

San Juan de Pasto 5 de diciembre de 2022

Señores

**COMITÉ CURRICULAR**

Departamento de Física

Universidad de Nariño

Cordial saludo

Por medio de la presente quiero informar que del 28 de noviembre al 2 de diciembre del presente año participe en el **seventh Colombian Meeting on High Energy Physics (7th ComHEP)** que se realizó en la ciudad de Villa de Leyva. En este evento presenté el trabajo titulado: "*Chern-Simons on the Null Plane*".

Agradezco de antemano su atención

Atentamente



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# Chern-Simons on the Null Plane

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# Abelian Chern-Simons Theory

The abelian Chern-Simons theory is described by the following Lagrangian density:

$$\mathcal{L}_{CS} = \frac{\kappa}{4} \varepsilon^{\mu\nu\alpha} F_{\mu\nu} A_\alpha, \quad (1)$$

where  $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ . The Lagrangian above is invariant by the following transformations,

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \Lambda(x) \quad , \quad \mathcal{L}_{CS} \rightarrow \mathcal{L}_{CS} + \partial_\alpha \left( \frac{\kappa}{4e} \varepsilon^{\mu\nu\alpha} F_{\mu\nu} \Omega \right). \quad (2)$$

The field equations are:

$$\frac{\kappa}{2} \varepsilon^{\rho\sigma\alpha} F_{\sigma\alpha} = 0, \quad (3)$$

The null plane time  $x^+$  and longitudinal coordinate  $x^-$  are defined, respectively, as

$$x^+ \equiv \frac{x^0 + x^3}{\sqrt{2}} \quad x^- \equiv \frac{x^0 - x^3}{\sqrt{2}}, \quad (4)$$

with the transverse coordinate kept unchanged. In the null-plane coordinates the metric is given by,

$$\eta_{\mu\nu} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (5)$$

Explicitly,  $x^+ = x_-$  and the derivatives with respect to  $x^+$  and  $x^-$  are defined as:  $\partial_+ \equiv \frac{\partial}{\partial x^+}$  and  $\partial_- \equiv \frac{\partial}{\partial x^-}$ , with  $\partial^+ = \partial_-$ . The Levi-Civita tensor has the following components,

$$\varepsilon^{+-1} = \varepsilon_{+-1} = 1. \quad (6)$$

## Dirac Formulation

From (1) it is easy to write the first-order Lagrangian by introducing the momentum  $\pi^\mu$  with respect to the fields  $A_\mu$ ,

$$\pi^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_+ A_\mu)} = \frac{\kappa}{2} \varepsilon^{+\rho\alpha} A_\alpha. \quad (7)$$

The theory has the following set of primary constraints:

$$\Omega_1 \equiv \pi^+ \approx 0 \quad , \quad \Omega_{2,a} \equiv \pi^a - \frac{\kappa}{2} \varepsilon_{ab} A_b \approx 0, \quad a = -, 1, \quad (8)$$

where we have defined the following antisymmetric tensor,

$$\varepsilon_{-1} = -\varepsilon_{1-} = 1. \quad (9)$$

The canonical Hamiltonian density is defined by,

$$\mathcal{H}_C = \pi^\mu \partial_+ A_\mu - \mathcal{L} = -\kappa A_+ \varepsilon_{ab} \partial_a A_b. \quad (10)$$

The canonical Hamiltonian take the form,

$$H_C = -\kappa \int d^2x A_+ \varepsilon_{ab} \partial_a A_b, \quad (11)$$

where  $d^2x \equiv dx^- dx^1$ . Thus, the dynamics is determined by the primary Hamiltonian defined by,

$$H_P = H_C + \int d^2y \left[ u^1(y) \Omega_1(y) + u^{2,a}(y) \Omega_{2,a}(y) \right]. \quad (12)$$

The fundamental Poisson brackets of the theory are,

$$\left\{ A_\mu(x), \pi^\nu(y) \right\} = \delta_\mu^\nu \delta^2(x - y), \quad (13)$$

The consistence conditions on the constraints are,

$$\dot{\Omega}_1(x) = \left\{ \Omega_1(x), H_P \right\} = \kappa \varepsilon_{ab} \partial_a A_b \approx 0,$$

Then, a secondary constraint arise defined by,

$$\Omega_4 \equiv \varepsilon_{ab} \partial_a A_b \approx 0. \quad (14)$$

The consistence of  $\Omega_4$  determine,

$$\dot{\Omega}_4(x) = \left\{ \Omega_4(x), H_P \right\} = \varepsilon_{ab} \partial_a u^{2,b} \approx 0, \quad (15)$$

thus, no more constraints are generated from  $\Omega_4$ .

The consistence condition of  $\Omega_{2,a}$  determine,

$$\dot{\Omega}_{2,a}(x) = \left\{ \Omega_{2,a}(x), H_P \right\} = u^{2,a} - \varepsilon_{ab} \partial_b A_+ \approx 0. \quad (16)$$

This relations is consistente with (18).

The theory is characterized by the following set of constraints,

$$\begin{aligned}\Omega_1 &\equiv \pi^+ \approx 0, \\ \Omega_{2,a} &\equiv \pi^a - \frac{\kappa}{2} \varepsilon_{ab} A_b \approx 0, \\ \Omega_4 &\equiv \varepsilon_{ab} \partial_a A_b \approx 0.\end{aligned}\tag{17}$$

The theory has the following set of first class constraints.

$$\Theta_1 \equiv \pi^+ \approx 0 \quad , \quad \Theta_2 \equiv \partial_a \pi^a + \frac{\kappa}{2} \varepsilon_{ab} \partial_a A_b \approx 0.\tag{18}$$

and a set of second class constraints,

$$\Phi_a \equiv \pi^a - \frac{\kappa}{2} \varepsilon_{ab} A_b \approx 0, \quad a = -, 1,\tag{19}$$

where,

$$\left\{ \Phi_a(x), \Phi_b(y) \right\} = -\kappa \varepsilon_{ab} \delta^2(x - y).\tag{20}$$

We are going to eliminate  $\Phi_a$  using the following Dirac Brackets between two dynamical variables,

$$\left\{ A_m(x), B_n(y) \right\}_{D1} = \left\{ A_m(x), B_n(y) \right\} - \int d^2u d^2v \left\{ A_m(x), \Phi_a(u) \right\} C_{ab}^{-1}(u, v) \left\{ \Phi_b(v), B_n(y) \right\}, \quad (21)$$

where  $C_{ab}^{-1}$  is the inverse of the second class constraint matrix defined by,

$$C_{ab}(u, v) \equiv \left\{ \Phi_a(x), \Phi_b(y) \right\} = -\kappa \varepsilon_{ab} \delta^2(x - y), \quad (22)$$

where  $C_{ab}^{-1}$  is determined by,

$$\int d^2z C_{ac}(x, z) C_{cb}^{-1}(z, y) = \delta_{ab} \delta^2(x - y). \quad (23)$$

It is possible to show,

$$C_{ab}^{-1}(x, y) = \frac{1}{\kappa} \varepsilon_{ab} \delta^2(x - y), \quad (24)$$

Under this DB, the constraint  $\Phi_a$  is a strong identity, thus, we can assume,

$$\pi^a = \frac{\kappa}{2} \varepsilon_{ab} A_b. \quad (25)$$

then  $A_b$  are the freedom degree. Thus, it is possible determine:

$$\left\{ A_a(x), A_b(y) \right\}_{D1} = \frac{1}{\kappa} \varepsilon_{ab} \delta^2(x - y). \quad (26)$$

The first class constraints can be written,

$$\Theta_1 = \pi^+ \approx 0 \quad , \quad \Theta_2 = \kappa \varepsilon_{ab} \partial_a A_b. \quad (27)$$

To eliminate the first class constraints it is necessary to introduce two gauge condition,

$$\Theta_3 = A_+ \approx 0 \quad , \quad \Theta_4 = A_- \approx 0, \quad (28)$$

and define a matrix with the following elements,

$$D_{ab}(x, y) \equiv \left\{ \Theta_a(x), \Theta_b(y) \right\}_{D1}. \quad a = 1, 2, 3, 4 \quad (29)$$

Now, we can define the final set of DB of the theory for two dynamical variables,

$$\left\{ A_m(x), B_n(y) \right\}_D = \left\{ A_m(x), B_n(y) \right\}_{D1} - \int d^2 u d^2 v \quad (30)$$

$$\left\{ A_m(x), \Theta_a(u) \right\}_{D1} D_{ab}^{-1}(u, v) \left\{ \Theta_b(v), B_n(y) \right\}_{D1}.$$

where

$$D^{-1}(z, y) \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{\partial_-^x} \\ -1 & 0 & 0 & 0 \\ 0 & -\frac{1}{\partial_-^x} & 0 & 0 \end{pmatrix} \delta^2(x - y). \quad (31)$$

It is possible to show

$$\left\{ A_1(x), A_1(y) \right\}_D = \frac{1}{\kappa} \frac{\partial_1^x}{\partial_-^x} \delta^2(x - y). \quad (32)$$

# Conclusions

- We have calculated the complete set of constraints of the theory.
- The constraints have been classified and it was shown that one of the first class constraints result of a combination of constraints.
- The inverse of the second class constraint matrices have been uniquely derived.
- The degrees of freedom have been determined and their corresponding DB calculated.