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Modeling a quasi-passive bipedal walker: when and where to kick

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Modeling a quasi-passive bipedal walker: when and where to kick

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Abstract. Walking is one of the most complex tasks a human being can perform. It takes several years to develop the patterns that enable a person to walk and run. Although it is now possible to replicate these patterns in machines such as robots, actuators are needed to control the pace and compensate for the energy loss of walking. However, in small robots (in the scale of centimeters) the size and weight of the actuators could be a limiting factor for their use. To solve this problem, quasi-passive walkers have been recently proposed. These walkers consist of several coupled rigid bodies, usually forming a biped, that move on flat surfaces in the presence of a gravitational field. It is well known that these bipeds can descend on small slope planes without the need for actuators. Walking on horizontal planes requires the presence of actuators to initiate and maintain the pace, however, these actuators could be small enough to "kick" the walker at the right point at the right time. In this work, we present a physical-mathematical model of a bipedal walker composed of five rigid bodies (two legs, two feet, and the hip) moving in a horizontal plane. The model focuses on the natural motion of the biped under certain initial conditions with the ultimate goal of determining the best position of the actuators and the actuation time to maintain the gait. This model could be used as a guide for the construction of small laboratory-scale walkers and as a teaching tool in biomedical engineering courses.

1. Introduction

Modeling the walking process involves the study of many active components interacting with a rigid surface in the presence of gravity. To address this problem, simple models have been proposed that, with a minimum of components, can show the main characteristics and requirements of a steady gait. This is the case of bipedal models, structures composed of one hip, two legs, and two feet [1, 2].

Several studies have shown that different versions of a biped walker can exhibit a stable gait on slightly inclined planes (powered by gravity) and under a certain set of initial conditions, without the need for actuators or any kind of active elements (passive walkers) [3, 4]. Recently, bipeds that can walk on horizontal surfaces have been studied, these walkers require the presence of small actuators that stimulate the legs with various types of forces (quasi-passive walkers) [5, 6]. In the quest for smaller bipeds [7] the size and number of the actuators must be reduced. In particular, at the centimeter scale, the actuators must be efficiently located so as to compensate for the losses originating from interaction with the floor [8, 9].

With this in mind and inspired by [9], in this paper, we propose a minimal model of a quasi-passive bipedal walker composed of one hip (massive), two feet (massive), and two legs of constant length and negligible mass. We will first analyze the natural dynamics of the biped seen as a set of coupled rigid bodies in the presence of the gravitational field (inverted double pendulum [10]). We will look for the initial conditions that allow a first stable step. Subsequently, the heel strike with the floor will be analyzed to determine the energy losses and the possibility of a second step. From this, we follow [9] and propose the presence of actuators that slightly modify the length of the legs in a harmonic manner. Although actuators are not considered directly as dynamic elements, the leg length variation is taken into account in the equations of motion for the biped. Under these conditions, we show that a steady gait is possible with, at least, a value of the frequency and amplitude of the length variation.

2. Description of the model and methodology

A schematic of the model can be seen in Figure 1. The biped is modeled as three-point particles joined by two rigid rods. The hip is modeled as a point particle of mass M. Feet are also point particles of mass m. During a stable step, we assume that a foot is in contact with the floor and no slip is allowed; the corresponding leg is the stance leg. The other leg (swinging leg) does not touch the floor during the step. Essentially, the biped is a rigid double pendulum working on the upper side of the support point. The state of the legs is completely determined by the angles ϕ_1 and ϕ_2. At the beginning of the first stable step, the foot of the supporting leg (black color in Figure 1) has just touched the floor. The swing leg (red color in Figure 1) has lost contact with the floor. At this point $\phi_1 = \phi_2$. The foot of the stance leg does not slip on the floor, so the stance leg rotates about the point O, while the swing leg rotates about de hip. In order to get a first stable step $\mid \phi_2 \mid \geqslant \mid \phi_1 \mid$ where the equality holds just for $\phi_{1,2} = 0$. At the end of the first step, the swing leg hits the floor at O' becoming the new stance leg.

To obtain the kinematics of both legs, we first derive the Lagrangian of the biped; two scenarios are considered: first, the length of the legs is constant and no actuators are considered (pure passive biped), second the length of the legs depends on time due to the presence of actuators (quasi-passive biped). From the Lagrangian equations, we derive the equation of motion for both legs in both scenarios and analyze the conditions for a stable gait.

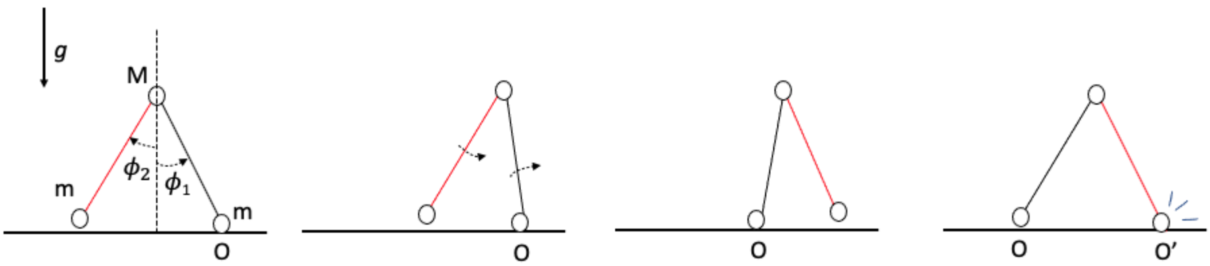

Figure 1. Schematic of the first stable step, the foot of the stance leg (black) does not slip, the swing leg (red) does not touch the floor; during the stable step $\mid \phi_2 \mid > \mid \phi_1 \mid$.

3. Results

The cases for obtaining the equations of motion are described below.

3.1. Case 1, $l = l_0$

To determine the initial conditions leading to a stable step we find the equations of motion of the biped using the Lagrangian formalism; the Lagrangian of the system is Equation (1).

$$L = \frac{1}{2}Ml^2\dot{\phi_1}^2 + \frac{1}{2}ml^2(\dot{\phi_1}^2 + \dot{\phi_2}^2 + 2\dot{\phi_1}\dot{\phi_2}cos(\phi_1 + \phi_2)) - Mglcos(\phi_1) - mgl(cos(\phi_1) - cos(\phi_2)), \quad (1)$$

where l is the length of the legs and g is the magnitude of the gravitational field. From the Lagrange equations, we find the equation of motion for ϕ_1 (Equation (2)).

$$
\begin{aligned}
(M+m)l^2\ddot{\phi_1} = & -ml^2(\ddot{\phi_2}cos(\phi_1+\phi_2) - \dot{\phi_2}(\dot{\phi_1}+\dot{\phi_2})sin(\phi_1+\phi_2)) \\
& - ml^2\dot{\phi_1}\dot{\phi_2}sin(\phi_1+\phi_2) + gl(M+m)sin(\phi_1),
\end{aligned} \tag{2}
$$

and for ϕ_2 (Equation (3)).

$$
\begin{aligned}
ml^2\ddot{\phi_2} = & -ml^2(\ddot{\phi_1}cos(\phi_1+\phi_2) - \dot{\phi_1}(\dot{\phi_1}+\dot{\phi_2})sin(\phi_1+\phi_2)) \\
& - ml^2\dot{\phi_1}\dot{\phi_2}sin(\phi_1+\phi_2) + mglsin(\phi_2),
\end{aligned} \tag{3}
$$

Equation (2) and Equation (3) were solved numerically for a wide set of initial conditions. we choose $M = 0.1$ kg, $m = 1.0$ kg, $l = 1.0$ m and $g = 9.8$ m/s^2. In general, the rigid double pendulum does not reproduce a stable gait for a wide set of initial conditions; for example Figure 2 shows $\phi_1(t)$ and $\phi_2(t)$ with the conditions $\phi_1(0) = 0.1$, $\dot{\phi_1}(0) = -1.5$ s^{-1}, $\phi_2(0) = 0.1$, $\dot{\phi_2}(0) = 0$. Clearly, under these values, there is no stable gait.

Stable gait were detected with the conditions: $\phi_1(0) = 0.2$, $\dot{\phi_1}(0) = -2.165$ s^{-1}, $\phi_2(0) = 0.2$, $\dot{\phi_2}(0) = 0$; the solution can be seen in Figure 3.

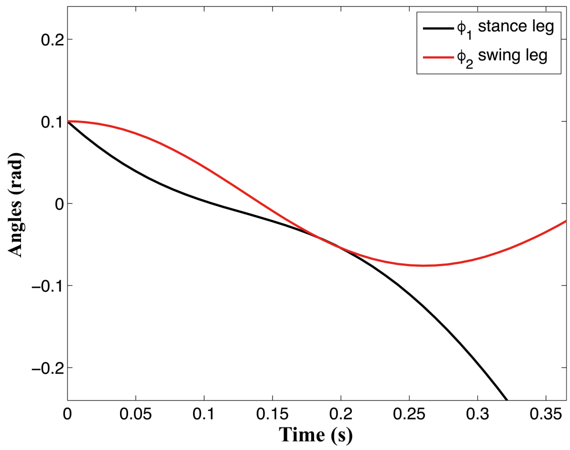

Figure 2. Solutions for ϕ_1 and ϕ_2 for initial conditions that do not generate a stable gait.

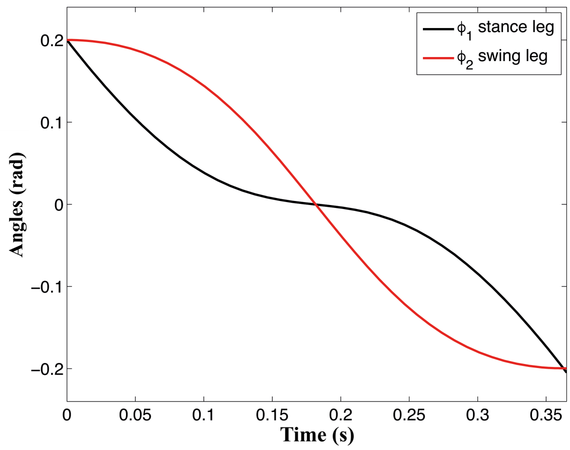

Figure 3. Solutions for ϕ_1 and ϕ_2 during a first stable step

The heel strike of the swing leg produces an inelastic collision. We assume no slipping and no impulsive forces at the former stance leg [11, 12]. The collision is defined by its restitutive coefficient (η), in this case, we choose $\eta = 0.5$. Under these assumptions, we found the initial conditions for the second step. In Figure 4 can be seen the results. Figure 4 shows that in the second step $\phi_1 = \phi_2$ for $\phi_{1,2} \neq 0$, which implies that the biped collapses.

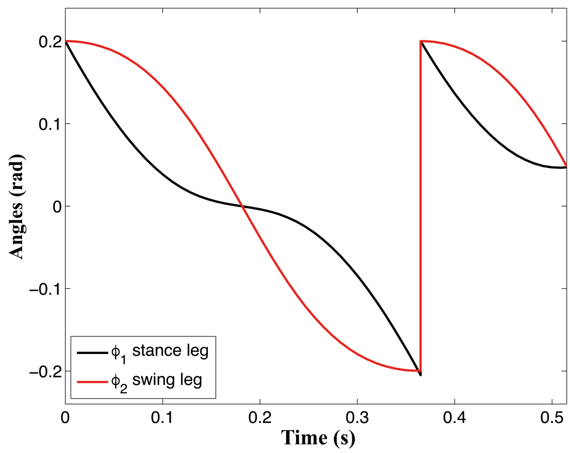

Figure 4. Solutions for ϕ_1 and ϕ_2 for the first (stable) step and the second (non-stable) step.

3.2. Case 2, $l = l(t)$

To regain stability, we propose to generate a positive impulse at the moment of impact. This could be done with various external excitation mechanisms, but a direct and simpler way is to consider an actuator that slightly lengthens and contracts the legs with a given frequency [9]. In this way the length of the legs is no longer constant and should be expressed as $l(t) = l_0 + Asin(\omega t)$, where A is the amplitude of the length oscillations. Now, in the Lagrangian of Equation (1), l depends on time; the new equation of motion for ϕ_1 is Equation (4).

$$\begin{aligned}(M+m)l^2\ddot{\phi_1} =& - ml^2(\ddot{\phi_2}cos(\phi_1+\phi_2) - \dot{\phi_2}(\dot{\phi_1}+\dot{\phi_2})sin(\phi_1+\phi_2))+ \\ & - ml^2\dot{\phi_1}\dot{\phi_2}sin(\phi_1+\phi_2) + gl(M+m)sin(\phi_1)+ \\ & + 2Ml\dot{l}\dot{\phi_1} + 2ml\dot{l}(\dot{\phi_1}+\dot{\phi_2}cos(\phi_1+\phi_2)).\end{aligned} \tag{4}$$

And for ϕ_2 is Equation (5).

$$\begin{aligned}ml^2\ddot{\phi_2} =& - ml^2(\ddot{\phi_1}cos(\phi_1+\phi_2) - \dot{\phi_1}(\dot{\phi_1}+\dot{\phi_2})sin(\phi_1+\phi_2))+ \\ & - ml^2\dot{\phi_1}\dot{\phi_2}sin(\phi_1+\phi_2) + mglsin(\phi_2)+ \\ & + 2ml\dot{l}(\dot{\phi_2}+\dot{\phi_1}cos(\phi_1+\phi_2)),\end{aligned} \tag{5}$$

where $\dot{l} = A\omega cos(\omega t)$. In order to see the effect of the actuators we solve Equation (4) and Equation (5) numerically with the same initial conditions of Figure 2; in this case, we take $A = 0.1$ m and $\omega = 10\ s^{-1}$. Figure 5 shows ϕ_1 and ϕ_2 for this case.

At the moment of the impact, the radial velocity of the former stance leg is $A\omega \neq 0$ which produces an energy input in the biped along with a length extension; all this explains the energy compensation after the heel strike and the generation of a stable gait. Evidently, the chosen values for A and ω are not unique and there is the possibility of other sets of values which could lead to stable gaits and other important behaviors.

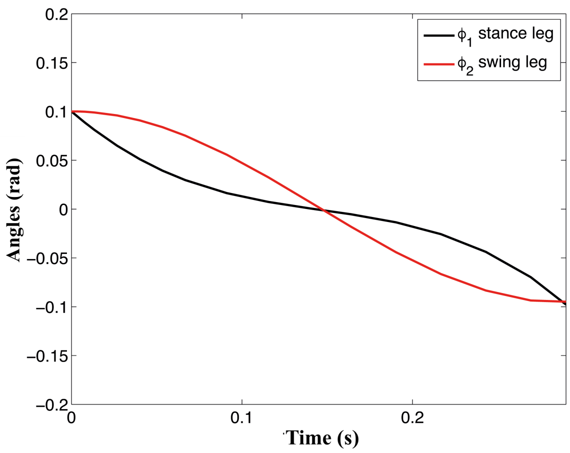

Figure 5. Solutions of Equation (4) and Equation (5) for ϕ_1 and ϕ_2; the initial conditions coincide with those of Figure 2.

4. Perspectives

The results obtained with this simple model and with restricted parameters suggest that it is possible to obtain the stable gait of the biped with the action of actuators that, in some way, compensate for the loss of energy on impact; it is likely that the presence of these actuators will expand the set of initial conditions that lead to a stable gait. Also, other types of excitations could be analyzed, for example, hip rotation mechanisms that affect the contact forces at impact and generate positive impulses or vibrating platforms that restore part of the energy losses at heel strike.

5. Conclusions

We modeled a quasi-passive biped with one hip, two legs, and two feet; from the Lagrangian formulation, we determined and solved the equations of motion for the angular variables of the supporting leg and the swing leg. Most of the initial conditions do not lead to a stable gait, however, we found initial conditions that lead to a stable first step; by modifying the biped so that its leg lengths change slightly harmonically we were able to obtain a stable gait for the conditions where the purely passive biped collapsed.

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