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Five Non-Fritzsch Texture Zeros for Quarks Mass Matrices in the Standard Model

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Abstract

In the Standard Model, we obtain a non-Fritzsch like configuration with five texture zeros for the quark mass matrices. This matrix generates the quark masses, the inner angles of the CKM unitary triangle, and the CP-violating phase in the quark sector. This work can be applied to the PMNS matrix in the lepton sector, by assuming Dirac masses for the neutrinos, where non-trivial predictions for the neutrino masses and mixing angles are expected.

1 Introduction

In models such as the Standard Model (SM) or its extensions, where the right-handed fields are singlets under $SU(2)$, it is always possible to choose a suitable basis for the right-handed quarks, so that the resultant quark mass matrices become hermitian [1, 2, 3, 4]. Consequently, without loss of generality, the mass matrices for the up- or down-type quarks can be written as: $M_u^\dagger = M_u$ and $M_d^\dagger = M_d$. Another consequence for models like the SM where there is a freedom to make independent unitary transformations for left- and right-handed quarks, while keeping the gauge currents invariants, is the existence of infinite representations for the mass matrices, all of them unitarily equivalent. In this analysis the same "weak-basis" (WB) transformation must be applied to M_u and M_d in order to keep the charged current invariant [5, 2], i.e., : $M_u \to M_u' = U^\dagger M_u U$, $M_d \to M_d' = U^\dagger M_d U$, where U is an arbitrary unitary matrix. To ensure that the charged currents are invariant, it is enough to keep the Cabibbo-Kobayashi-Maskawa CKM matrix invariant. Note that the observed parameters for the CKM mixing matrix are fitted at the electroweak scale $\mu = m_Z$, so, it is necessary to use the quark masses (in MeV units) at the same scale [6, 7]:

$$m_u = 1.38^{+0.42}_{-0.41}\,,\ m_c = 638^{+43}_{-84},\ m_t = 172100 \pm 1200\,, m_d = 2.82 \pm 0.48\,,\ m_s = 57^{+18}_{-12}\,,\ m_b = 2860^{+160}_{-60}. \quad (1)$$

The CKM mixing matrix [6] is a 3×3 unitary matrix, which can be parametrized by three mixing angles and the CP-violating phase. Usually it has the following standard choice

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix}, \quad (2)$$

where $s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$, and the angles are chosen to lie in the first quadrant, so $s_{ij}, c_{ij} \geq 0$. And δ is the phase responsible for all CP-violating phenomena in flavor-changing processes in the SM. The Wolfenstein parametrization exhibits the experimental hierarchy $s_{13} \ll s_{23} \ll s_{12} \ll 1$. The fit results for the values of all nine CKM elements are: $s_{12} = 0.22537 \pm 0.00061$, $s_{23} = 0.0413 \pm 0.0012$, $s_{13}e^{i\delta} = (0.00355 \pm 0.00016)e^{i(1.250\pm0.056)}$.

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2 Five numeric texture zeros

By performing WB transformations on the quark mass matrices, it is always possible to obtain one of the following arrangements [8]:

$$M_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix}, \ M_d = V D_d V^\dagger, \ \text{or}, \ M_u = V^\dagger D_u V, \ M_d = D_d = \begin{pmatrix} \lambda_{1d} & 0 & 0 \\ 0 & \lambda_{2d} & 0 \\ 0 & 0 & \lambda_{3d} \end{pmatrix}. \quad (3)$$

Therefore, because of their simplicity, it is convenient to consider them as the initial representations for the quark mass matrices [5, 9, 3, 8] —where V is the CKM mixing matrix, and the eigenvalues $|\lambda_{iq}|$ $(i = 1, 2, 3)$ are the up- $(q = u)$ and down- $(q = d)$ quark masses— and

$$|\lambda_{1u}| = m_u, |\lambda_{2u}| = m_c, |\lambda_{3u}| = m_t, |\lambda_{1d}| = m_d, |\lambda_{2d}| = m_s, |\lambda_{3d}| = m_b, \ \text{where} \ |\lambda_{1q}| \ll |\lambda_{2q}| \ll |\lambda_{3q}|. \quad (4)$$

The properties of the WB transformations allow us to use the bases (3) as the initial matrices to generate any physical structure in the quark mass matrix sector. If there are texture zeros, this transformation can find them. Since some texture zeros are in the diagonal elements of the hermitian mass matrices, it implies that at least one and at most two of their eigenvalues are negative [5]. Also, in the case of two negative eigenvalues, these mass matrices can be reduced to having one by adding a negative sign in the bases (3), as follows: $M_u = -(-M_u)$ or $M_d = -(-M_d)$, such that WB transformations for terms in parentheses can be implemented. Therefore, without losing generality, the texture zeros in the models can be obtained by assuming that each quark mass matrix, M_u and M_d, contains precisely one single negative eigenvalue [3], i.e.,

$$\lambda_{iq} \ \text{is negative for one value of} \ i \ \text{and positive for the others.} \quad (5)$$

Each *realistic* quark mass matrix can contain, at most, three texture zeros. Also, there are only two possible patterns according to the distribution of the three zeros in the elements of the mass matrix. In the first case, the mass matrix has a single zero in the diagonal elements, while in the other case, it has only two zeros in the diagonal entries. The two respective basic patterns are as follows:

$$M_{1q} = \begin{pmatrix} 0 & |\xi_q| & 0 \\ |\xi_q| & \gamma_q & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}, \quad M_{2q} = \begin{pmatrix} 0 & |\xi_q| & 0 \\ |\xi_q| & 0 & |\beta_q| \\ 0 & |\beta_q| & \alpha_q \end{pmatrix}, \quad (6)$$

where ($q = u$ or d), and we can observe that by making WB transformations of the form $p_i M_{1,2q} p_i^T$ and considering all permutation matrices p_i $\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}, \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}, \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}, \begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}, \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}$, we get as many viable cases as possible for each pattern considered. All viable three-zero textures for quark mass matrices are summarized here. These patterns are general, and including phases is not necessary, as they can be absorbed by the other mass matrix (u or d) through a WB transformation. Let us start with the standard representation of the pattern of two zeros in the diagonal entries, M_{2q}, expression (6), which coincides with the matrix (13) by making $\gamma_q = 0$. Its diagonalization matrix, U_{2q}, satisfies Eq. (14), from which the results (15) are derived; so, we have the following:

$$\alpha_q = \lambda_{1q} + \lambda_{2q} + \lambda_{3q}, \ |\xi_q| = \sqrt{\frac{-\lambda_{1q}\lambda_{2q}\lambda_{3q}}{\alpha_q}}, \ |\beta_q| = \sqrt{-\frac{(\lambda_{1q} + \lambda_{2q})(\lambda_{1q} + \lambda_{3q})(\lambda_{2q} + \lambda_{3q})}{\alpha_q}}. \quad (7)$$

The result (7) for $|\xi_q|$ must be a real number, and because only an eigenvalue λ_{iq} is assumed negative, Eq. (5), we have that $\alpha_q > 0$, where, together with (7) for β_q, and hierarchy (4), only one possibility is allowed:

$$\lambda_{1q}, \lambda_{3q} > 0 \quad \text{and} \quad \lambda_{2q} < 0, \quad \text{with} \quad \alpha_q > 0. \quad (8)$$

According to (16), the matrix that diagonalizes M_{2q} is

$$U_{2q} = \begin{pmatrix} e^{ix}\sqrt{\frac{\lambda_{2q}\lambda_{3q}(\alpha_q-\lambda_{1q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & -e^{iy}\sqrt{\frac{\lambda_{1q}\lambda_{3q}(\lambda_{2q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{1q}\lambda_{2q}(\alpha_q-\lambda_{3q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ e^{ix}\sqrt{\frac{\lambda_{1q}(\lambda_{1q}-\alpha_q)}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & e^{iy}\sqrt{\frac{\lambda_{2q}(\alpha_q-\lambda_{2q})}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\lambda_{3q}-\alpha_q)}{(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ -e^{ix}\sqrt{\frac{\lambda_{1q}(\alpha_q-\lambda_{2q})(\alpha_q-\lambda_{3q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & -e^{iy}\sqrt{\frac{\lambda_{2q}(\alpha_q-\lambda_{1q})(\lambda_{3q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\alpha_q-\lambda_{1q})(\alpha_q-\lambda_{2q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \end{pmatrix}, \tag{9}$$

where α_q is in Eq. (7). Performing a WB transformation on the second base of (3), using, in this case, the unitary matrix given in (9) with $q = d$ (i.e., U_{2d}), we have

$$M'_d = U_{2d}(D_d)U^\dagger_{2d} = \begin{pmatrix} 0 & |\xi_d| & 0 \\ |\xi_d| & 0 & |\beta_d| \\ 0 & |\beta_d| & \alpha_d \end{pmatrix}, \quad M'_u = U_{2d}\,(V^\dagger D_u V)\,U^\dagger_{2d}\,, \tag{10}$$

where the equation (14) was taken into account. According to (8), in this case λ_{1d}, $\lambda_{3d} > 0$, $\lambda_{2d} < 0$, and $\alpha_d = \lambda_{1d} + \lambda_{2d} + \lambda_{3d} > 0$. The calculations are simplified if we define the following variables for the phases introduced in (9):

$$e^{ix} = x_1 + ix_2, \quad e^{iy} = y_1 + iy_2, \tag{11}$$

where $x_1^2 + x_2^2 = 1$ and $y_1^2 + y_2^2 = 1$, and it is satisfied that $|x_1|, |x_2|, |y_1|, |y_2| \le 1$. With these definitions, and using experimental data (1) and (2), the elements of the matrix M'_u, in (10) become *surfaces* for the points (x_1, x_2, y_1, y_2) in $\mathbb{R}^4$ for each case considered: $\lambda_{1u} = -m_u$ or $\lambda_{2u} = -m_c$ or $\lambda_{3u} = -m_t$. Taking (11) into account, the analysis of these surfaces shows that only elements (1,2) and (1,3) of M'_u can give solutions equal to zero (texture zeros). Let's take the case $\lambda_{1u} = -m_u$ as an example; we obtain the best results considering the following masses for quarks (in MeV units): $m_u = 1.7160$, $m_d = 2.9042$, $m_s = 65.0$, $m_c = 567.0$, $m_b = 2860.0$, $m_t = 172100$, which are close to the central values and are within the range allowed by (1). The solutions are: $x_1 = 0.68499$, $y_1 = -0.50043$, $x_2 = 0.72855$, $y_2 = -0.86578$, with the component $M'_u(1,1) = 0$. The corresponding numerical matrices obtained for the quark masses with five texture zeros are the following [1]

$$M'_u = \begin{pmatrix} 0 & 0 & -79.323 + 154.72i \\ 0 & 5539.2 & 28126 + 6112.8i \\ -79.323 - 154.72i & 28126 - 6112.8i & 167130 \end{pmatrix} \text{MeV}, \tag{12a}$$

$$M'_d = \begin{pmatrix} 0 & 13.891 & 0 \\ 13.891 & 0 & 421.41 \\ 0 & 421.41 & 2797.9 \end{pmatrix} \text{MeV}, \tag{12b}$$

and their diagonalization matrices are respectively

$$U'_u = \begin{pmatrix} 0.67626 + 0.73481i & -0.050244 + 0.013892i & -0.00044950 + 0.00088826i \\ 0.027505 - 0.043184i & -0.49704 - 0.84931i & 0.16658 + 0.035285i \\ -0.0034086 + 0.0092482i & 0.11507 + 0.12514i & 0.98538 - 0.00519999i \end{pmatrix},$$

$$U'_d = \begin{pmatrix} 0.67018 + 0.71279i & 0.10352 + 0.17909i & 0.00070804 \\ 0.14011 + 0.14902i & -0.48439 - 0.83802i & 0.14578 \\ -0.021126 - 0.022469i & 0.071301 + 0.12335i & 0.98932 \end{pmatrix},$$

which gives the correct CKM mixing matrix with a precision level of 1σ [10, 11, 12], $V = U'^\dagger_u U'_d$, including the phase responsible for the CP violation. The case $\lambda_{2u} = -m_c$ has already been done in article [3]. We also note that the first diagonal base (3) and the pattern with a zero in the diagonal (6) do not give additional consistent solutions with five texture zeros.

3 Conclusions

We have made a complete study of the texture zeros in the quark sector of the SM, starting from general quark mass matrices, based on the WB transformation property [3], to generate any possible mass matrix configuration. This result allowed us to use specific basis, (3), to reproduce as many texture zeros as possible. In this way, we discovered a numerical texture pattern consisting of five zeros, including permutations, whose matrix representation is in (12); the pattern is not Fritzsch type [2] because of the way texture zeros are present. It suggests the construction of a model with nine parameters that involves mass and mixing relations, while, at the same time, gives precise masses for quarks, CKM mixing angles, and the phase responsible for the CP violation at a confidence level. There are no additional representations of five texture zeros apart from the one given in article [3]. By assuming Dirac masses for the neutrinos, next step is to apply our results to the PMNS matrix in the lepton sector where non-trivial results are expected [13, 14].

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A Matrix with two texture zeros

Consider the following structure for the up- $(q = u)$ and down- $(q = d)$ quark mass matrix:

$$M_q = \begin{pmatrix} 0 & |\xi_q| & 0 \\ |\xi_q| & \gamma_q & |\beta_q| \\ 0 & |\beta_q| & \alpha_q \end{pmatrix}. \tag{13}$$

Owing to the hermiticity of M_q, γ_q and α_q are real numbers. The phases of the parameters outside the diagonal can be included later using a WB transformation. By diagonalizing the mass matrix M_q, we have

$$U_q^\dagger M_q U_q = D_q, \tag{14}$$

where the λ_{iq} $(i = 1, 2, 3)$ are defined in (4) and D_q in (3). The parameters γ_q, $|\xi_q|$ and $|\beta_q|$ can be expressed in terms of λ_{iq} and α_q. For this, we apply the invariant matrix functions $\mathrm{tr}M_q$, $\mathrm{tr}M_q^2$ and $\det M_q$, on (14). Results

$$\gamma_q = \lambda_{1q} + \lambda_{2q} + \lambda_{3q} - \alpha_q, \;\; |\xi_q| = \sqrt{\frac{-\lambda_{1q}\lambda_{2q}\lambda_{3q}}{\alpha_q}}, \;\; |\beta_q| = \sqrt{\frac{(\alpha_q - \lambda_{1q})(\alpha_q - \lambda_{2q})(\lambda_{3q} - \alpha_q)}{\alpha_q}}. \tag{15}$$

The expressions (15) are real, so the parameter α_q is confined to one interval. Let's see the different possibilities: If $\lambda_{1q} < 0$, $\lambda_{2q} > 0$ and $\lambda_{3q} > 0$ then $|\lambda_{2q}| < \alpha_q < |\lambda_{3q}|$, if $\lambda_{1q} > 0$, $\lambda_{2q} < 0$ and $\lambda_{3q} > 0$ then $|\lambda_{1q}| < \alpha_q < |\lambda_{3q}|$, if $\lambda_{1q} > 0$, $\lambda_{2q} > 0$ and $\lambda_{3q} < 0$ then $|\lambda_{1q}| < \alpha_q < |\lambda_{2q}|$. In the previous analysis, we take into account the hierarchy (4) and the assumption (5). The exact analytical result for the matrix (U_q) that diagonalizes to M_q in (13) is [2, 3, 15]

$$U_q = \begin{pmatrix} e^{ix}\frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{2q}\lambda_{3q}(\alpha_q-\lambda_{1q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & e^{iy}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}\lambda_{3q}(\lambda_{2q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{1q}\lambda_{2q}(\alpha_q-\lambda_{3q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ -e^{ix}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}(\lambda_{1q}-\alpha_q)}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & e^{iy}\sqrt{\frac{\lambda_{2q}(\alpha_q-\lambda_{2q})}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{3q}(\lambda_{3q}-\alpha_q)}{(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ e^{ix}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}(\alpha_q-\lambda_{2q})(\alpha_q-\lambda_{3q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & -e^{iy}\frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{2q}(\alpha_q-\lambda_{1q})(\lambda_{3q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\alpha_q-\lambda_{1q})(\alpha_q-\lambda_{2q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \end{pmatrix}, \tag{16}$$

where we have added phases to make the CKM matrix compatible with the convention chosen in (2), something that was justified in [8].

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