Non-Fritzsch Like Five-Zero Texture for Quark Mass Matrices in the Standard Model

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Abstract

We will consider a non-Fritzsch like five-zero texture that is completely valid and generates all the physical quantities involved, including the quark masses, the Jarlskog invariant quantity and the inner angles of the Cabibbo-Kobayashi-Maskawa unitarity triangle, and it explains the charge parity violation phenomenon at 1σ confidence level. To achieve this, non-physical phases must be included in the unitary matrices used to diagonalize the quark mass matrices, in order to put the Cabibbo-Kobayashi-Maskawa matrix in standard form. Besides, these phases can be rotated away so they do not have any physical meaning. Thus, the model has a total of nine parameters to reproduce ten physical quantities, which implies physical relationships between quark masses and/or mixings.

Introduction

- Models like the Standard Model (SM) or its extensions, where the right-handed fields are $SU(2)$ singlets, it is always possible to choose a suitable basis for the right-handed quarks by using the unitary matrix coming from the *polar decomposition theorem* of matrix algebra, such that the resultant up- and down-type quark mass matrices become hermitian.

$$M_u^\dagger = M_u, \quad \text{and} \quad M_d^\dagger = M_d.$$

- In the SM, the left- and right-handed quarks can be transformed unitarily, such that the gauge currents remains invariants, and as a result quark mass matrices are transformed into new equivalent ones. This process consists basically in a common unitary transformation applied on M_u and M_d known as a "Weak Basis" (WB) Transformation [1], as follows

$$M_u \to M_u' = U^\dagger M_u U, \quad M_d \to M_d' = U^\dagger M_d U,$$

where U is an arbitrary unitary matrix which preserves hermiticity of the quark mass matrices.

- Making a WB transformation, any physical viable quark mass matrices can be derived from specific quark mass matrices.

Quark masses and CKM

The quark masses and observed CKM matrix parameters $|V_{ij}|$ are given at $\mu = m_Z$ [2]:

$$m_u = 1.38^{+0.42}_{-0.41}\,, \ m_c = 638^{+43}_{-84}, \ m_t = 172100 \pm 1200\,,$$
$$m_d = 2.82 \pm 0.48\,, \ m_s = 57^{+18}_{-12}\,, \ m_b = 2860^{+160}_{-60}\,.$$

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix},$$

and the Jarlskog invariant is

$$J = \left(3.06^{+0.21}_{-0.20}\right) \times 10^{-5}.$$

1. The initial quark mass matrices

The u-diagonal representation [3, 4]:

$$M_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix},$$
$$M_d = V D_d V^\dagger, \quad \text{where} \quad V = U_u^\dagger U_d.$$

The d-diagonal representation.

Numerical Five-Zero Textures

2. One- and two-zero diagonal pattern

Permutation matrices	two-zero diagonal pattern $(p_i\, M_q\, p_i^T)$	one-zero diagonal pattern $(p_i\, M_q\, p_i^T)$
$p_1 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & 0 \\ \|\xi_q\| & 0 & \|\beta_q\| \\ 0 & \|\beta_q\| & \alpha_q \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & 0 \\ \|\xi_q\| & \gamma_q & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}$
$p_2 = \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \|\xi_q\| \\ 0 & \alpha_q & \|\beta_q\| \\ \|\xi_q\| & \|\beta_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \|\xi_q\| \\ 0 & \alpha_q & 0 \\ \|\xi_q\| & 0 & \gamma_q \end{pmatrix}$
$p_3 = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} \alpha_q & \|\beta_q\| & 0 \\ \|\beta_q\| & 0 & \|\xi_q\| \\ 0 & \|\xi_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & 0 \\ 0 & \gamma_q & \|\xi_q\| \\ 0 & \|\xi_q\| & 0 \end{pmatrix}$
$p_4 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & \|\beta_q\| \\ \|\xi_q\| & 0 & 0 \\ \|\beta_q\| & 0 & \alpha_q \end{pmatrix}$	$\begin{pmatrix} \|\gamma_q\| & \|\xi_q\| & 0 \\ \|\xi_q\| & 0 & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}$
$p_5 = \begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & \|\beta_q\| \\ 0 & 0 & \|\xi_q\| \\ \|\beta_q\| & \|\xi_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & 0 \\ 0 & 0 & \|\xi_q\| \\ 0 & \|\xi_q\| & \gamma_q \end{pmatrix}$
$p_6 = \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} 0 & \|\beta_q\| & \|\xi_q\| \\ \|\beta_q\| & \alpha_q & 0 \\ \|\xi_q\| & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \gamma_q & 0 & \|\xi_q\| \\ 0 & \alpha_q & 0 \\ \|\xi_q\| & 0 & 0 \end{pmatrix}$

3. Numerical quark masses (In MeV units)

$$M_u' = \begin{pmatrix} 0 & 0 & -79.32 + 154.72i \\ 0 & 5539.2 & 28125.9 + 6112.8i \\ -79.323 - 154.72i & 28125.9 - 6112.8i & 167126.0 \end{pmatrix},$$

$$M_d' = \begin{pmatrix} 0 & 13.891097 & 0 \\ 13.891097 & 0 & 421.41405 \\ 0 & 421.41405 & 2797.9042 \end{pmatrix}.$$

Analytical Five-Zero Textures and the CKM Mixing Matrix

The five-zero texture matrix derived above has the following standard form:

$$M_u = P^\dagger \begin{pmatrix} 0 & 0 & |\xi_u| \\ 0 & \alpha_u & |\beta_u| \\ |\xi_u| & |\beta_u| & \gamma_u \end{pmatrix} P, \quad M_d = \begin{pmatrix} 0 & |\xi_d| & 0 \\ |\xi_d| & 0 & |\beta_d| \\ 0 & |\beta_d| & \alpha_d \end{pmatrix},$$

where $P = \text{diag}(e^{-i\phi_{\xi_u}}, e^{-i\phi_{\beta_u}}, 1)$ (with $\phi_{\beta_u} \equiv \arg(\beta_u)$ and $\phi_{\xi_u} \equiv \arg(\xi_u)$). We have nine free parameters, to reproduce ten physical quantities: 6 quark masses and 3 mixing angles and 1 phase from the CKM matrix, which implies physical relationships between the quark masses and/or mixings.

4. The Mixings

$$|V_{ud}| \approx |V_{cs}| \approx |V_{tb}| \approx 1,$$

$$|V_{us}| \approx \left|\sqrt{\frac{\alpha_u - m_c}{\alpha_u}}\sqrt{\frac{m_u}{m_c}} - e^{i(\phi_{\beta_u} - \phi_{\xi_u})}\sqrt{\frac{m_d}{m_s}}\right|,$$

$$|V_{cd}| \approx \left|\sqrt{\frac{\alpha_u - m_c}{\alpha_u}}\sqrt{\frac{m_u}{m_c}} - e^{i(\phi_{\xi_u} - \phi_{\beta_u})}\sqrt{\frac{m_d}{m_s}}\right|,$$

$$|V_{cb}| \approx \left|\sqrt{\frac{m_s}{m_b}} - e^{i\phi_{\beta_u}}\sqrt{\frac{\alpha_u - m_c}{m_t}}\right|,$$

$$|V_{ts}| \approx \left|\sqrt{\frac{m_s}{m_b}} - e^{-i\phi_{\beta_u}}\sqrt{\frac{\alpha_u - m_c}{m_t}}\right|,$$

$$\frac{|V_{ub}|}{|V_{cb}|} \approx \sqrt{\frac{m_u}{m_c}}\left|\frac{\sqrt{\frac{\alpha_u}{m_t}} - e^{-i\phi_{\beta_u}}\sqrt{\frac{\alpha_u - m_c}{\alpha_u}}\sqrt{\frac{m_s}{m_b}}}{\sqrt{\frac{\alpha_u - m_c}{m_t}} - e^{-i\phi_{\beta_u}}\sqrt{\frac{m_s}{m_b}}}\right|,$$

$$\frac{|V_{td}|}{|V_{ts}|} \approx \sqrt{\frac{m_d}{m_s}},$$

where $\alpha_u \ll m_t$ be assumed. We shall consider $\alpha_u \gtrsim m_c$ in order to fit experimental data, which gives $(\phi_{\beta_u} - \phi_{\xi_u}) \sim -\pi/2$, which it is an important contribution term for CP-violation.

Conclusions

The main conclusions of this work are:

- We have found only two different numerical five-zero texture patterns.
- We have nine free parameters to reproduce ten physical quantities: 6 quark masses, 3 mixing angles and 1 phase from the CKM matrix, which implies physical relationships between the quark masses and/or mixings.
- The GST relation is maintained, and an important contribution for CP violation is still exhibited in the context of the model.

References

[1] G.C. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe, Phys.Lett.B477, 2000 [hep-ph/9911418].

[2] K. Nakamura et al. (Particle Data Group), JP G 37, 075021 (2010) and 2011 partial update for the 2012 edition (URL: http://pdg.lbl.gov).

[3] Yithsbey Giraldo, Phys.Rev.D86,093021(2012).

[4] Yithsbey Giraldo, Phys.Rev.D91,038302(2015).