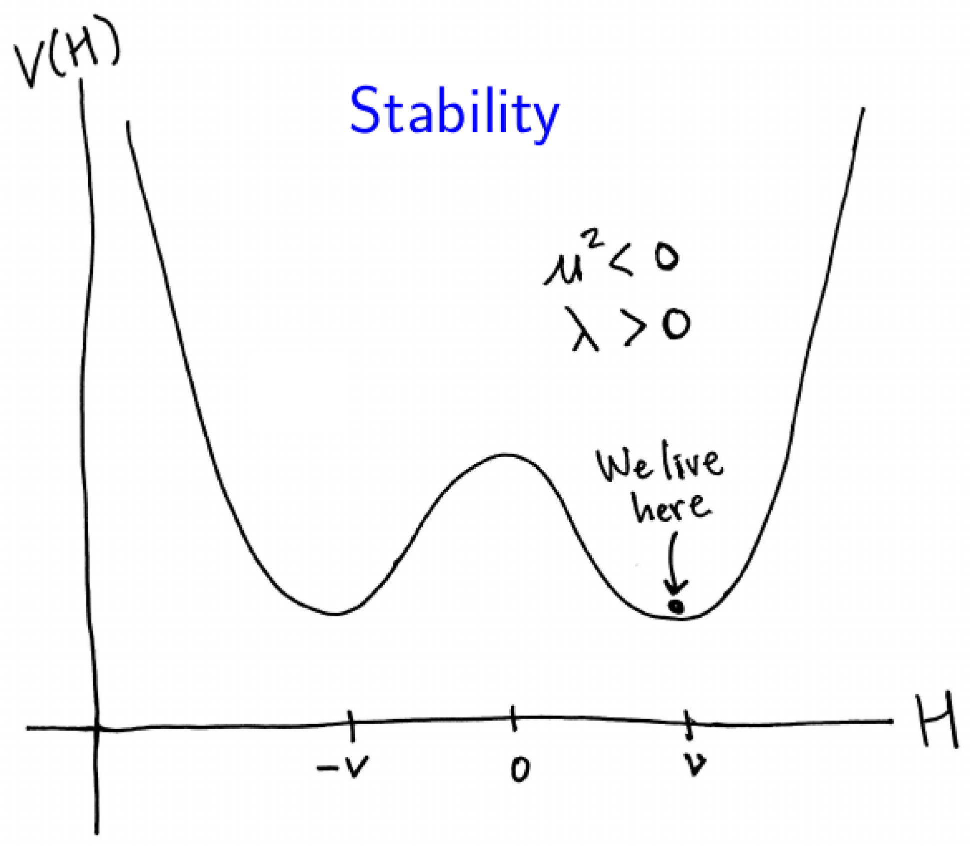
Abstract

We will revise one of the methods given in the literature to determine the necessary and sufficient conditions that the parameters must satisfy to have a stable scalar potential in the general two-Higgs doublet model (THDM). We will give a procedure that facilitates finding the conditions for stability of a scalar potential. The stability guarantees that the scalar potential has a global minimum, that is, the potential is bounded from below, which is a necessary condition to implement the spontaneous gauge-symmetry breaking in the models.

Introduction

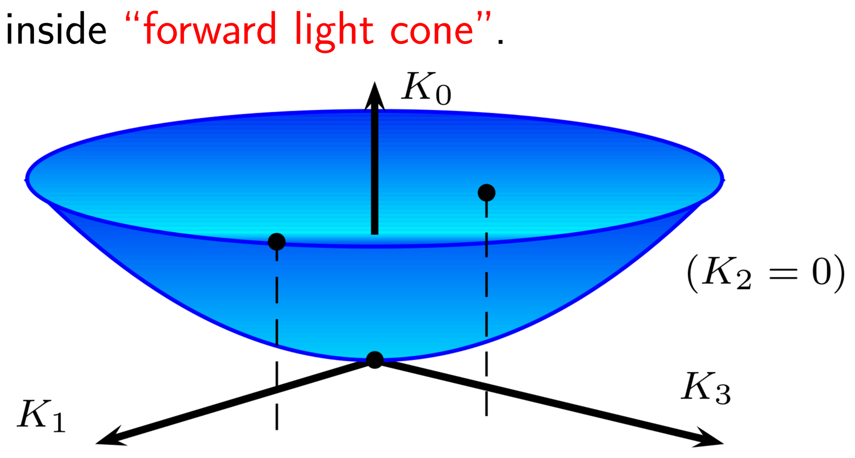


THDM Higgs Potential

- Two complex Higgs-doublet fields with hypercharge $y = +1/2$:
$$\varphi_1(x) = \begin{pmatrix} \varphi_1^+(x) \\ \varphi_1^0(x) \end{pmatrix}, \qquad \varphi_2(x) = \begin{pmatrix} \varphi_2^+(x) \\ \varphi_2^0(x) \end{pmatrix}.$$
- Renormalisable, gauge invariant potential contains only
$\varphi_i^\dagger \varphi_j, \qquad (\varphi_i^\dagger \varphi_j)(\varphi_k^\dagger \varphi_l), \quad i, j, k, l \in \{1, 2\}$
Definition: orbit variables K_0, K_1, K_2, K_3:
$$\begin{pmatrix} \varphi_1^\dagger \varphi_1 & \varphi_2^\dagger \varphi_1 \\ \varphi_1^\dagger \varphi_2 & \varphi_2^\dagger \varphi_2 \end{pmatrix} \equiv \frac{1}{2}(K_0 \mathbb{1} + K_a \sigma^a) \Leftrightarrow$$
$$\begin{cases} K_0 = \varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2, \ K_1 = 2\,\mathrm{Re}\,\varphi_1^\dagger \varphi_2, \\ K_3 = \varphi_1^\dagger \varphi_1 - \varphi_2^\dagger \varphi_2, \ K_2 = 2\,\mathrm{Im}\,\varphi_1^\dagger \varphi_2 \end{cases}$$
- General THDM Higgs potential :
$V(\varphi_1, \varphi_2) = V_2 + V_4$
with $\begin{cases} V_2 = \xi_0 K_0 + \xi_a K_a \\ V_4 = \eta_{00} K_0^2 + 2K_0 \eta_a K_a + K_a \eta_{ab} K_b \end{cases}$
- No gauge d.o.f. in this scheme, reduced powers

Orbit Variables

- Domain($\mathbf{K} \equiv (K_1\ K_2\ K_3)^T$):
$K_0 = ||\varphi_1||^2 + ||\varphi_2||^2 \quad \geq 0$
$K_0^2 - K^2 = 4\left(||\varphi_1||^2 ||\varphi_2||^2 - |\varphi_1^\dagger \varphi_2|^2\right) \quad \geq 0$
- Change of doublet basis by $U \in U(2)$
$$(\varphi_1'\ \varphi_2')^T = U\,(\varphi_1\ \varphi_2)^T,$$
means for orbit variables $K_0' = K_0$,
$\mathbf{K}' = R(U)\mathbf{K}$, $R(U) \in SO(3)$,
$U^\dagger \sigma^a U = R_{ab}(U)\sigma^b$.
- Minkowski type structure: $(K_0, \mathbf{K})$ on and inside "forward light cone".



Stability

- stable potential: bounded from below
- stability determined by V in limit $K_0 \to \infty$,
- consider V_4 for $K_0 > 0$, define: $\boldsymbol{k} \equiv \boldsymbol{K}/K_0$, with $|\boldsymbol{k}| \leq 1$.
$$V_4 = K_0^2 J_4(\boldsymbol{k}), \quad J_4(\boldsymbol{k}) \equiv \eta_{00} + 2\eta^T \mathbf{k} + \boldsymbol{k}^T E \mathbf{k}$$
- stability guaranteed by $V_4 \Leftrightarrow J_4(\boldsymbol{k}) > 0$ for all $|\boldsymbol{k}| \leq 1$
- domain of J_4 is unit ball: compact
$J_4 > 0 \Leftrightarrow J_4|_{stat} > 0$ for all its stationary points
- stationary points of $J_4(\boldsymbol{k}) \equiv \eta_{00} + 2\eta^T k + k^T E k$ on domain $\mathbf{k}^2 \leq 1$

$|k| < 1$: solve $\nabla_{\boldsymbol{k}} J_4(\boldsymbol{k}) = 0 \quad \Leftrightarrow Ek = -\eta$,
with $\quad 1 - \boldsymbol{k}^2 > 0$,

$|k| = 1$: define $F_4(\boldsymbol{k}, u) \equiv J_4(\boldsymbol{k}) + u(1 - \boldsymbol{k}^2)$,
with Lagrange multiplier $\quad u$,
solve $\nabla_k F_4(\boldsymbol{k}, u) = 0 \Leftrightarrow (E - u)k = -\eta$,
with $\quad 1 - \boldsymbol{k}^2 = 0$.

Revised criteria for stability

The deepest minimum

- Suppose we find two solutions $\mathbf{p}$ and $\mathbf{q}$ with their respective Lagrange multipliers u_p and u_q such that
$$(E - u_p)\mathbf{p} = -\boldsymbol{\eta},$$
$$(E - u_q)\mathbf{q} = -\boldsymbol{\eta},$$
where
$$|\mathbf{p}| = 1, \qquad |\mathbf{q}| = 1 \quad \text{and} \quad u_p \neq u_q.$$
- is concluded that
$$u_p < u_q \Longleftrightarrow J_4(\mathbf{p}) < J_4(\mathbf{q}). \qquad (1)$$
- it makes it easier to find the conditions of the parameters to have a stable scalar potential:
  - computed all the "regular" and "exceptional" Lagrange multipliers,
  - takes, from them, the smallest value, or assuming that each one of them is the lowest value.
  - if the smallest value is a regular solution, the condition $J_4(\mathbf{p}_i) > 0$ would guarantee the stability of the scalar potential.
  - If the smallest value is an exceptional solution, you must first verify that it gives a valid stationary point. In the case of being satisfied, impose the condition $J_4(\mathbf{p}_j) > 0$, which would guarantee the stability of the potential.
- conditions coming from regular solutions are necessary.
- conditions arising from the exceptional solutions may not be necessary, they are sufficient.

Example: Stability for THDM

THDM of Gunion et al., with the Higgs potential
$$\begin{aligned} V(\varphi_1, \varphi_2) &= \lambda_1 \left(\varphi_1^\dagger \varphi_1 - v_1^2\right)^2 + \lambda_2 \left(\varphi_2^\dagger \varphi_2 - v_2^2\right)^2 \\ &+ \lambda_3 \left(\varphi_1^\dagger \varphi_1 - v_1^2 + \varphi_2^\dagger \varphi_2 - v_2^2\right)^2 \\ &+ \lambda_4 \left((\varphi_1^\dagger \varphi_1)(\varphi_2^\dagger \varphi_2) - (\varphi_1^\dagger \varphi_2)(\varphi_2^\dagger \varphi_1)\right) \\ &+ \lambda_5 \left(\mathrm{Re}(\varphi_1^\dagger \varphi_2) - v_1 v_2 \cos\xi\right)^2 \\ &+ \lambda_6 \left(\mathrm{Im}(\varphi_1^\dagger \varphi_2) - v_1 v_2 \sin\xi\right)^2 \\ &+ \lambda_7 \left(\mathrm{Re}(\varphi_1^\dagger \varphi_2) - v_1 v_2 \cos\xi\right)\left(\mathrm{Im}(\varphi_1^\dagger \varphi_2) - v_1 v_2 \sin\xi\right), \end{aligned}$$

Stability conditions

Lagrange multipliers
$u_1 = \frac{1}{4}(2\lambda_1 - \lambda_4), u_2 = \frac{1}{4}(2\lambda_2 - \lambda_4),$
$u_3 = 0, \mu_4 = \frac{1}{4}(\kappa - \lambda_4),$
$\mu_5 = \frac{1}{8}\left(-2\lambda_4 + \lambda_5 + \lambda_6 + \sqrt{(\lambda_5 - \lambda_6)^2 + \lambda_7^2}\right),$
where $\kappa = \frac{1}{2}\left(\lambda_5 + \lambda_6 - \sqrt{(\lambda_5 - \lambda_6)^2 + \lambda_7^2}\right)$.

$$\left.\begin{array}{l} \text{If } u_1 \text{ is the smallest value} \Rightarrow \lambda_1 + \lambda_3 > 0 \\ \text{If } u_2 \text{ is the smallest value} \Rightarrow \lambda_2 + \lambda_3 > 0 \end{array}\right\} \text{necessary}$$

$$\left.\begin{array}{l} \text{If } u_3 = 0 < u_1, u_2, \mu_4, \mu_5, \text{ then} \\ \quad \lambda_4 > -2\lambda_3 - 2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} \\ \text{If } \mu_4 < u_1, u_2, u_3, \mu_5, \text{ then} \\ \quad \kappa > -2\lambda_3 - 2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} \end{array}\right\} \text{sufficient}$$

Conclusions

We can see that the application of the result (1) is essential to get a consistent model and be able to derive sufficient conditions to have a stable scalar potential. It allows us to identify either necessary conditions (for regular solutions) or conditions that may not be necessary, coming from exceptional solutions (including 0). Both conditions generate sufficient inequalities that guarantee the stability of a scalar potential. As an example, we can appreciate it, in the expression (152) of Ref. [1], where $u_2 < u_1, u_3$, so for stability conditions, only u_2 is considered. In this sense, it may happen that some Lagrange multipliers, although not being the smallest values, must be taken into account for stability conditions. You can appreciate it from Gunion's potential (Eq. (79) of Ref. [1]), since if μ_4 were not a valid stationary point, we would have had to analyze μ_5. In that way, we can reduce the number of sufficient conditions arising from exceptional solutions (including 0) provided that $J'(\boldsymbol{k}_j) < 0$ (or $J'(0) \leq 0$).

References

[1] Maniatis M, von Manteuffel A, Nachtmann O and Nagel F 2006 Eur. Phys. J. C48 805–823 (Preprint hep-ph/0605184)

[2] Nagel F 2004 New aspects of gauge-boson couplings and the Higgs sector Ph.D. thesis Heidelberg U. URL http://www.ub.uni-heidelberg.de/archiv/4803